Chapter 5

Compression Member

This chapter starts with the behaviour of columns, general discussion of buckling, and determination of the axial load needed to buckle. Followed by the assumption of Euler's Theory and the calculation with the different types of support in the column. At the end of the chapter, Secant formula will be discussed when the axial load acted at the offset from centroid.

After successfully completing this chapter you should be able to:

- Determine the type of failure in compression member
- Determine the shape of buckling in compression member
- Analyse the compression member using Euler's theory and Secant formula

5.1 Introduction

The selection of the column is often a very critical part of the design of structure because the failure of the column usually has catastrophic effects. If a column is long compared to its width, it may fail by buckling (bending and deflection laterally). The buckling may be either elastic or inelastic depends upon the slenderness of the column.

5.2 Critical load and Euler Theory

In this section we discuss a theory of a straight column that is simply supported at either side. This theory was first developed by Leonard Euler and is named after him. The assumptions for this theory are:

a) The column is perfectly straight;
b) The cross section of the column is uniform;
c) The column material is homogeneous;
d) The column behave elastically;
e) The compression force acted on the centroid of the section.

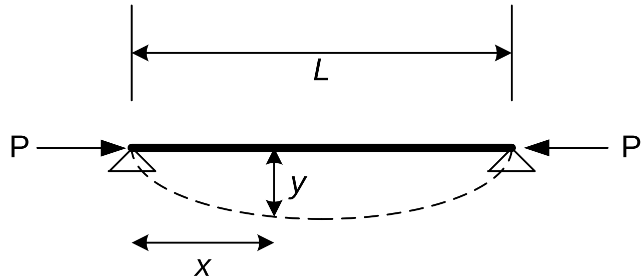


Figure 5.1

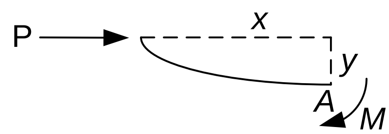


Figure 5.2

Figure 5.1 shows a simply supported column that is axially loaded with force P. Let the bending deflection at any location x be given by y as in Figure 5.2. By balancing the moment at Point A, we obtain

$$M + Py = 0$$

The differential equation for moment curvature relationship is given by

$$EI\,\frac{d^2y}{dx^2} = M$$

Therefore

$$EI\,\frac{d^2y}{dx^2} + Py = 0$$

Where

$$\frac{d^2y}{dx^2} + \frac{Py}{EI} = 0$$

By substitution $\lambda = \sqrt{\frac{P}{EI}}$

$$\frac{d^2y}{dx^2} + \lambda^2 y = 0$$

The solution to the differential equation is:

$$y = A\ kos\ \lambda x + B\ sin\ \lambda x$$

With a boundary condition $x = 0$ $y = 0$ we obtain $A = 0$, thus $y = B \sin \lambda x$

And with $x = L$, $y = 0$; we obtain $B \sin \lambda L = 0$

If $B = 0$, than we obtain a trivial solution, For a nontrivial solution, the sun function mest equal to zero

$$\sin \lambda L = 0 \rightarrow \textit{is called buckling equation}$$

$$\lambda L = n\pi \text{ (n = 1,2,..)}$$

Therefore

$$\lambda^2 L^2 = n^2 \pi^2$$

$$\left(\frac{P}{EI}\right) L^2 = n^2 \pi^2$$

The critical buckling load is $P_{cr} = n^2 \pi^2 \frac{EI}{L^2}$ (5.1)

P_{cr} the critical buckling load is also caller Euler load. Buckling will occur about the axis that has minimum area of moment of inertia.

The importance of each buckled mode shape is shown in Figure 5.3 as n is increased; the deflection curve has more and more inflection points.

