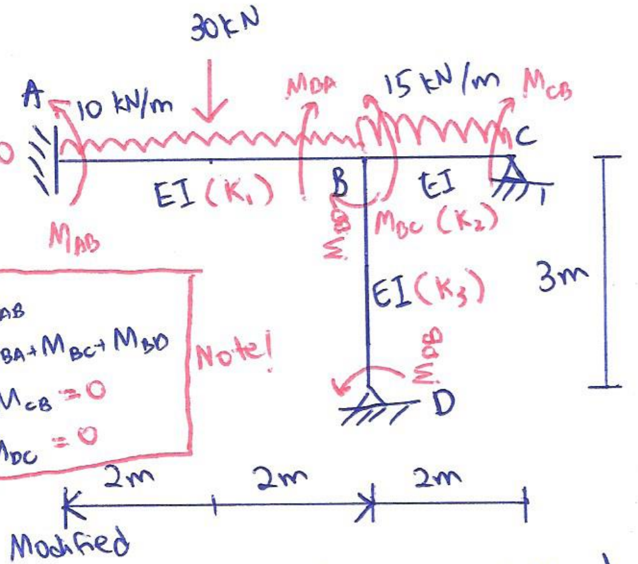
MOMENT DISTRIBUTION METHOD FOR RIGID NON-SWAY FRAME



$M_A = M_{AB}$
$M_B = M_{BA} + M_{BC} + M_{BD}$
$M_C = M_{CB} = 0$
$M_D = M_{DC} = 0$

Note!

Using Modified Moment Distribution Method,

Determine :

1) Reaction at A, B and C

Sketch :

1) SFD & BMD for the frame.

Step 1 : Distribution Factor (D.F)

$DF_{AB} = 0$ (because point A is a fixed connection)

$DF_{CB} = 1$
$DF_{DB} = 1$ } (point C and D is pinned connection)

* refer to the calculation!

Joint	Member	Stiffness (K)	ΣK	D.F
B	BA	$\frac{4EI}{L} = \frac{4EI}{4} = EI$	3.5EI	$\frac{1}{3.5} = \boxed{0.286}$
	BC	$\frac{3EI}{L} = \frac{3EI}{2} = 1.5EI$	3.5EI	$\frac{1.5}{3.5} = \boxed{0.429}$
	BD	$\frac{3EI}{L} = \frac{3EI}{3} = EI$	3.5EI	$\frac{1}{3.5} = \boxed{0.286}$

