UTHM
Universiti Tun Hussein Onn Malaysia

www.uthm.edu.my

Semester 2 2017/2018

# RC COLUMN DESIGN

By
Dr. Zainorizuan Bin Mohd Jaini
Department of Structure and Material Engineering

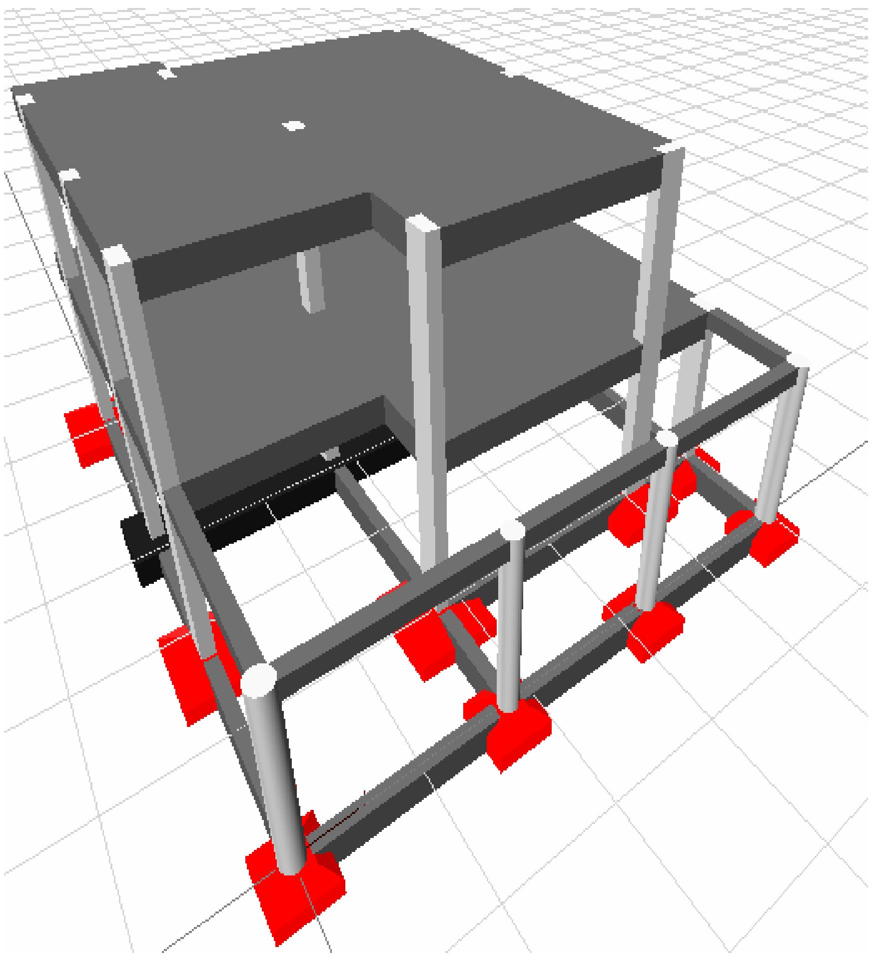
FKAAS
FAKULTI KEJURUTERAAN AWAM DAN ALAM SEKITAR

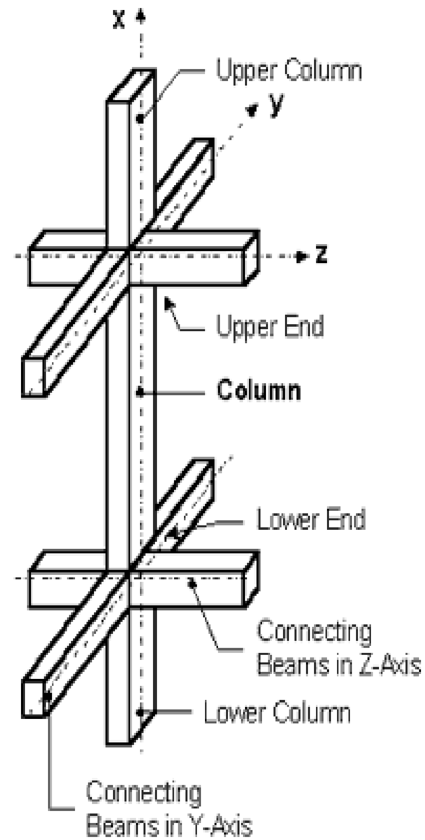
With Wisdom We Explore

# Introduction

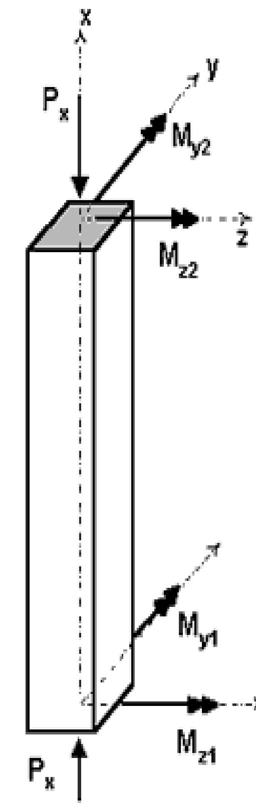
- Columns act as vertical supports to beams and slabs, and to transmit the loads to the foundations.
- Columns are primarily compression members, although they may also have to resist bending moment transmitted by beams.
- Columns may be classified as braced or unbraced, short or slender, depending on various dimensional and structural factors.
- EC2 in term of column design provides:
  1) New guideline to determine the effective height of column, slenderness ratio and limiting slenderness.
  2) Design for second order (so-called P-Δ)
  3) Design moment method based on nominal curvature.
  4) Geometry imperfection: Deviations in cross-section dimensions are normally taken into account in the material factors and should not be included in structural analysis.

# Introduction






a) Basic Model




a) Column Loads

# Introduction

- Classification of column:

| Braced | Unbraced |
|---|---|
| Where the lateral loads are resisted by shear wall or other form of bracing capable of transmitting all horizontal loading to the foundations. | Where horizontal loads are resisted by the frame action of rigidity connected columns, beams and slabs |
| With a braced structure, the axial forces and moments in the columns are induced by the permanent and variable actions (vertical) only. | With an unbraced structure, the loading arrangement which include the effects of lateral load must also be considered |

# Failure of Column

Damage of circular column during earthquake due to poor detailing of transverse reinforcement