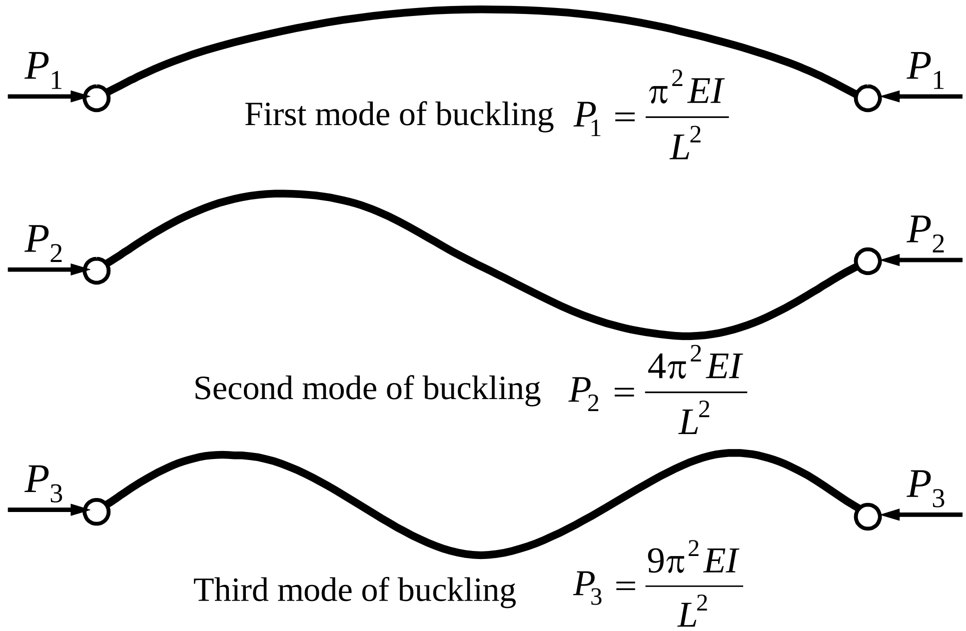


Figure 5.3: The value of *n* defines the buckling mode shape.

For case 1 where n = 1, the value of *L* that should be used is depends on the support condition at both ends of the column. So the equation 5.1 can be written as:

$$P_{cr} = \pi^2 \frac{EI}{{L_e}^2} \qquad (5.2)$$

Where

i) If both ends are pinned; $L_e = L$

ii) If one end fixed, other end free; $L_e = 2L$

iii) If both end are fixed; $L_e = L/2$

iv) If one end fixed, other end pinned; $L_e = 0.7L$

Pinned/Pinned	Fixed/Free	Fixed/Fixed	Pinned/Fixed
$L_e = L$	$L_e = 2L$	$L_e = L/2$	$L_e = 0.7\ L$