Using Modified Stiffness Method

$M_{AB} = 2K_1(2\theta_A + \theta_B)$

since $\theta_A = 0$ $\quad M_{AB} = 2K_1(2(0) + \theta_B)$

$$\boxed{M_{AB} = 2K_1\theta_B}$$

$M_{BA} = 2K_1(2\theta_B + \theta_A) = 2K_1(2\theta_B + 0)$

$$\boxed{M_{BA} = 4K_1\theta_B}$$

$M_{CB} = 2K_2(2\theta_C + \theta_B)$

since $M_C = 0$ $\quad M_{CB} = 2K_2(2\theta_C + \theta_B) = 0$

$2\theta_C + \theta_B = 0$

$$\boxed{\theta_C = -\frac{\theta_B}{2}}$$

$M_{BC} = 2K_2(2\theta_B + \theta_C) = 2K_2(2\theta_B - \frac{\theta_B}{2})$

$$\boxed{M_{BC} = 3K_2\theta_B}$$

$M_{DB} = 2K_3(2\theta_D + \theta_B)$

since $M_D = 0$ $\quad M_{DB} = 2K_3(2\theta_D + \theta_B) = 0$

$2\theta_D + \theta_B = 0$

$$\boxed{\theta_D = -\frac{\theta_B}{2}}$$

$M_{BD} = 2K_3(2\theta_B + \theta_D) = 2K_3(2\theta_B - \frac{\theta_B}{2})$

$$\boxed{M_{BD} = 3K_3\theta_B}$$

$\Sigma M_B = M_{BA} + M_{BC} + M_{BD}$

$= 4K_1\theta_B + 3K_2\theta_B + 3K_3\theta_B$

$= (4K_1 + 3K_2 + 3K_3)\theta_B$

$\theta_B = \frac{M_B}{\boxed{4K_1 + 3K_2 + 3K_3}}$ → Total stiffness

Step 2 : Fixed End Moment

$FEM_{AB} = -\left(\frac{wL^2}{12} + \frac{Pab^2}{L^2}\right) = -28.33$ kNm

$FEM_{AB} = \frac{wL^2}{12} + \frac{Pa^2b}{L^2} = 28.33$ kNm

$FEM_{BC} = -\frac{wL^2}{12} = -5$ kNm

$FEM_{CB} = \frac{wL^2}{12} = 5$ kNm

$FEM_{CD} = 0$ kNm
$FEM_{DC} = 0$ kNm } No load is applied along the span!

* For confirmation, please do your own calculation! (for FEM)

Step 3 : Distribute moments to members.

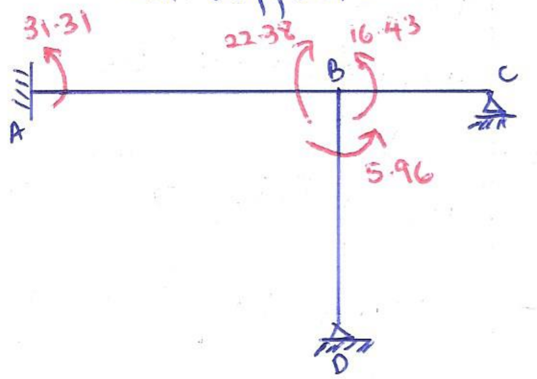
Joint	A	B			C	D
Member	AB	BA	BC	BD	CB	DB
Carry Over Factor (C.F)	0	0.5	0	0	0.5	0.5
Distribution Factor (D.F)	0	0.286	0.429	0.286	1	1
① F.E.M	−28.33	28.33	−5	0	5	0
Distribution	0	$\Sigma M \times DF \times -1$ −6.67	−10.01	−6.67	−5	0
② Carry Over	−3.34 $(-3.34 \times 0 \times -1)$	0	−2.5	0	0	0
Distribution	0	0.715	1.073	0.715	0	0
③ Carry Over	0.358	0	0	0	0	0
Distribution	0	0	0	0	0	0
TOTAL	−31.31	22.38	−16.43	−5.96	0	0

ΣM (sum of BA, BC, BD in rows ① and ②)

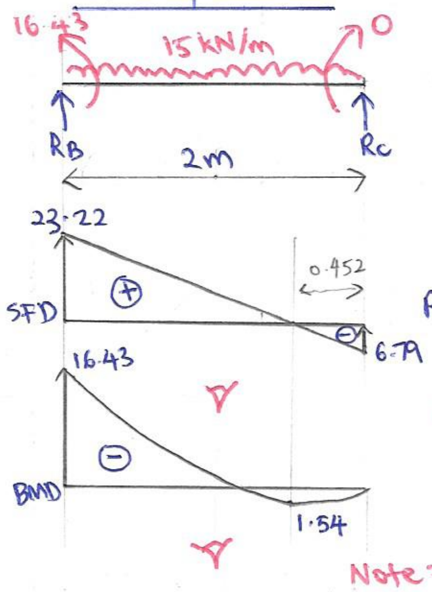
NOTE

From now on, there are no more moments to be distributed. The calculation stops here!

Step 4 : Determine reaction at all support.



For span BC



$\Sigma F_y = 0$

$R_B + R_C = 30$ (15 × 2)

$\Sigma M_C = 0$

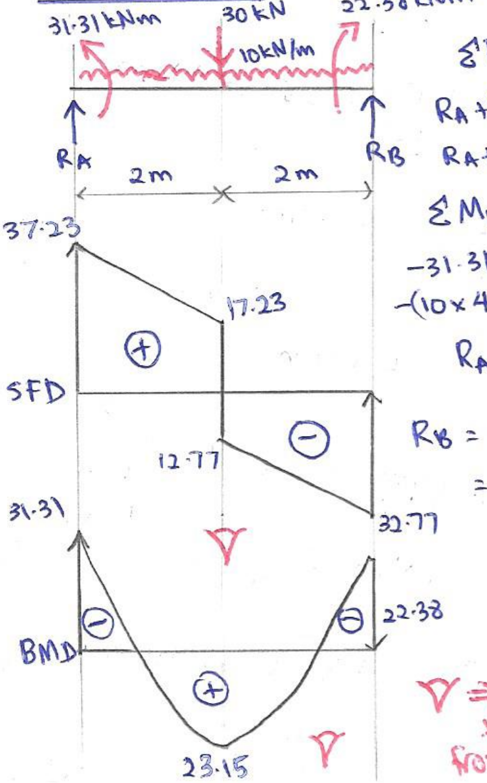
$-16.43 - (15 \times 2 \times 1) + R_B \cdot 2 = 0$

