Virtual Work Method

The definition of work :

WORK = FORCE x DISPLACEMENT

(W) = (N) x (m)

We know that work is the multiplication of force and displacement. In plastic moment, work is defined as :

WORK = MOMENT x ROTATION ANGLE

(W) = (Nm) x (rad)

So, work is defined by moment and the angle produced by the rotated body.

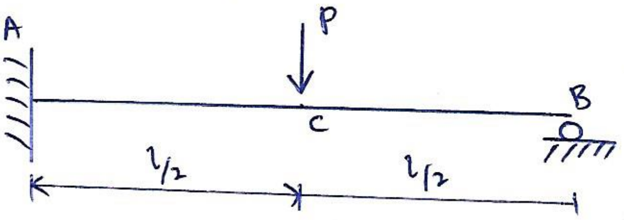
In Virtual Work Method, it also known that ;

EXTERNAL WORK = INTERNAL WORK

So that a balance state is achievable for a static analysis.

Example :

Determine the plastic moment (M_p) and the load required to cause collapse using the virtual work method.



As we know, hinge will appear at

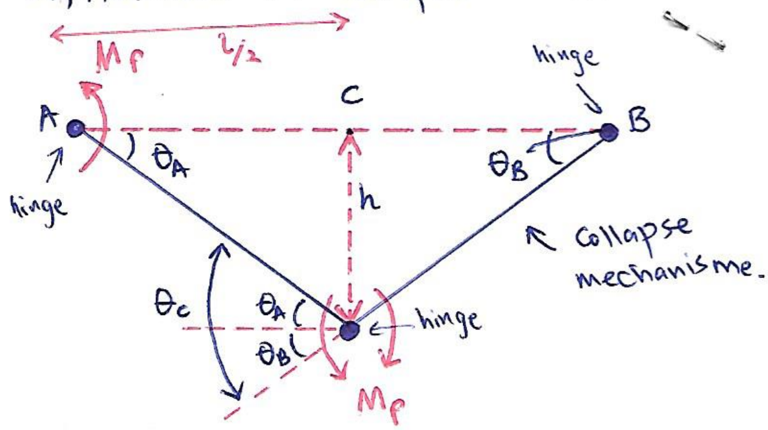
① the point where force is acting

② fixed connection

or

③ Point where bending moment is maximum.

So, illustrate the collapse mechanisme:



Because A is a fixed connection ($\theta_A \approx 0$), so we assume $\tan \theta_A = \theta$ (because the value is so small)

$$\tan \theta_A = \theta$$

$$\frac{h}{l/2} = \theta \text{, therefore } h = \frac{l\theta}{2}$$

$$\text{Also, } \theta_B = \frac{h}{l/2} = \frac{\frac{l\theta}{2}}{l/2} = \theta$$

So, to calculate θ_c ;

$$\theta_c = \theta_A + \theta_B$$

$$\theta_c = \theta + \theta = 2\theta \ \#$$

External Work = Internal Work

Force x displacement = Moment x angle

$$P \times h = (\text{Total Moment in the mechanisme } \Sigma M_p)$$

$$P \times h = M_p \theta_A + M_p \theta_c$$

Here, $h = \frac{l\theta}{2}$, $\theta_A = \theta$ and $\theta_c = 2\theta$. So,

$$P \times \frac{l\theta}{2} = M_p \cdot \theta + M_p \cdot 2\theta$$

$$\frac{Pl\theta}{2} = 3M_p \cdot \theta$$

So, the load required to cause collapse is ;

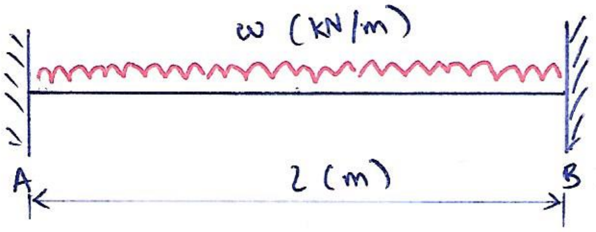
$$\boxed{P = \frac{6M_p}{l}} \ \#$$

In the case of uniformly Distributed load (UDL); we assume the acting force times the area produced by the collapse mechanisme is the total work!

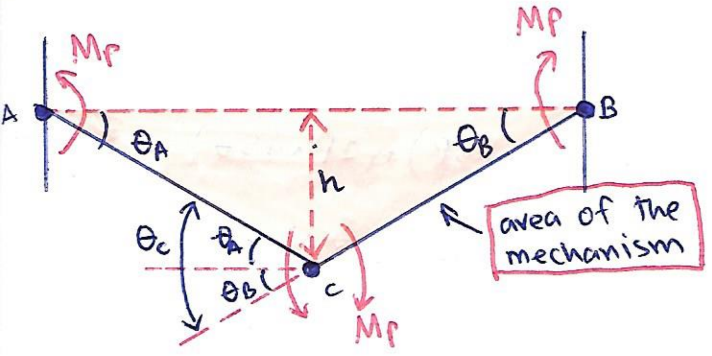
Work, W = UDL × area of mechanisme

Example:

Determine the plastic moment, M_p for the following structure using virtual work method.



So, we illustrate the collapse mechanisme. Here, we will have 3 hinges, two at ~~each~~ the supports and one in the middle of the structure.