Buckling of freeway support columns, San Fernando Valley

Shear failure of a column of the Shinkansen bridge

# Failure of Column

Kukup Laut, Johor
9 January 2014



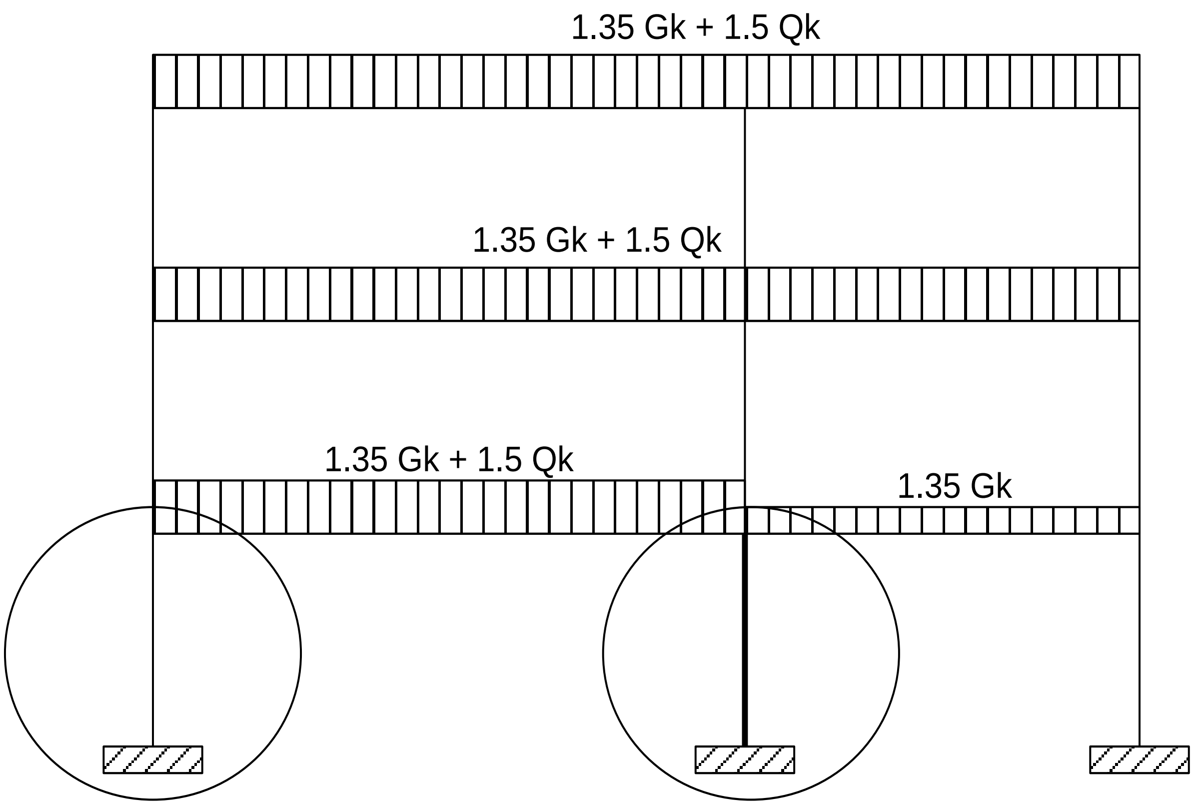
Batu Pahat, Johor
13 July 2014

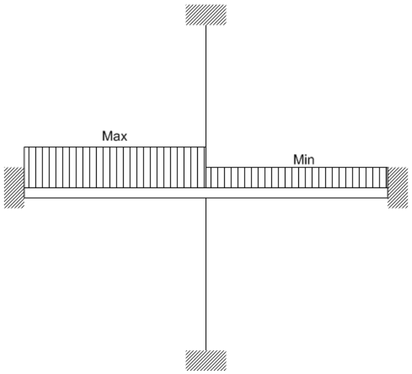
# Design Procedure

| Step | Task | Standard |
|---|---|---|
| 1 | Determine design life | UK NA to BS EN 1990 Table NA.2.1 |
| 2 | Assess actions on the column | BS EN 1991 (10 parts) and UK National Annexes |
| 3 | Determine which combinations of actions apply | UK NA to BS EN 1990 Tables NA.A1.1 and NA.A1.2 (B) |
| 4 | Assess durability requirements and determine concrete strength | BS 8500: 2002 |
| 5 | Check cover requirements for appropriate fire resistance period | Approved Document B. BS EN 1992–1–2 |
| 6 | Calculate min. cover for durability, fire and bond requirements | BS EN 1992–1–1 Cl. 4.4.1 |
| 7 | Analyse structure to obtain critical moments and axial forces | BS EN 1992–1–1 section 5 |
| 8 | Check slenderness | BS EN 1992–1–1 section 5.8 |
| 9 | Determine area of reinforcement required | BS EN 1992–1–1 section 6.1 |
| 10 | Check spacing of bars | BS EN 1992–1–1 sections 8 and 9 |

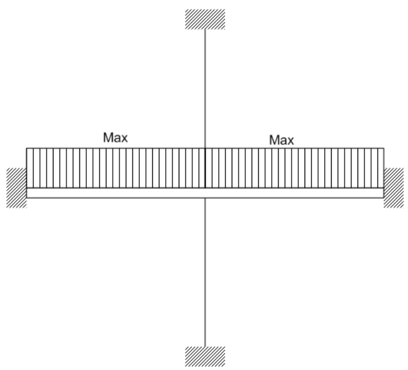
# Action and Response

- For a braced structure, the critical arrangement of the ultimate load is usually that which causes the largest moment in the column together with a larger axial load.
- The critical loading arrangement:




Load arrangement for maximum moment




Load arrangement for maximum axial load

# Action and Response

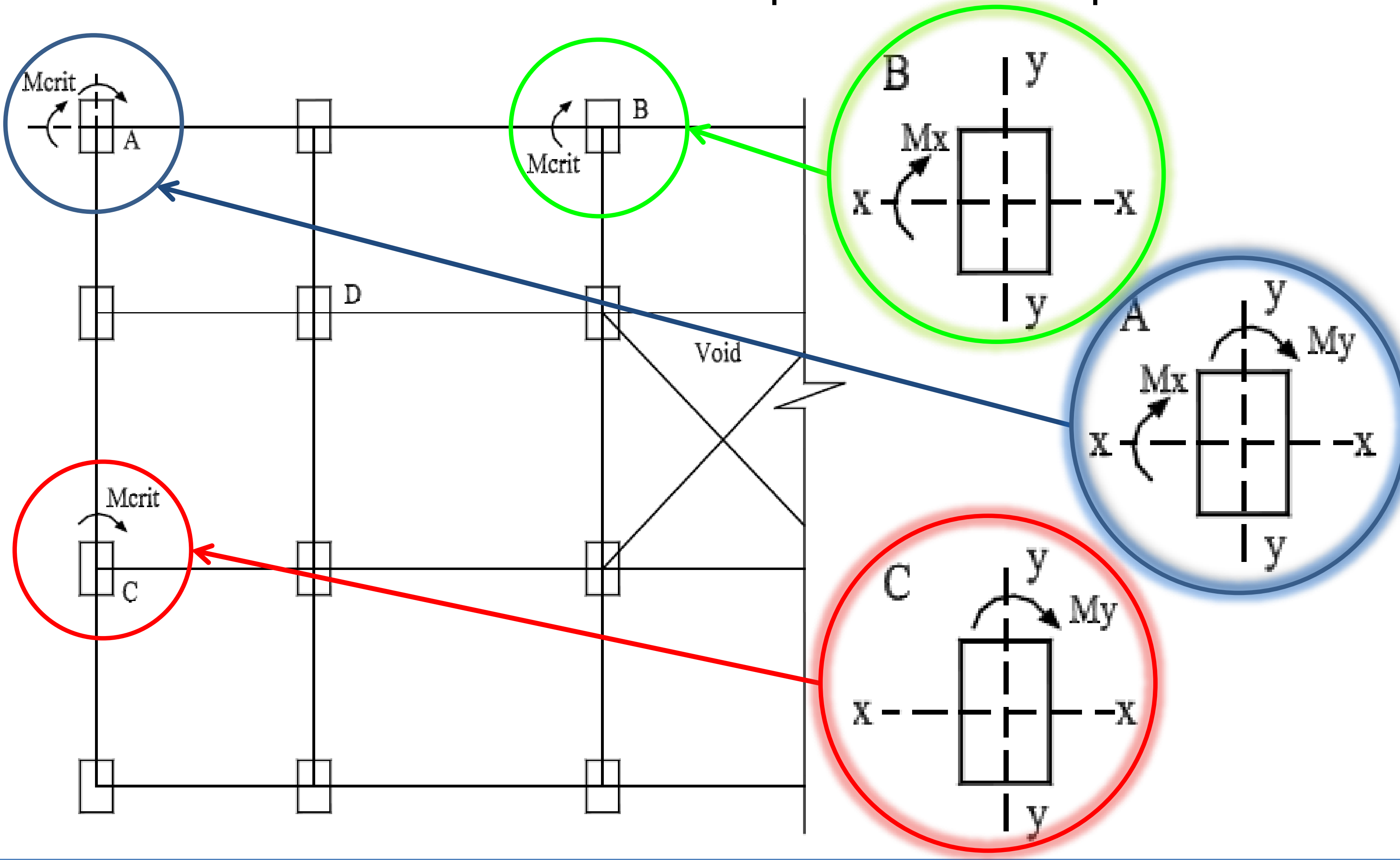
- Bending moment and deflected profile of column

# Design Moment

- In general, the magnitude and direction of moment that react on a column are dependant upon the following factors:
  1. Position of column in building, either located at internal, side or corner.
  2. Type of column, either short or slender.
  3. Shape of column, either square, rectangular or circular.
  4. Arrangement of beam supported by column, either symmetrical or unsymmetrical.
  5. Span of beam at both sides of column.
  6. Difference of load for beam at both sides of column.
- The above mentioned factors determine whether the induced moments are about single axis or double axis and about major or minor axis.
- Moment calculated from frame analysis is not the ultimate value of design moment and must not be used directly in column design.