$R_B = 23.22$ kN

$R_C = 30 - 23.22 = 6.79$ kN

▽ = view from this sight

For span AB



$\Sigma F_y = 0$

$R_A + R_B = 30 + 10(4)$

$R_A + R_B = 70$

$\Sigma M_B = 0$

$-31.31 + 22.38 - 30(2) - (10 \times 4 \times 2) + R_A \cdot 4 = 0$

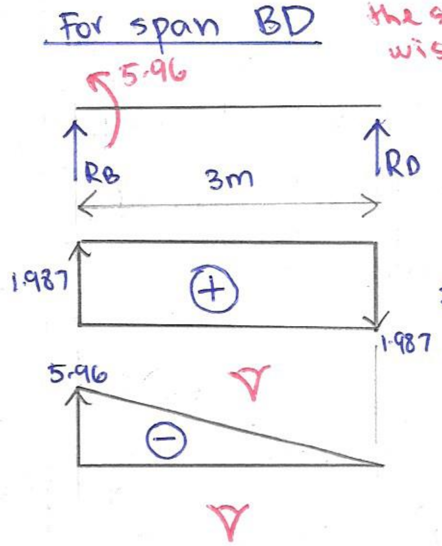
$R_A = 37.23$ kN

$R_B = 70 - 37.23 = 32.77$ kN

▽ ⇒ imagine your eyesight, view the SFD & BMD from this direction

Note: for span BD, I rotated the span 90° counter clockwise for a better view for SFD & BMD

For span BD



$\Sigma F_y = 0$

$R_B + R_D = 0$

$\Sigma M_B = 0$

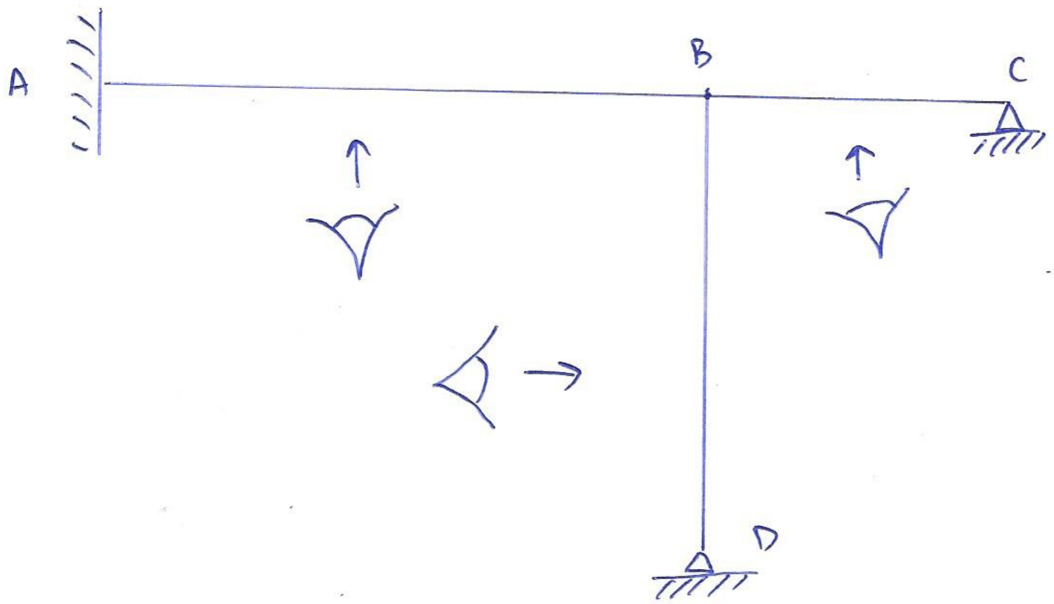
$-5.96 - R_D \cdot 3 = 0$

$R_D = -1.987$ kN (←)