Assume $\tan\theta_A = \theta$

$$\theta_A = \frac{h}{l/2} = \theta \;; \quad h = \frac{l\theta}{2}$$

$$\theta_B = \frac{h}{l/2} = \frac{\frac{l\theta}{2}}{l/2} = \theta$$

$$\theta_C = \theta_A + \theta_B = \theta + \theta = 2\theta$$

According to the equilibrium state of a static body,

EXTERNAL WORK = INTERNAL WORK

$$\underbrace{\frac{1}{2}\times h \times l}_{\text{The area of the mechanisme}} \times \underbrace{\omega}_{\text{UDL}} = \underbrace{\Sigma M_p}_{\text{total of moment plastic in the mechanisme}}$$

$$\frac{\omega h l}{2} = M_p\theta_A + M_p\theta_B + M_p\theta_C$$

$h = \frac{l\theta}{2}$

$$\frac{\omega h l}{2} = M_p\theta + M_p\theta + 2M_p\theta$$

$$\frac{\omega l^2 \cancel{\theta}}{4} = 4M_p\cancel{\theta}$$

$$M_p = \frac{\omega l^2}{16} \quad \#$$

→ the answer is similar to a fixed end moment

Note:

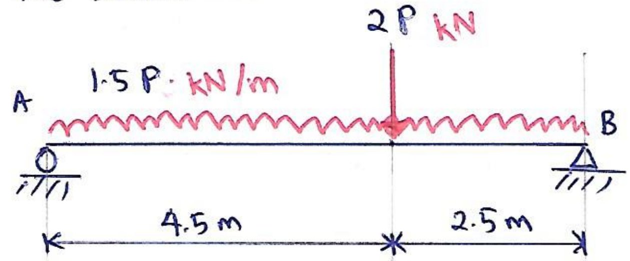
1) If there are more than one load applied, pick the biggest value for the M_p.
2) For a frame with sway, the total mechanisme depends on the following equation. **$M_e = N - d$**

M_e = total of mechanisme
N = total number of plastic hinge that may occur
d = number of redundant $d = R - 3$
∴ which R = number of reaction.

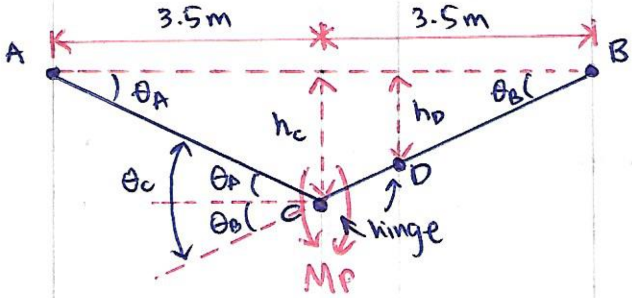
For the case of multiple loads on a structure, observe the following example.

EXAMPLE

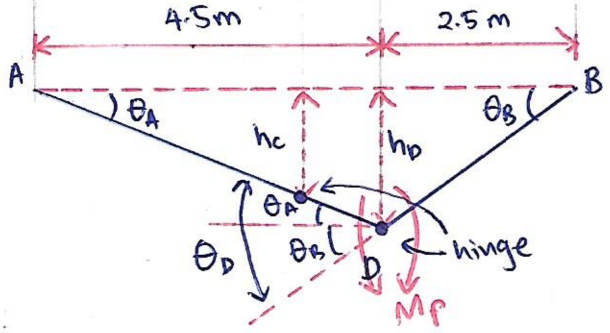
An UDL and a point load are subjected on the structure. Investigate the load which requires to cause collapse for the structure.



Mode failure ① : When considering the UDL as main load



Mode failure ② : when considering the point load as main load



※ Notice the difference of point where the hinges may occur for ① and ②

for mode failure ①

Assume $\tan \theta_A = \theta$

$\theta_A = \frac{h_c}{3.5} = \theta \quad \therefore h_c = 3.5\theta$

$\theta_B = \frac{h_c}{3.5} = \frac{3.5\theta}{3.5} = \theta$

$\theta_B = \frac{h_D}{2.5} \quad \therefore h_D = 2.5\theta_B = 2.5\theta$

$\theta_c = \theta_A + \theta_B = \theta + \theta = 2\theta$

EXTERNAL WORK = INTERNAL WORK

$\left(\frac{1}{2} \times l \times h_c \times w\right) + \left(2P \times h_D\right) = M_p \theta_c$

UDL — Point Load

$\left(\frac{1}{2} \times 7 \times 3.5\theta \times 1.5P\right) + \left(2P \times 2.5\theta\right) = 2M_p\theta$

$18.375P\theta + 5P\theta = 2M_p\theta$

$23.375P\theta = 2M_p\theta$

The required load $P = 0.0856 M_p$ //

for mode failure ②

Assume $\tan \theta_A = \theta$

$\theta_A = \frac{h_D}{4.5} = \theta \quad \therefore h_D = 4.5\theta$

$\theta_B = \frac{h_D}{2.5} = \frac{4.5\theta}{2.5} = 1.8\theta$

$\theta_D = \theta_A + \theta_B = \theta + 1.8\theta$
$= 2.8\theta$

EXTERNAL WORK = INTERNAL WORK

$\left(\frac{1}{2} \times l \times h_D \times w\right) + \left(2P \times h_D\right) = M_p \theta_D$

UDL — Point Load

$\left(\frac{1}{2} \times 7 \times 4.5\theta \times 1.5P\right) + \left(2P \times 4.5\theta\right) = 2.8M_p\theta$

$23.625P\theta + 9P\theta = 2.8M_p\theta$

$32.625P\theta = 2.8M_p\theta$

The required load $P = 0.0858 M_p$

By comparing mode failure ① $P = 0.0856 M_p$

② $P = 0.0858 M_p$

We can assume the failure mode is ②, when the point load is the main ② primary load.

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