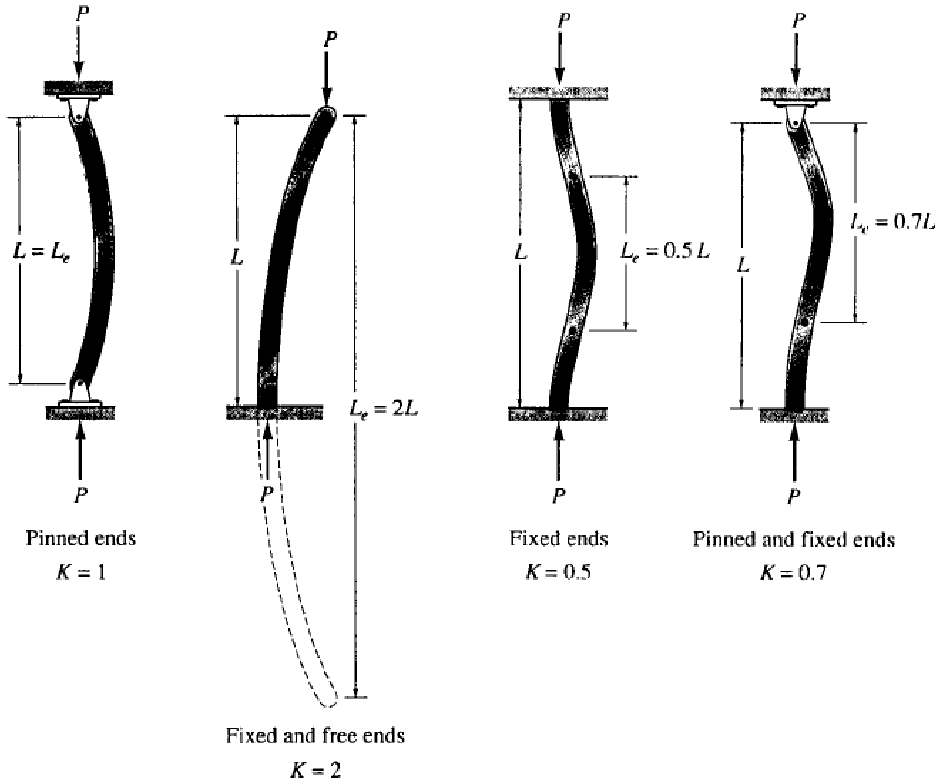


Figure 5.3: Effect of support condition

EXAMPLE 1

A 2 m long pin ended column of square cross section is to be made of wood. Assuming E = 13 GPa, σ_{all} = 12 MPa and using a factor of safety of 2.5 in computing Euler's critical load for buckling. Determine the size of cross section if the column is to safely support (a) 100 kN load and (b) 200 kN load.

Solution

(a) 100 kN load

P_{cr} = 2.5 (100) = 250 kN, L = 2 m, E = 13 GPa.

In Euler's Equation 5.2 and solve for I, we have

$$I = \frac{P_{cr}L^2}{\pi^2 E} = \frac{(250x10^3)(2^2)}{\pi^2(13x10^9)} = 7.794x10^{-6}\,m^4$$

For a square of side a, we have $I = a^4/12$, we write

$$\frac{a^4}{12} = 7.794x10^{-6}$$ a = 98.3 mm $\approx$ 100 mm

We check value of normal stress in the column:

$$\sigma = \frac{P}{A} = \frac{100}{0.1^2} = 10MPa$$

Since σ is smaller than the allowable stress, a 100 x 100 mm cross section is acceptable.

(b) For the 200 kN load

Solving again Equation 5.2 for I, but making now P_{cr} = 2.5(200) = 500 kN, we have I = 15.588 x 10^{-6} m^4.

$$\frac{a^4}{12} = 15.588x10^{-6}$$ a = 116.95 mm

The value of normal stress is

$$\sigma = \frac{P}{A} = \frac{200}{0.11695^2} = 14.62MPa$$

Since this value is larger than the allowable stress, the dimension obtained is not acceptable and we must select the cross section on the basis of its resistance to compression. We write

$$A = \frac{P}{\sigma_{all}} = \frac{200}{12} = 16.67x10^{-3}m^2$$

a^2 = 16.67 x 10^{-3} m^2 a = 129.1 mm

A 130 x 130 mm cross section is acceptable.

EXAMPLE 2

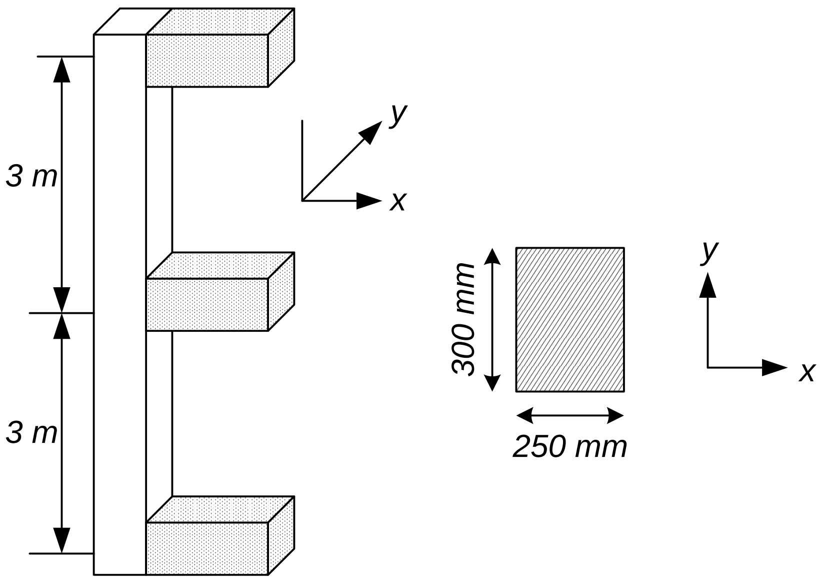


Figure 5.4

A column with 6 meters height is connected to the beam as in Figure 5.4. With the connection between beam and column is pinned, Determine the critical buckling load of the column. E = 5000 MPa