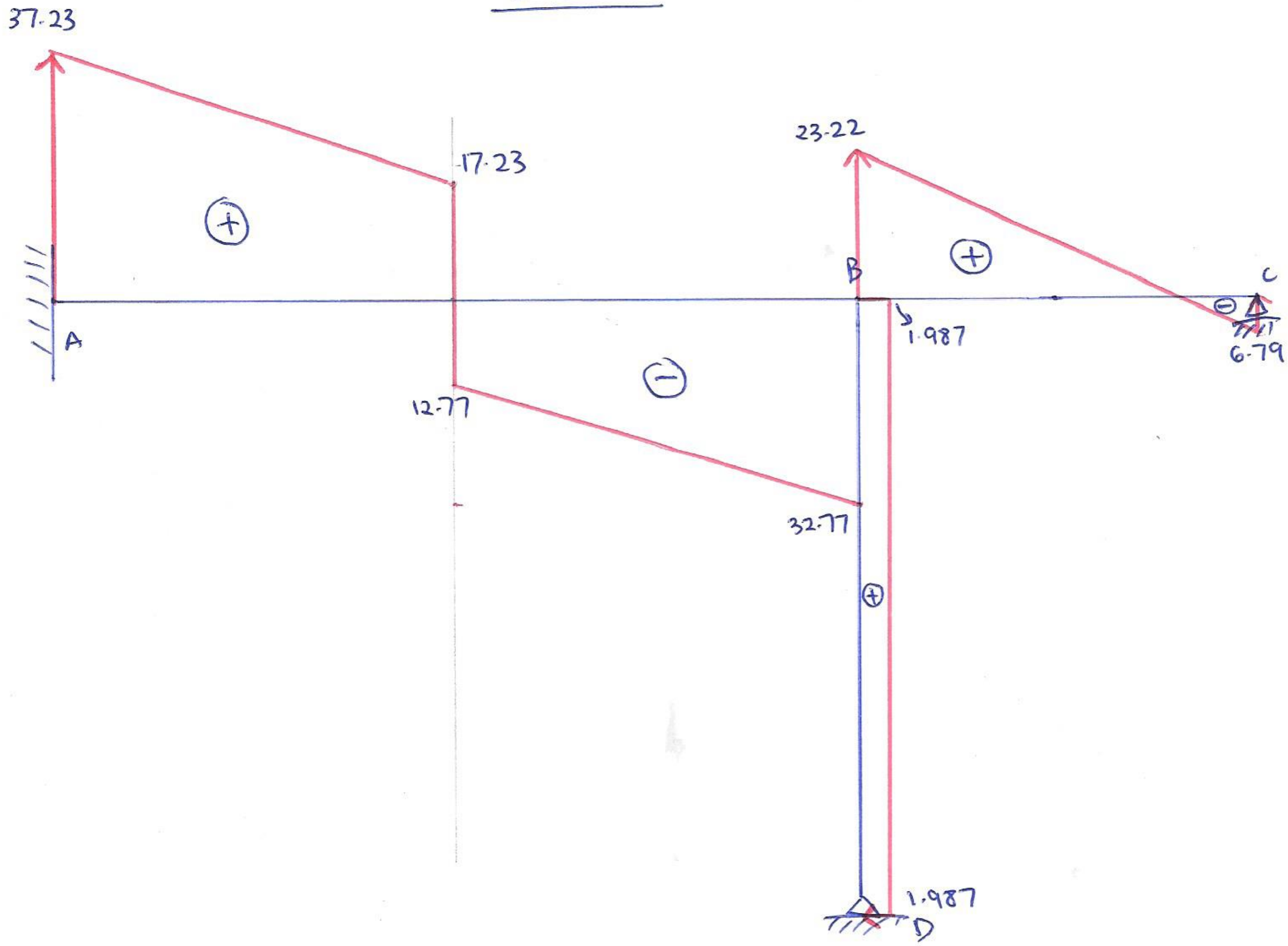
$R_B = 1.987$ kN (→)