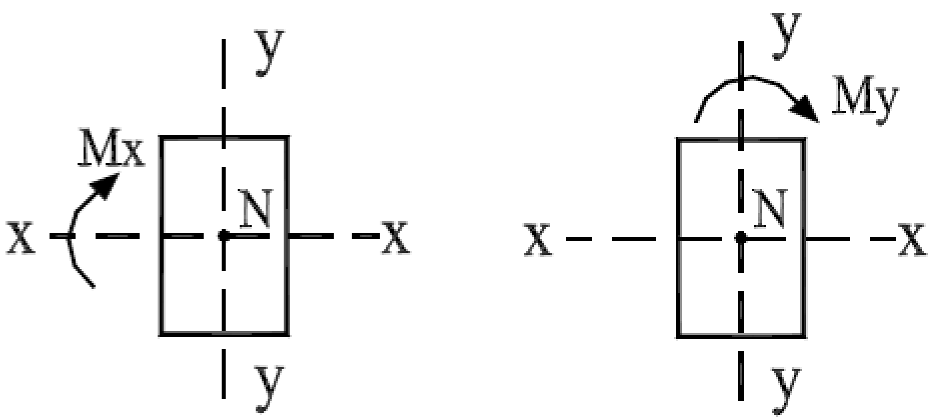
# Design Moment

- Different moment conditions with respect to column positions

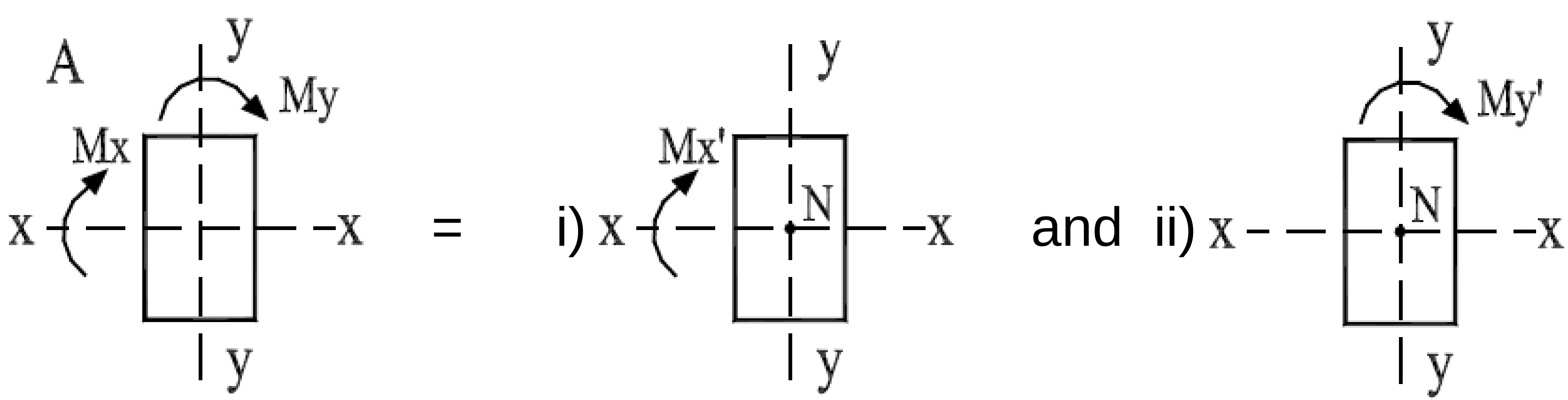


# Design Moment

- Column with bending at one axis




- Column with biaxial bending



# Durability and Fire Resistance

| Standard fire resistance | Minimum dimensions (mm) Column width $b_{min}$/axis distance, a, of the main bars | | |
|---|---|---|---|
| | Column exposed on more than one side | | Column exposed on one side ($\mu_{fi}$ = 0.7) |
| | $\mu_{fi}$ = 0.5 | $\mu_{fi}$ = 0.7 | |
| R 60 | 200/36<br>300/31 | 250/46<br>350/40 | 155/25 |
| R 90 | 300/45<br>400/38[a] | 350/53<br>450/40[a] | 155/25 |
| R 120 | 350/45[a]<br>450/40[a] | 350/57[a]<br>450/51[a] | 175/35 |
| R 240 | 450/75[a] | [b] | 295/70 |

**Key**

a Minimum 8 bars

b Method B may be used which indicates 600/70 for R 240 and $\mu_{fi}$ = 0.7. See BS EN 1992–1–2 Table 5.2b

**Note**

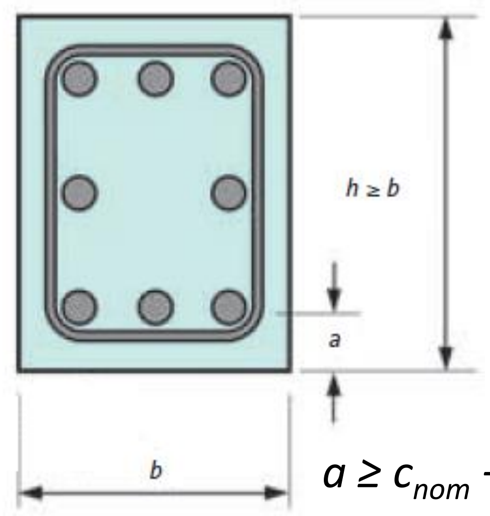
The table is taken from BS EN 1992–1–2 Table 5.2a (method A) and is valid under the following conditions:

1 The effective length of a braced column under fire conditions $l_{o,fi} \leq$ 3m. The value of $l_{o,fi}$ may be taken as 50% of the actual length for intermediate floors and between 50% and 70% of the actual length for the upper floor column.

2 The first order eccentricity under fire conditions should be ≤ 0.15$b$ (or $h$). Alternatively use method B (see Eurocode 2, Part 1–2, Table 5.2b). The eccentricity under fire conditions may be taken as that used in normal temperature design.

3 The reinforcement area outside lap locations does not exceed 4% of the concrete cross section.

4 $\mu_{fi}$ is the ratio of the design axial load under fire conditions to the design resistance of the column at normal temperature conditions. $\mu_{fi}$ may conservatively be taken as 0.7.




$$a \geq c_{nom} + \varnothing_{link} + \varnothing_{bar}/2$$

# Slenderness

- EC2 states that second order effects may be ignored if they are less than 10% of the first order effects.
- As an alternative, if the slenderness ratio ($\lambda$) is less than the slenderness limit ($\lambda_{lim}$), then second order effects can be ignored.
- The slenderness ratio ($\lambda$) of a column bent about an axis is given by Cl.5.8.3.2(1) as:

$$\lambda = \frac{l_0}{i} = \frac{l_0}{\sqrt{I / A}}$$

where:

$l_o$ = effective height of the column

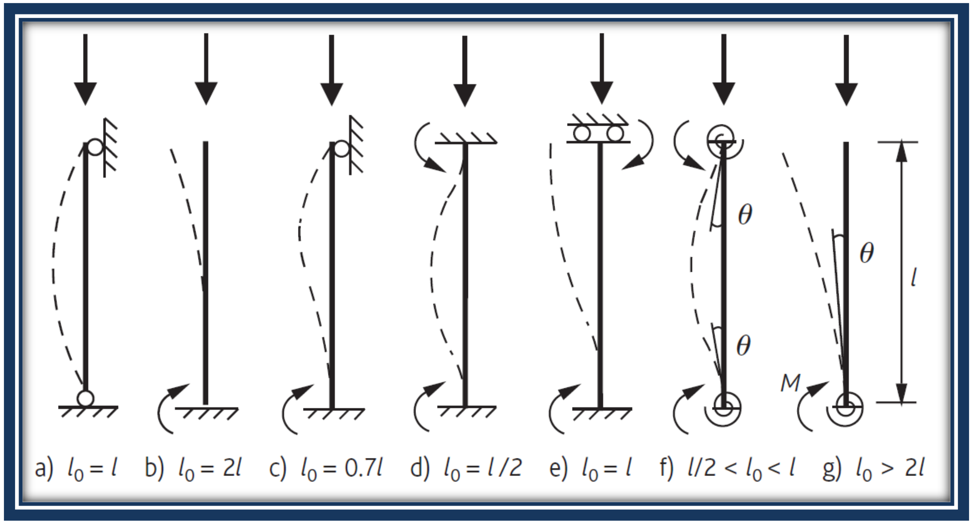
$i$ = radius of gyration about the axis

$I$ = the second moment of area of the section about the axis

$A$ = the cross section area of the column

# Slenderness

- Effective height
  - $l_o$ is the height of a theoretical column of equivalent section but pinned at both ends.
  - This depends on the degree of fixity at each end and of the column.
  - Depends on the relative stiffness of the column and beams connected to either end of the column under consideration




EC2: Cl.5.8.3.2

Different buckling modes and corresponding effective height for isolated column

# Slenderness

- Formula to calculate the effective height (Cl.5.8.3.2)
  - For braced member:

$$l_0 = 0.5l\sqrt{\left(1+\frac{k_1}{0.45+k_1}\right)\left(1+\frac{k_2}{0.45+k_2}\right)}$$

  - For unbraced member (which one maximum)

$$l_0 = l\sqrt{\left(1+10\frac{k_1k_2}{k_1+k_2}\right)} \quad \text{or} \quad l_0 = l\left(1+\frac{k_1}{1+k_1}\right)\left(1+\frac{k_2}{1+k_2}\right)$$

where, $k_1$ and $k_2$ relative flexibility of the rotational restrains at end '1' and '2' of the column respectively.

At each end $k_1$ and $k_2$ can be taken as:

# Slenderness

$$k = \frac{\text{column stiffness}}{\sum \text{beam stiffness}}$$

$$= \frac{(EI/L)_{column}}{\sum 2(EI/L)_{beam}}$$

$$= \frac{(I/L)_{column}}{\sum 2(I/L)_{beam}}$$

For a typical column in a symmetrical frame with span approximately equal length, $k_1$ and $k_2$ can be calculated as:

$$k_1 = k_2 = \frac{(I/L)_{column}}{4(I/L)_{beam}}$$

** It is also generally accepted that Table 3.19 of BS 8110 may conservatively be used to determine the effective length factor.

# Slenderness

## Limiting λ – Short or Slender Column

- Slenderness ratio, $\lambda = \dfrac{l_o}{i}$
- Slenderness limit, $\lambda_{\text{lim}} = \dfrac{20ABC}{\sqrt{n}} \leq \dfrac{15.4C}{\sqrt{n}}$

*Minimum limit

$$\lambda_{\text{lim}} = \frac{26.2}{\sqrt{N_{ED} / (A_c f_{cd})}}$$

where

$A = 1/(1+0.2\varphi_{ef})$ (if $\varphi_{ef}$ is not known, $A$ = 0.7 may be used)

$B = (1+2w)^{1/2}$

$w$ = reinforcement ratio (if $w$ is not known, $B$ = 1.1 may be used)

$C = 1.7 - r_{\text{m}}$ (if $r_{\text{m}}$ is not known, $C$ = 0.7 may be used)

$n = (N_{ED})/(A_c f_{cd})$ ; $f_{cd}$=design compressive strength

$r_{\text{m}} = (M_{01}/ M_{02})$ $M_{01}$, $M_{02}$ are the first order end moments, in which $|M_{02}| \geq |M_{01}|$. If the end moments $M_{01}$ and $M_{02}$ give tension on the same side, $r_m$ should be taken positive.