Solution

Determine moment of inertia, *I*

$$I_{xx} = \frac{bh^3}{12} = \frac{250(300^3)}{12} = 5.6x10^8 \text{ mm}^4$$

$$I_{yy} = \frac{bh^3}{12} = \frac{300(250^3)}{12} = 3.9x10^8 \text{ mm}^4$$

Deremine critical buckling load

$$P_{cr_y} = \frac{\pi^2 EI}{Le^2} = \frac{\pi^2 (5000)(5.6x10^8)}{(1.0x6000)^2} = 771\text{kN}$$

$$P_{cr_y} = \frac{\pi^2 EI}{Le^2} = \frac{\pi^2 (5000)(3.9x10^8)}{(1.0x3000)^2} = 2138\text{kN}$$

Therefore the critical buckling load is 771 kN

5.3 Limitation of Euler Theory

Since P_{cr} is proportional to I, the column will buckle in the direction corresponding to the minimum value of I.

A column can either fail due to the material yielding, or because the column buckles, it is of interest to the engineer to determine when this point of transition occurs. Because of the large deflection caused by buckling, the least moment of inertia I can be expressed as $I = Ar^2$

where: A is the cross sectional area and r is the ***radius of gyration*** of the cross sectional area, i.e. .

Note that the *smallest* radius of gyration of the column, i.e. the *least* moment of inertia I should be taken in order to find the critical stress.

Dividing the buckling equation by A, gives:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/r)^2}$$

where:

σ_{cr} is the compressive stress in the column and must not exceed the yield stress σ_y of the material, i.e. $\sigma_{cr} < \sigma_y$,

L / r is called the ***slenderness ratio***, it is a measure of the column's flexibility.

5.4 Secant Formula for Column

When a column with simply supported is compressed by an eccentricity applied axial as in Figure 5.4, the maximum compressive stress in the column is:

$$\sigma_{max} = \frac{P}{A} + \frac{Pe}{S}\sec\frac{kL}{2} \qquad 5.3$$

The first term on the right-hand side of this equation represents the effect of direct compression and the second term represents the effect of bending of the column. Recalling that the section modulus *S* = *I/c*, where c is the distance from the neutral axis to the extreme fiber on the concave side of the column and also introducing the notation $r = \sqrt{I/A}$ for the radius of gyration, we can express equation 5.3 in the form

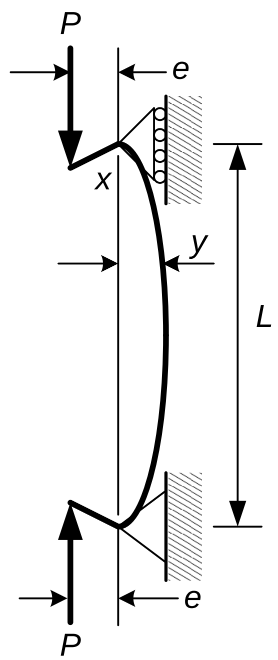


Figure 5.4: Column with eccentrically applied axial force

$$\sigma_{\max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\frac{kL}{2}\right)$$

Next replacing *k* by $\sqrt{P/EI}$, we obtain

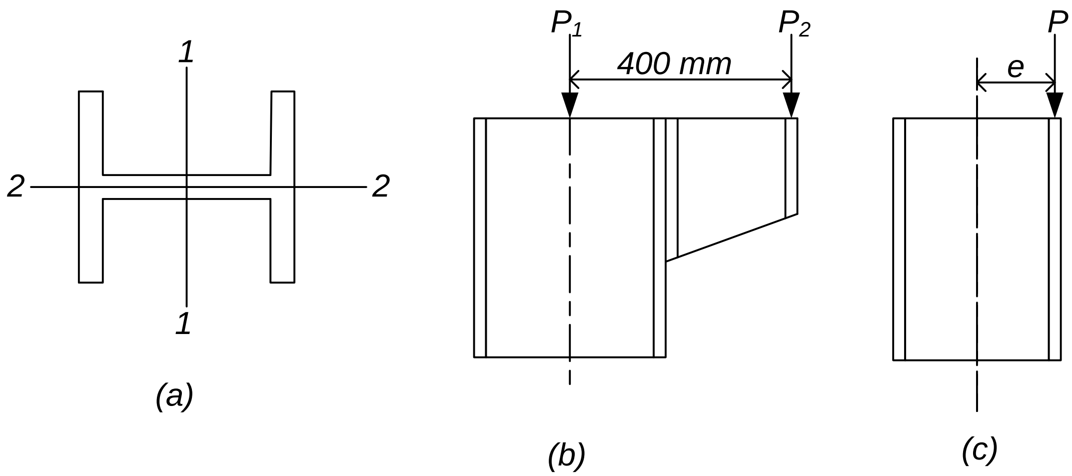
$$\sigma_{\max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\sec\frac{L}{2r}\sqrt{\frac{P}{EA}}\right) \qquad 5.4$$

