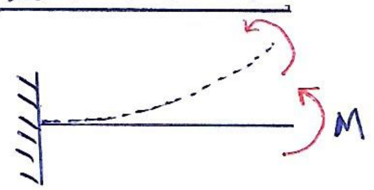
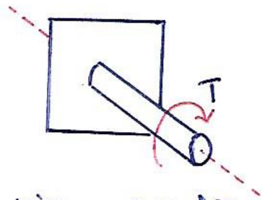
MECHANIC OF MATERIALS

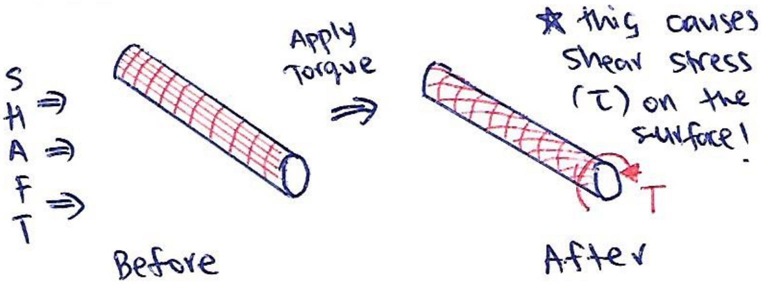
TORSION

- the symbol is T (Torque)
- unit is $\boxed{N.m}$

the difference with bending moment:

BENDING MOMENT	TORSION
	
- Bending Moment (B.M) creates $\boxed{\text{BENDING}}$	- Torsion creates $\boxed{\text{TWISTING}}$
- B.M creates deflection angle (θ)	- Torsion creates angle of twist (ϕ)
- angle unit is rad	- angle unit is rad
- Moment force is along the axis	- Torsion force is perpendicular (90°) to the axis
- Unit is N.m	- Unit is N.m

TORSION FOR CIRCULAR SECTION



Basic rule of a linear torque:

1) longitudinal axis remains straight.
2) The shaft does not increase or decrease in length.
3) Radial lines remain straight
4) Area of cross section rotate about the axis.

- Since torque will produce shear stress on the surface, the equation will be

$$\boxed{\tau = \frac{T r}{J}}$$

τ : Shear stress τ (N/mm^2)
T : Torque (N.m)
r : Radius (m)
J : Polar moment of inertia (m^4)

- The Polar Moment of Inertia (J) value is different from the ~~Ma~~ Secondary Moment of Inertia.

Area shape	Secondary Moment of inertia (I)	Polar moment of inertia (J)
circle, d	$\frac{\pi d^4}{64}$	$\frac{\pi d^4}{32}$
hollow circle, d_1, d_0	$\frac{\pi(d_0^4 - d_1^4)}{64}$	$\frac{\pi(d_0^4 - d_1^4)}{32}$
SQUARE, b, h	$I_x = \frac{bh^3}{12}$ $I_y = \frac{hb^3}{12}$	$\frac{1}{12} bh(b^2 + h^2)$
TRIANGLE, b, h	$I_x = \frac{bh^3}{36}$	$\frac{b^3 h}{36}$

Angle of Twist (ϕ)

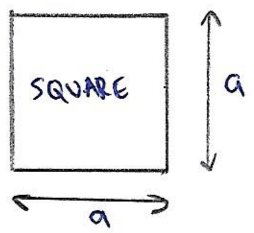
- Since the application of torque, it will produce the angle of twist, where it is defined by

$$\phi = \frac{TL}{JG}$$

(only for circular section)

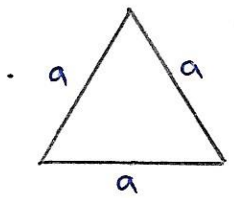
ϕ = angle of twist (rad)
T = Torque (N.m)
L = length (m)
J = Polar moment of inertia
G = Shear Modulus

- For non cylindrical member, the equations to determine maximum shear stress and angle of twist are stated as below



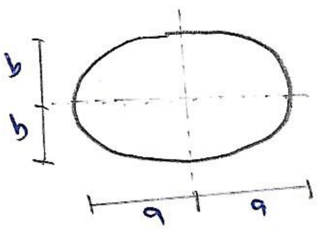
$$\tau_{max} = \frac{4.81\,T}{a^3}$$

$$\phi = \frac{7.10\,TL}{a^4 G}$$



$$\tau_{max} = \frac{20\,T}{a^3}$$

$$\phi = \frac{46\,TL}{a^4 G}$$

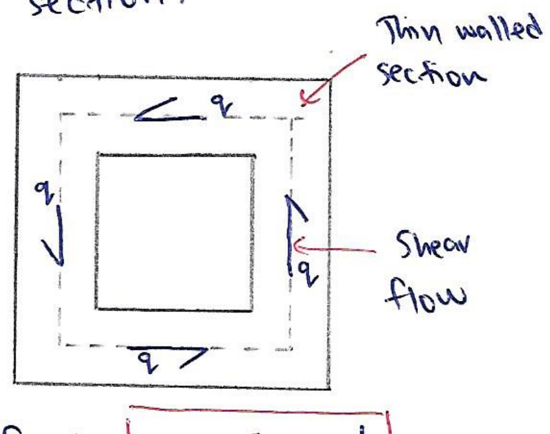


$$\tau_{max} = \frac{2T}{\pi a b^2}$$

$$\phi = \frac{(a^2+b^2)\,TL}{\pi a^3 b^3 G}$$

Thin Walled Having Closed Cross Section.

- For the case of thin walled section, there a term called shear flow (q), where the shear stress is assumed to be at the middle of a thin walled section.



Shear flow $\boxed{q = \tau_{ave} \cdot t}$

t = thickness of the thin walled section

Average shear stress

- In a non circular segment, the shear stress may differ at each section. Therefore, we assume an average value for the shear stress

$$\tau_{ave} = \frac{T}{2\,A_m \cdot t}$$

τ : average of shear stress (N/mm^2)

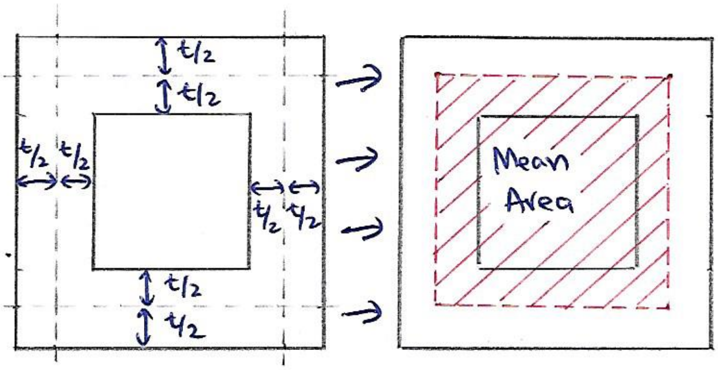
A_m : mean area (mm^2)

T : Torque (N-mm)

t : thickness (mm)

Mean area

- Mean area is defined as the area enclosed ~~by~~ from the middle of each section's thickness. Let say,



Since torque still produces angle of twist for this section, the equation is shown as follow

$$\phi = \frac{TL}{4\,A_m^2\,G} \oint \frac{ds}{t}$$