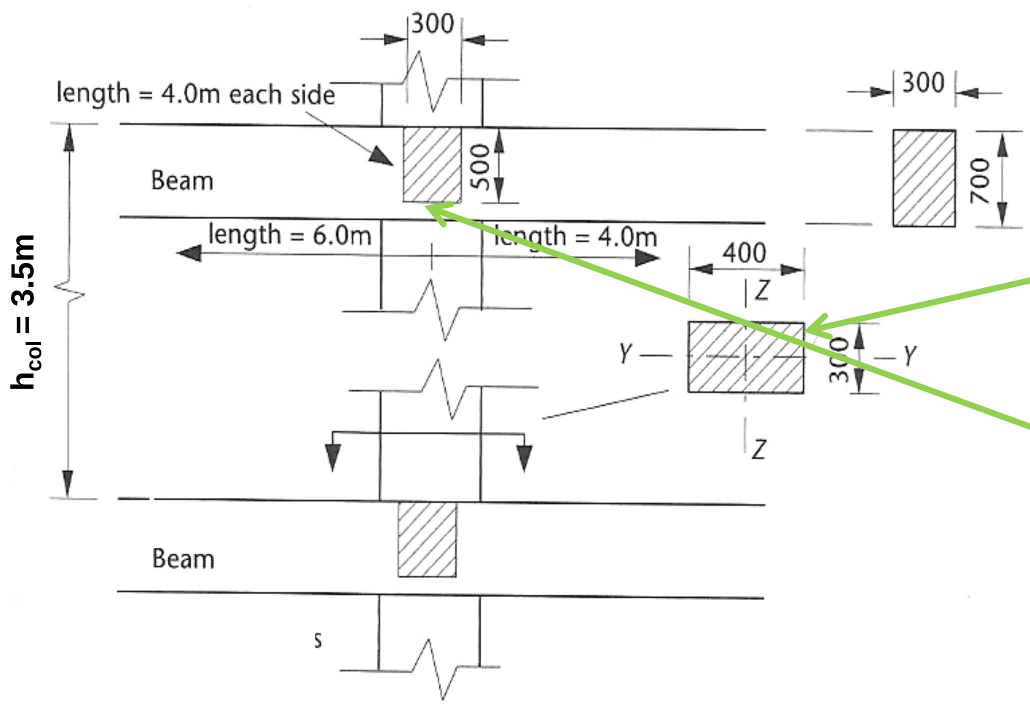
# Slenderness

- Of the three factors *A*, *B* and *C* ;*C* will has the largest impact on $\lambda_{lim}$ and is the simplest to calculate.
- An initial assessment of $\lambda_{lim}$ can therefore be made using the default values for A and B, but including a calculation for C.
- Care should be taken in determining C because the sign of the moments makes a significant difference. For unbraced members C should always be taken as 0.7.
- If the comparison yield the condition:

| | |
|---|---|
| $\lambda \leq \lambda_{lim}$ | Short column and the slenderness effect may be neglected. |
| $\lambda \geq \lambda_{lim}$ | Slender column and must be designed for an additional moment cause by its curvature at ultimate conditions. |

# Example 3.1

Determine if the column in the braced frame shown in the figure below is short or slender. The concrete strength $f_{ck}$ = 25 N/mm$^2$ and the ultimate axial load = 1280kN.




Highest slenderness ratio

- -Column with smallest h (about axis YY, h=300mm)
- -End restraints are less stiff (smallest dimension of beam, 300 x 500)

Note: The beams are continuous in both directions

# Example 3.1

Effective column height, $l_o$

$$I_{col} = \frac{400 \times 300^3}{12} = 900 \times 10^6 \text{ mm}^4$$

$$I_{beam} = \frac{300 \times 500^3}{12} = 3125 \times 10^6 \text{ mm}^4$$

$$k_1 = k_2 = \frac{I_{col} / l_{col}}{\sum(2I_{beam} / l_{beam})} = \frac{900 \times 10^6 / 3 \times 10^3}{2(2 \times 3125 \times 10^6 / 4 \times 10^3)} = 0.096$$

$$l_o = 0.5l\sqrt{\left(1 + \frac{k_1}{0.45 + k_1}\right)\left(1 + \frac{k_2}{0.45 + k_2}\right)} = 0.59 \times 3.0 = 1.77 \text{ mm}$$

Radius of gyration, $i = \sqrt{\frac{I_{col}}{A_{col}}} = \sqrt{\frac{bh^3/12}{bh}} = \frac{h}{3.46} = 86.6 \text{ mm}$

Slenderness ratio, $\lambda = \frac{l_0}{i} = \frac{1.77 \times 10^3}{86.6} = 20.4$

# Example 3.1

For braced column:

$$\lambda_{\text{lim}} = \frac{20ABC}{\sqrt{n}}$$

$$\lambda_{\text{lim}} = \frac{26.2}{\sqrt{N_{ED} / (A_c f_{cd})}}$$

$$f_{cd} = 0.85\left(\frac{f_{ck}}{1.5}\right)$$

$$\sqrt{N_{ED} / (A_c f_{cd})} = \sqrt{1280 \times 10^3 / (400 \times 300 \times 0.85 \times 25 / 1.5)} = 0.866$$

$$\lambda_{\text{lim}} = \frac{26.2}{0.866} = 30.25 > 20.4$$

Hence, compared with $\lambda_{lim}$, the column is short and second order moment effects would not have taken into account.

## Reinforcement Details

- **Longitudinal steel**
  A minimum of four bars is required in the rectangular column (one bar in each corner) and six bars in circular column. Bar diameter should not be less than 12mm.

  The minimum area of steel is given in Cl.9.5.2(2) as:

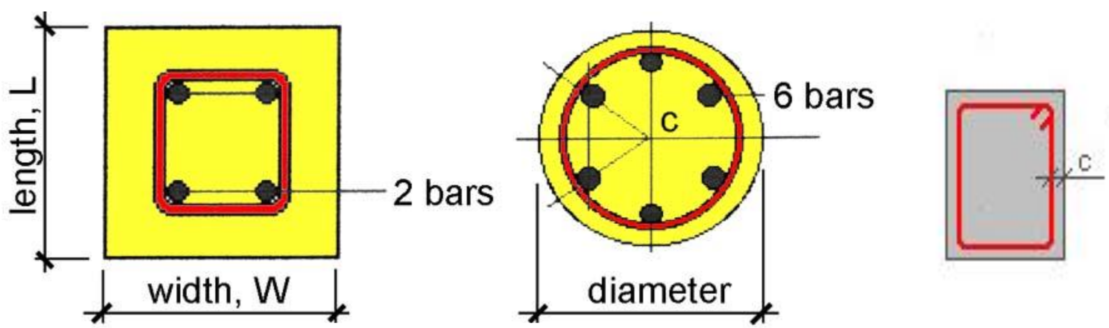
$$A_{s,\min} = \frac{0.10 N_{Ed}}{0.87 f_{yk}} \geq 0.002 A_c$$

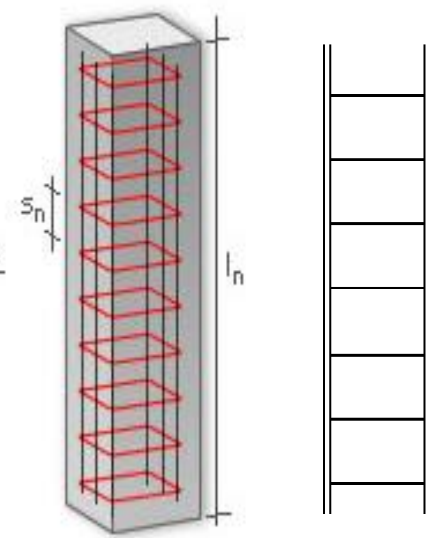
- **Links**
  The diameter of the transverse reinforcement should not be less than 6mm or one quarter of the maximum diameter of the longitudinal bars.

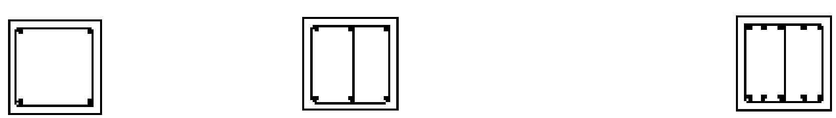
$$\phi_{link} = \max\{0.25\phi_{main}; 6\}$$

# Principle of Design

- **Spacing requirements**
  The maximum spacing of transverse reinforcement (i.e. links) in columns (Cl.9.5.3(1)) should not generally exceed:
  - ✓ 20 times the minimum diameter of the longitudinal bars.
  - ✓ the lesser dimension of the column.
  - ✓ 400 mm.