This equation is called the ***secant formula*** for an eccentrically loaded column. It gives the maximum stress in the column as a function of the average compressive stress *P/A*, the eccentricity ratio ec/r^2, and the slenderness ratio *L/r*.

EXAMPLE 3

A steel column of 254 x 254 x 107 kg UC section (Fig. 5.5a) with pinned ends is 8 m long. It carries a centrally applied load P_1 = 980 kN and an eccentrically applied load P_2 = 140 kN on axis 2-2 at a distance of 400 mm from axis 1-1 (Fig 5.5b).

a) Using a secant formula, calculate the maximum compressive stress in the column;



Solution

a) The two loads P1 and P2 acting as shown in Fig. 5.5(b) are statically equivalent to a single load P = 1120 kN acting with an eccentricity e = 50 mm (see Fig. 5.5(c). Using the table of properties, we find

$$\frac{P}{A} = \frac{1120x10^3}{13660} = 82N/mm^2 \qquad \frac{L}{r} = \frac{8000}{113} = 70.8$$

$$\frac{ec}{r^2} = \frac{eA}{S} = \frac{(50)(13660)}{1313x10^3} = 0.520$$

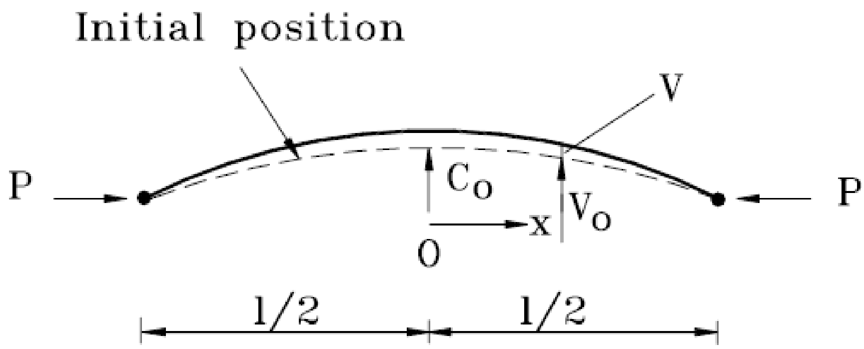
Substituiting into equation 5.4 using E = 200 kN/mm^2, we get

$\sigma_{\max} = 138.6$ N/mm^2.

5.5 Perry Robertson Formula

Several different formulae have been devised that give a more realistic estimate of buckling loads than the Euler equation.

The formula usually used for structural steelwork is the Perry-Robertson formula. Formulation is based on the assumption that the strut is initially bent with a maximum offset of c_0



Notes: Origin fixed at strut mid length.
v is increase in deflection due to *P*.

Perry formula

$$\sigma_p = \frac{1}{2}\left[\sigma_{yield} + (\eta+1)\sigma_E\right] - \sqrt{\left[\frac{1}{2}\sigma_{yield} + \frac{1}{2}(\eta+1)\sigma_E\right]^2 - \sigma_{yield}\sigma_E}$$

Based on test on circular mild steel column, Robertson proposed that a value of $\eta = 0.003\frac{l_e}{k}$ could be used for mild steel columns. If a value for initial out of straightness c_0 is known, this value should be used to calculate $\eta \Rightarrow \eta = \frac{c_0 v}{k^2}$