※ NOW, WHILE OBEYING THE WAY WE VIEW THE SFD & BMD FOR FRAME, COMBINE ALL THE DIAGRAM

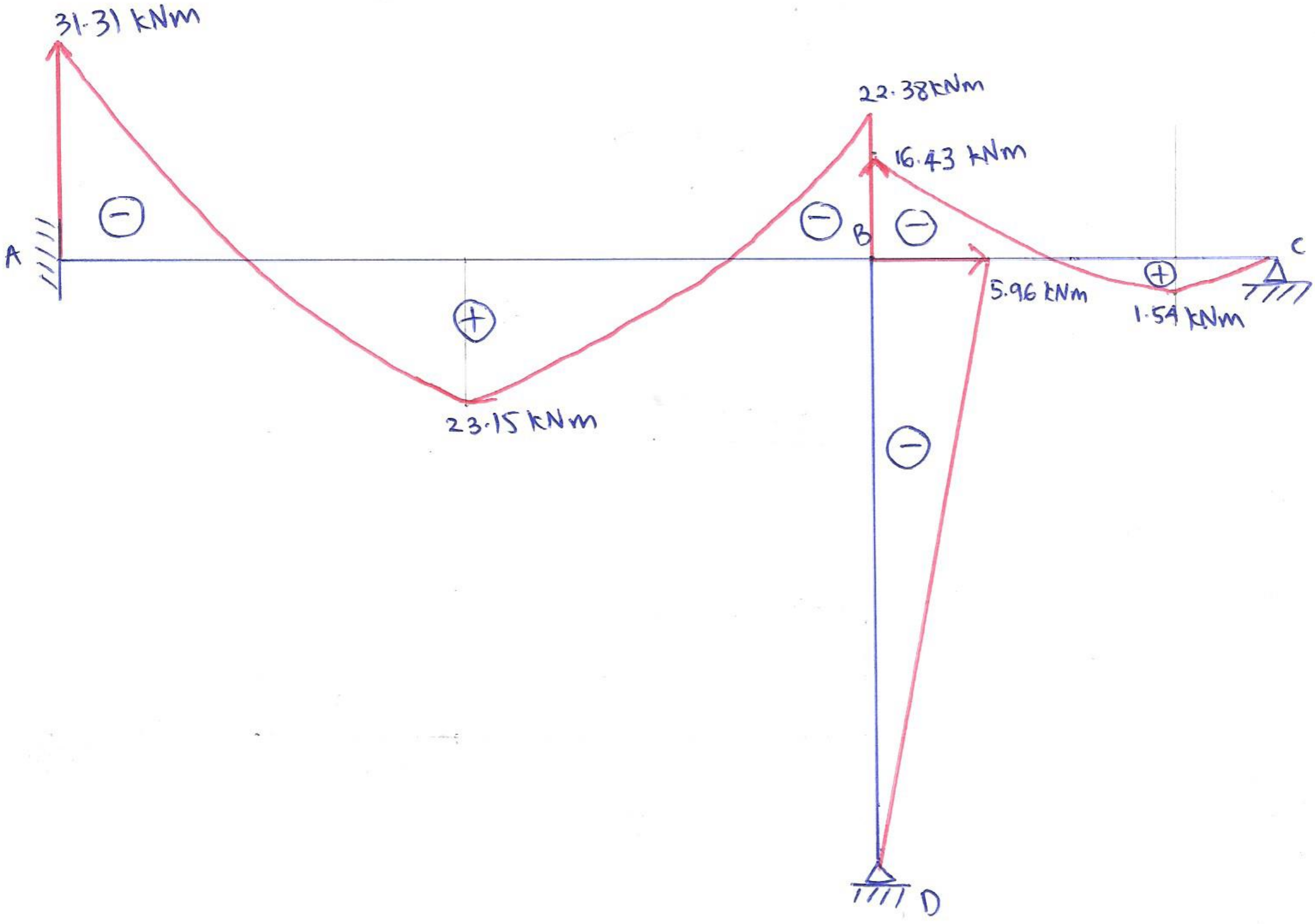
THE RULE STATED THAT, TO VIEW THE SFD & BMD FOR THIS TYPE OF FRAME, IT IS SOMETHING LIKE THIS ....



S.F.D



BMD



Easy, right? ^^