Normal    Spacing > 150mm    Spacing < 150mm

## Design Moment

- For braced slender column, the design bending moment is defined as given in Cl.5.8.8.2:

$$M_{\mathrm{Ed}} = \max \{M_{02} \; ; \; M_{0e} + M_2 \; ; \; M_{01} + 0.5\, M_2 \; ; \; N_{Ed}.e_0\}$$

- For unbraced slender column:

$$M_{\mathrm{Ed}} = \max \{M_{02} + M_2 \; ; \; N_{Ed}.e_0\}$$

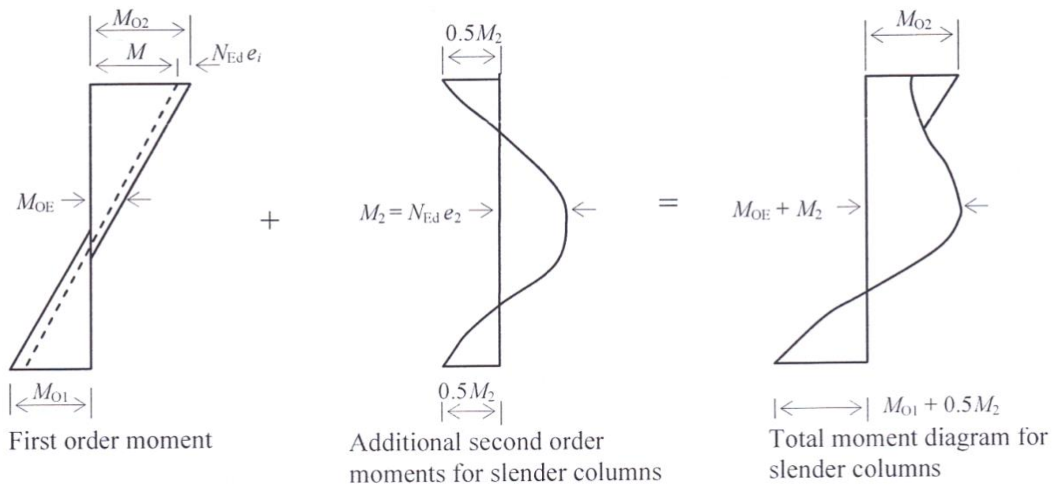
where:

$M_{01}$ = min $\{M_{\mathrm{top}} \; ; \; M_{\mathrm{bot}}\} + N_{Ed}.e_i$
$M_{02}$ = max $\{M_{\mathrm{top}} \; ; \; M_{bot}\} + N_{ed}.e_i$
$e_0$ = max {h/30; 20 mm}
$e_{\mathrm{i}}$ = $l_{\mathrm{o}}/400$
$M_{\mathrm{top}}$ = Moment at the top of the column
$M_{\mathrm{bot}}$ = Moment at the bottom of the column

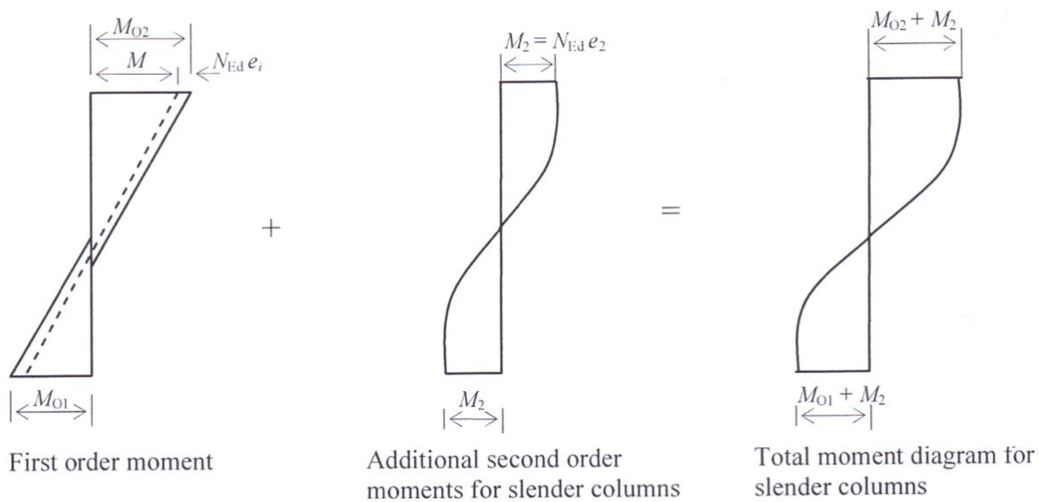
# Principle of Design

- Moment diagram (first and second orders):

Braced column




Unbraced column



# Principle of Design

$M_{0e} = 0.6\ M_{02} + 0.4\ M_{01} \geq 0.4\ M_{02}$

$M_{01}$ and $M_{02}$ should be positive if they give tension on the same side.

$M_2 = N_{Ed} \times e_2$ = The nominal second order moment

where:

$N_{Ed}$ = the design axial load

$e_2$ = Deflection due to second order effects

$$= \frac{1}{r}\left(\frac{l_0^{\,2}}{c}\right)$$

$l_o$ = effective length

$c$ = a factor depending on the curvature distribution, normally $\pi^2 \approx 10$

$1/r$ = the curvature = $Kr.K\varphi.1/r_0$

Phi (uppercase Φ, lowercase φ, or math symbol ϕ

# Principle of Design

Axial load correction factor, $Kr = (n_u - n) / (n_u - n_{bal}) < 1$

where, $n = N_{Ed} / \left( A_c f_{cd} \right)$, $n_u = 1 + w, n_{bal} = 0.4$

$$w = A_s f_{yd} / ( A_c f_{cd} )$$

Creep correction factor, $K\varphi = 1 + \beta\varphi_{ef} \geq 1$

where: $\varphi_{ef}$ = effective creep ratio = $jM_{0Eqp} / M_{0Ed}$

= 0, if ($\varphi$ < 2, M/N > h, $1/r_0$ < 75)

$\beta = 0.35 + f_{ck}/200 - \lambda/150$

$1/r_0 = \varepsilon_{yd} / (0.45d) = \left( f_{yd} / E_s \right) / 0.45d$

A non-slender column can be designed ignoring second order effects and therefore the ultimate design moment,

**$M_{Ed} = M_{02}$** ← **for short column**

## Method of Design

- Short column resisting moments and axial forces
- The area of longitudinal reinforcement is determined based on:

| | | |
|---|---|---|
| 1. | Design chart or construction M-N interaction diagram | rectangular or circular section and symmetrical arrangement of reinforcement (d'/h = 0.05-0.25 as provided by EC2) |
| 2. | A solution a basic design equation | Unsymmetrical arrangement of reinforcement, or cross section is non rectangular |
| 3. | An approximate method | |

A column should not be designed for a moment less than $N_{Ed}$ x $e_{min}$ where $e_{min}$ has a grater value of h/300 or 20 mm

# Design Chart

- Design chart for rectangular column

$d_2/h=0.05$

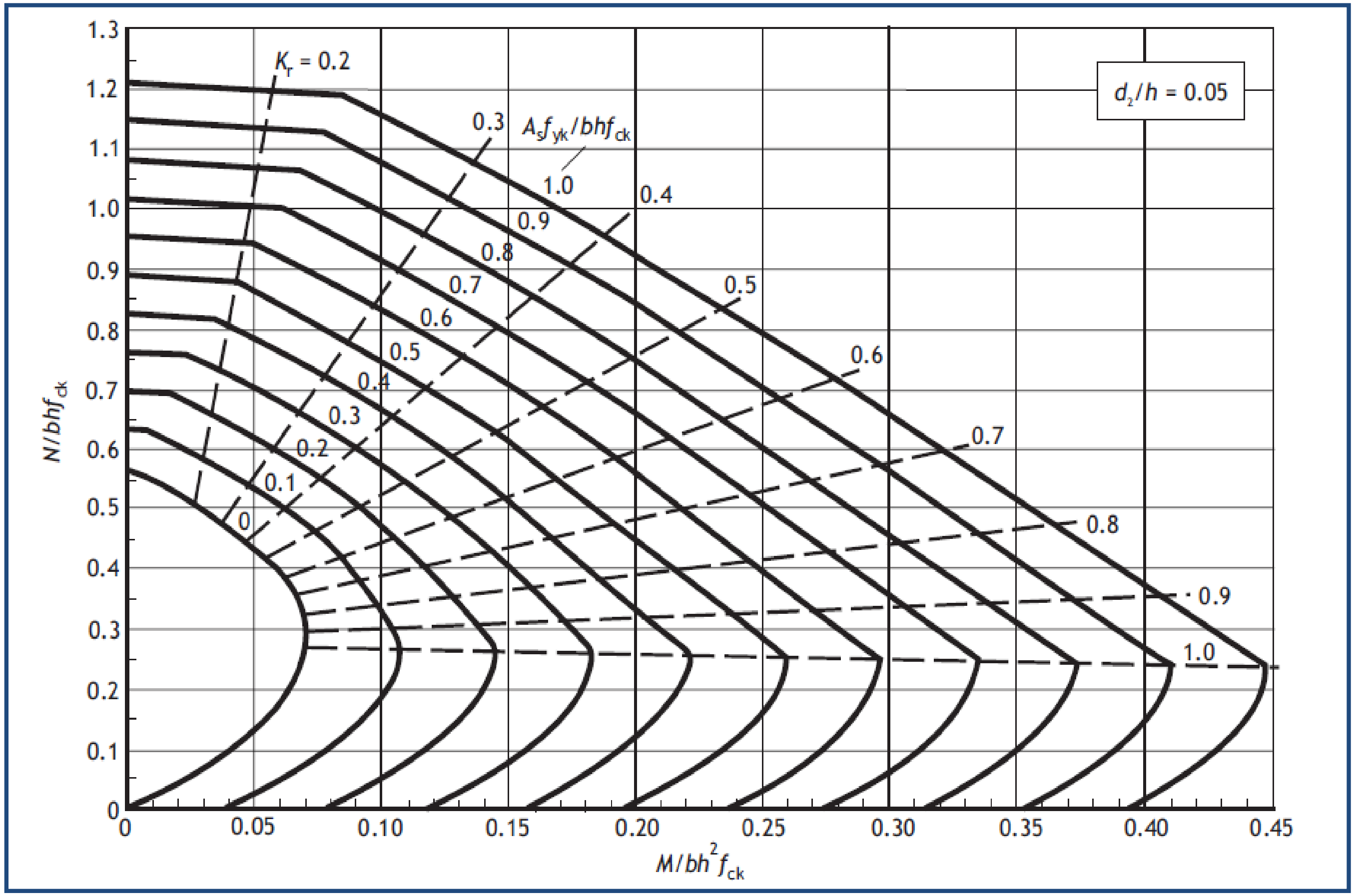
$d_2/h=0.10$

$d_2/h=0.15$

$d_2/h=0.20$

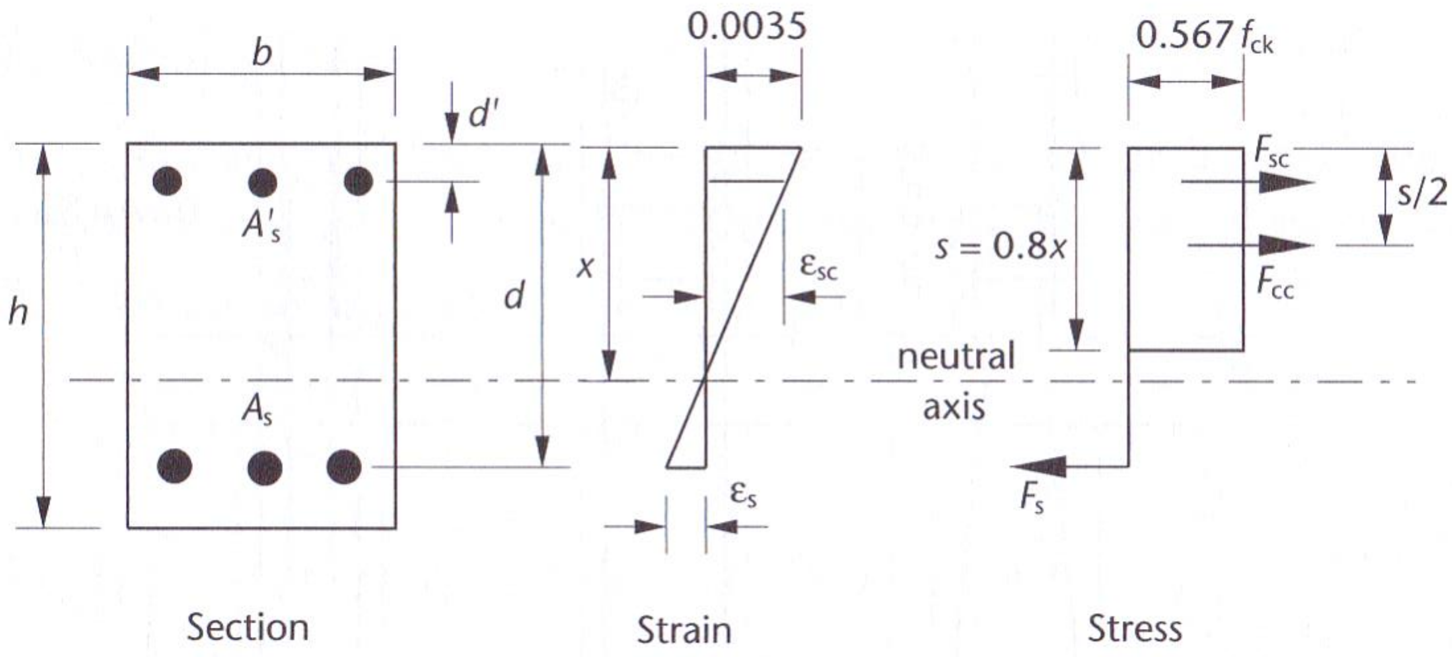
$d_2/h=0.25$

** $d_2$ or d'



# Principle of Design

- Two expressions can be derived for the area of steel required (based on a rectangular stress block), one for the axial loads and the other for the moments.
- Interaction diagrams can be constructed for any arrangement of cross section.



# Principle of Design

- Basic equation

$$N_{Ed} = F_{cc} + F_{sc} + F_s = 0.567 f_{ck} bs + f_{sc} A'_s + f_s A_s$$

$$M_{Ed} = F_{cc}\left(\frac{h}{2} - \frac{s}{2}\right) + F_{sc}\left(\frac{h}{2} - d'\right) - F_s\left(d - \frac{h}{2}\right)$$

where;

$N_{Ed}$ = design ultimate axial load
$M_{Ed}$ = design ultimate moment
$s$ = the depth of the stress block = 0.8$x$
$A'_s$ = the area of longitudinal reinforcement in the more highly compressed face
$A_s$ = the area of reinforcement in the other face
$f_{sc}$ = the stress in reinforcement $A'_s$
$f_s$ = the stress in reinforcement $A_s$, negative when tensile

## Basic Equation

- Area of reinforcement required to resist axial load:

$$A_{sN}/2 = \left[\left(N_{Ed} - f_{cd} b d_c\right)\right] / \left[\left(\sigma_{sc} - \sigma_{st}\right)\gamma_c\right]$$

- While the total are of reinforcement required to resist moment:

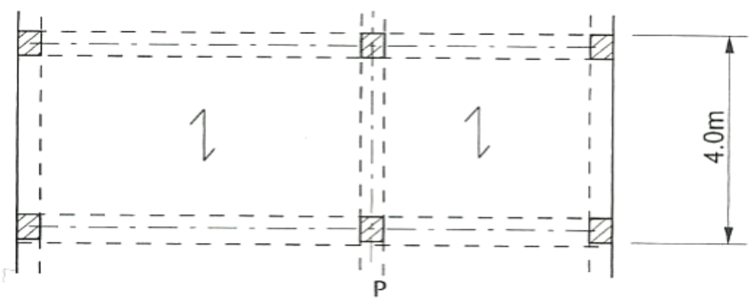
$$A_{sM}/2 = \left[M - f_{cd} b d_c \left(h/2 - d_c/2\right)\right] / \left[\left(h/2 - d_2\right)\left(\sigma_{sc} + \sigma_{st}\right)\gamma_c\right]$$

where;
$N_{Ed}$ = Axial load
$f_{cd}$ = Design value of concrete compressive strength
$\sigma_{sc}(\sigma_{st})$ = Stress in compression (and tension) reinforcement
$b$ = Breadth of section; $h$ = Height of section
$\gamma_c$ = Partial factor for concrete (1.5)
$d_c$ = Effective depth of concrete in compression = $\lambda x \leq h$
$\lambda$ = 0.8 for ≤ C50/60
$x$ = Depth to neutral axis

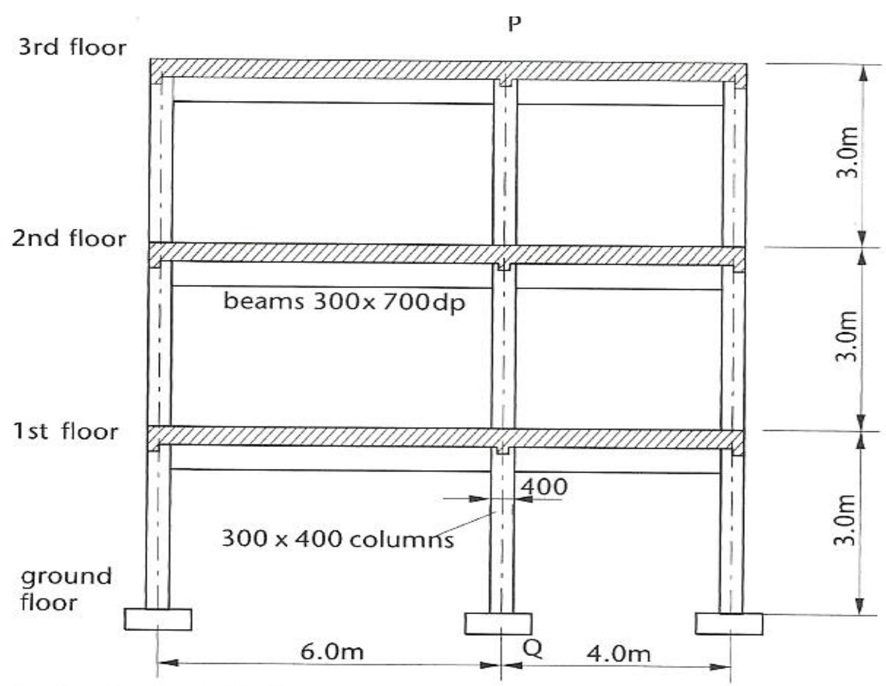
# Example 3.2

Figure 3.2 shows a frame of heavily loaded industrial structure for which the centre column along line PQ are to be designed in this example. The frame at 4m centres are braced against lateral forces and support the following floor loads:

$g_k$ = 10 kN/m$^2$
$q_k$ = 15 kN/m$^2$
$f_{ck}$ = 25 N/mm$^2$
$f_{yk}$ = 500 N/mm$^2$




Plan




Section through the frame

# Example 3.2

Maximum ultimate load at each floor:
$= 4.0\ (1.35g_k + 1.5q_k)$ per meter length of beam
$= 4.0\ (1.35 \times 10 + 1.5 \times 15)$
$= 144$ kN/m

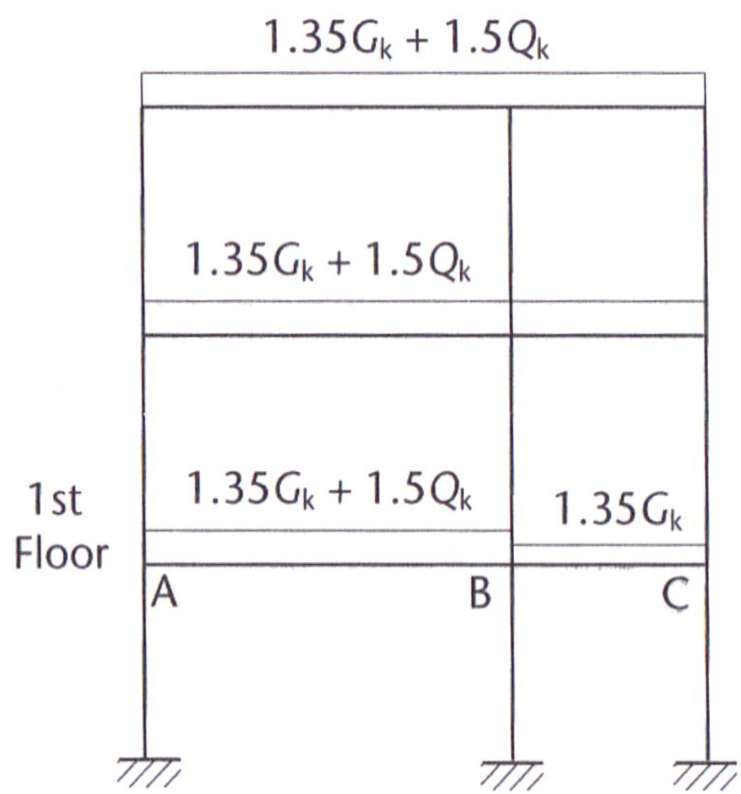
Minimum ultimate load at each floor:
$= 4.0 \times 1.35g_k$
$= 4.0 \times (1.35 \times 10)$
$= 54$ kN per meter length of beam

Column load:

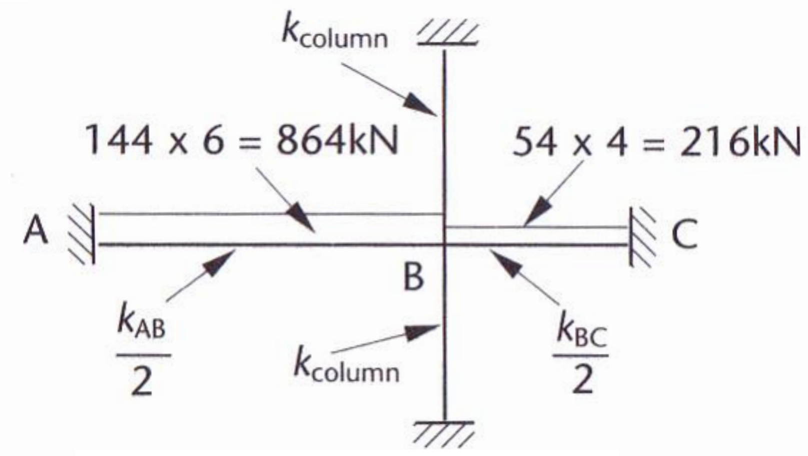
| | | |
|---|---|---|
| 1st floor | $= 144 \times 6/2 + 54 \times 4/2$ | $= 540$ kN |
| 2nd and 3rd floor | $= 2 \times 144 \times 10/2$ | $= 1440$ kN |
| Column self weight | $= 2 \times 14$ | $= 28$ kN |
| | $N_{Ed}$ | **= 2008 kN** |

Selfweight of column
=
(0.3x0.4 x3.0x24) x 3
= 26 kN

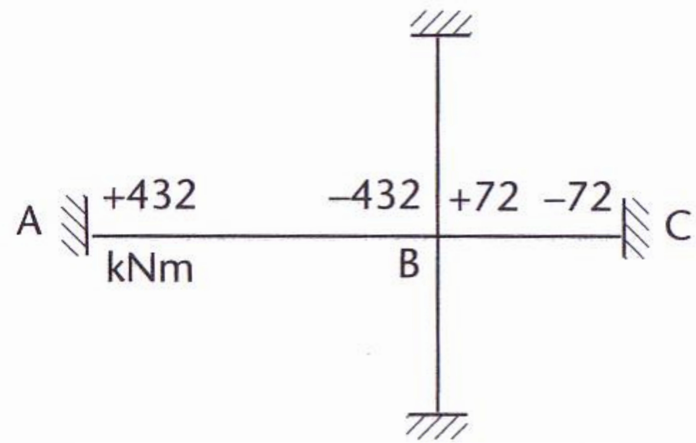
# Example 3.2




a) Critical loading arrangement for centre columns at 1st floor




b) Substitute frame




c) Fixed end moments

# Example 3.2

Member stiffness are

$$\frac{k_{AB}}{2} = \frac{1}{2} \times \frac{bh^3}{12L_{AB}} = \frac{1}{2} \times \frac{0.3 \times 0.7^3}{12 \times 6} = 0.71 \times 10^{-3}$$

$$\frac{k_{BC}}{2} = \frac{1}{2} \times \frac{0.3 \times 0.7^3}{12 \times 4} = 1.07 \times 10^{-3}$$

$$k_{col} = \frac{0.3 \times 0.4^3}{12 \times 3.0} = 0.53 \times 10^{-3}$$

Therefore

$$\sum k = (0.71 + 1.07 + 2 \times 0.53)10^{-3}$$

Distribution factor for the column

$$DF_{col} = \frac{k_{col}}{\sum k} = \frac{0.53}{2.84} = 0.19$$

# Example 3.2

Fixed end moment at B

$$FEM_{BC} = \frac{144 \times 6^2}{12} = 432\text{kNm}$$

$$FEM_{CB} = \frac{54 \times 4^2}{12} = 72\text{kNm}$$

Thus,

Column moment

$$M = 0.19(432 - 72) = 68.4\text{kNm}$$

Design moment allowing for geometric imperfections

$$M_{Ed} = M + \frac{N_{Ed} l_o}{400}$$

# Example 3.2

For underside of 1st floor

$$M_{Ed} = 68.4 + \frac{2008 \times 2.34}{400} = 68.4 + 11.75 = 80.15\text{kNm}$$

For topside of 1st floor

$$M_{Ed} = 68.4 + \frac{1468 \times 1.8}{400} = 68.4 + 6.61 = 75.01\text{kNm}$$

The minimum moment in both cases is $N_{Ed} \times e_{min}$ where $e_{min}$ = 20mm (>h/30=400/30=13.3mm) which in neither case is critical.

# Example 3.2

At the 3rd floor

$$\sum k = (0.71 + 1.07 + 0.53)10^{-3} = 2.31 \times 10^{-3}$$

and Column moment

$$M = \frac{0.53}{2.31}(432 - 72) = 82.6\text{kNm}$$

$$M_{Ed} = M + \frac{N_{Ed} l_o}{400}$$

$$M_{Ed} = 82.6 + \frac{734 \times 1.8}{400} = 82.6 + 3.30 = 85.90\text{kNm}$$

$$M_{Ed} > N_{Ed} \times e_{min}$$

# Example 3.2

Using the design chart:

Assume cover = 50mm

d'/h= 80/400 = 0.2

Ground to 1st floor:

$N_{Ed}/bhf_{ck}$= 0.67 , $M_{Ed}/bh^2f_{ck}$= 0.067 → $A_sf_{yk}/bhf_{ck}$ = 0.3

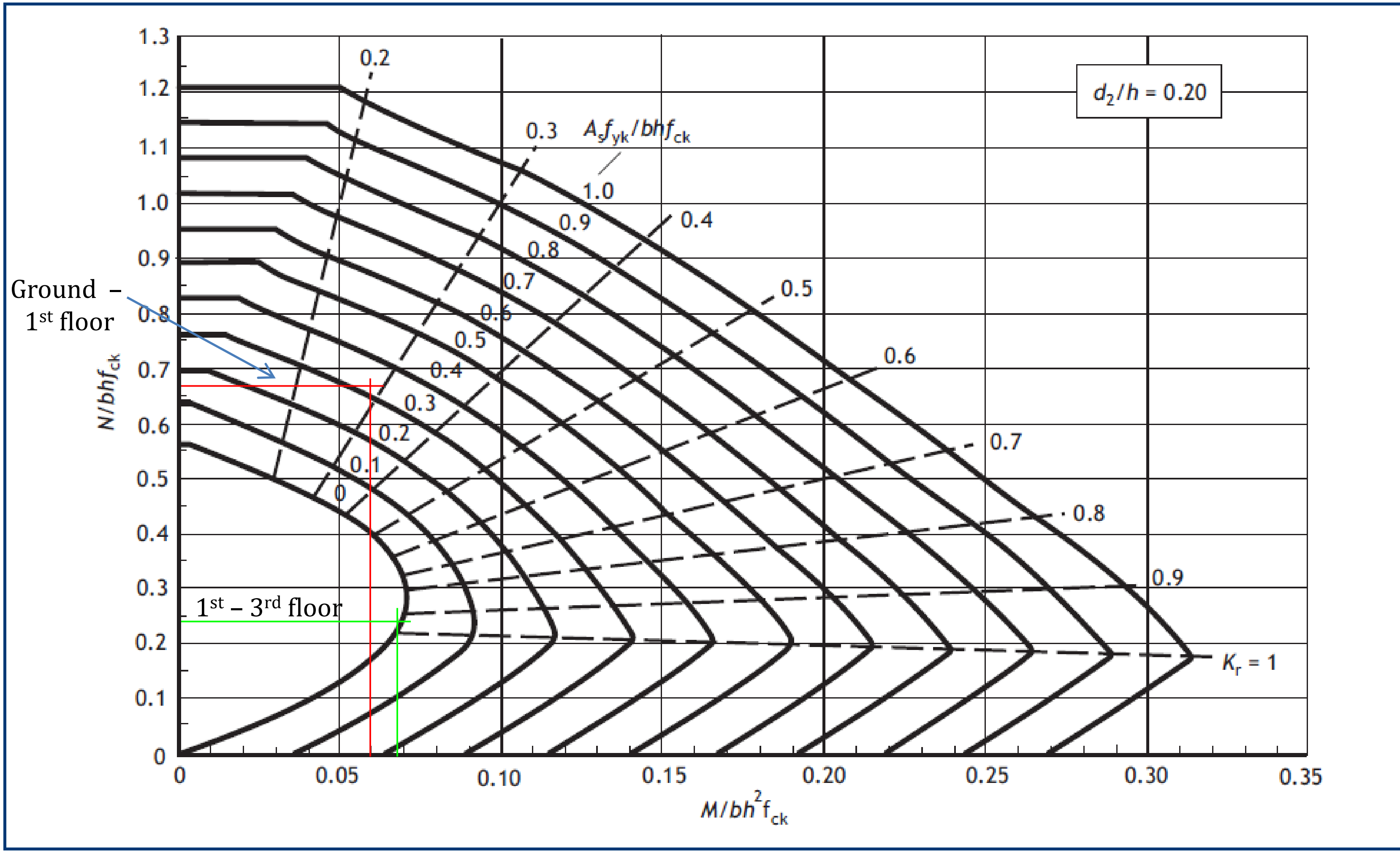
→ $A_s$= 1800mm²

1st to 3rd floor:

$N_{Ed}/bhf_{ck}$= 0.24 , $M_{Ed}/bh^2f_{ck}$= 0.07 → $A_sf_{yk}/bhf_{ck}$ = 0.0

→ $A_{s,min}$= 240mm²

# Example 3.2


$d_2/h = 0.20$
$A_s f_{yk}/bhf_{ck}$
$N/bhf_{ck}$
$M/bh^2 f_{ck}$
$K_r = 1$
Ground – 1st floor
1st – 3rd floor
0
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3

# Example 3.2

The min. area allowed:

$A_{s,min} = 0.002bh$

$A_{s,min} = 0.002 \times 300 \times 400$

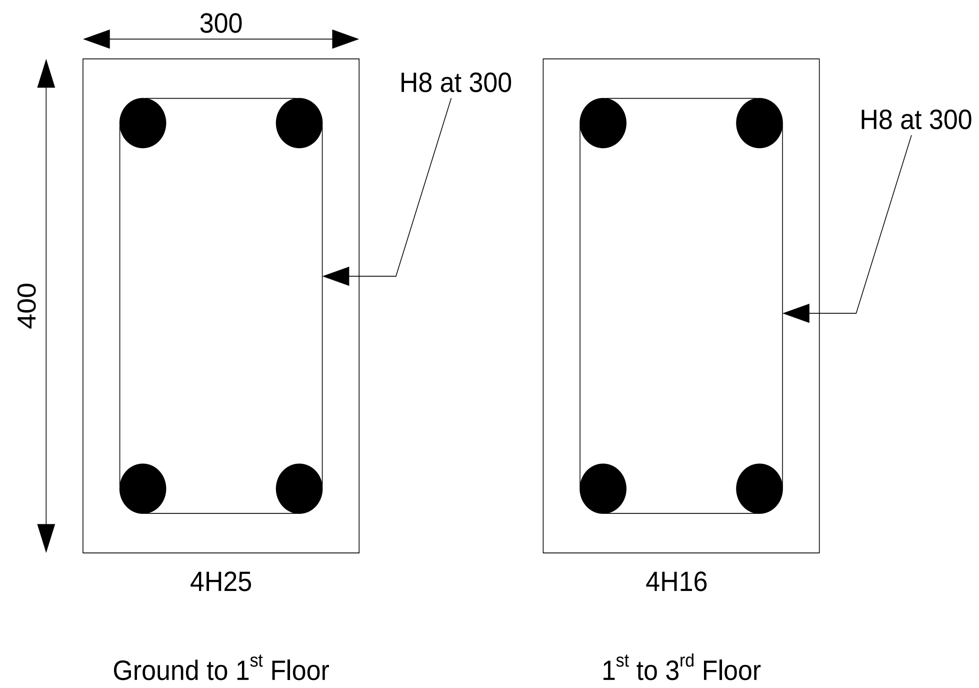
$A_{s,min} = 240mm^2$

The max. area:

$A_{s,max} = 0.08bh$

$A_{s,max} = 0.08 \times 300 \times 400$

$A_{s,max} = 9600mm^2$




Although EC2 permits the use of 12mm main steel, 16mm bars have been used to ensure adequate rigidity of the reinforcing cage.

# Example 3.2

| Floor | $N_{Ed}$ (kN) | $M_{Ed}$ (kNm) | $\frac{N_{Ed}}{bhf_{ck}}$ | $\frac{M_{Ed}}{bh^2f_{ck}}$ | $\frac{A_sf_{yk}}{bhf_{ck}}$ | As (mm²) |
|---|---|---|---|---|---|---|
| 3rd u.s | 540 | 82.6 | 0.18 | 0.07 | 0 | 240 |
| 2nd t.s | 734 | 68.4 | 0.24 | 0.06 | 0 | 240 |
| | + 540 | | | | | |
| 2nd u.s | 1274 | 68.4 | 0.42 | 0.06 | 0 | 240 |
| 1st t.s | 1468 | 68.4 | 0.49 | 0.06 | 0.10 | 600 |
| | + 540 | | | | | |
| 1st u.s | 2008 | 68.4 | 0.67 | 0.06 | 0.30 | 1800 |

u.s – under side
t.s – top side

1st floor = 144 x 6/2 + 54 x 4/2 = 540 kN
2nd and 3rd floor = 2 x 144 x 10/2 = 1440 kN
Column self weight = 2 x 14 = 28 kN

**Total = 2008 kN**

# Design Under Biaxial Bending

- The effects of biaxial bending may be checked using Eq.(5.39), Cl.5.8.9, which was first developed by Breslaer:

$$\left(\frac{M_{Edz}}{M_{Rdz}}\right)^a + \left(\frac{M_{Edy}}{M_{Rdy}}\right)^a \leq 1.0$$

where;

$M_{Edz,y}$ = Design moment in the respective direction including second order effects in a slender column

$M_{Rdz,y}$ = Moment of resistance in the respective direction

$a$ = 2 for circular and elliptical sections; refer to Table 1 for rectangular sections

$N_{Rd}$ = $A_c f_{cd} + A_s f_{yd}$

| $N_{Ed}/N_{Rd}$ | 0.1 | 0.7 | 1.0 |
|---|---|---|---|
| a | 1.0 | 1.5 | 2.0 |

Note
Linear interpolation may be used.

Value of *a* for rectangular sections

# Design Under Biaxial Bending

Either $\frac{e_z}{b} / \frac{e_y}{h} \leq 0.2$ or $\frac{e_y}{h} / \frac{e_z}{b} \leq 0.2$

Must bigger than 0.2 for biaxial moments

where $e_y$ and $e_z$ are the first-order eccentricities in the direction of the section dimensions b and h respectively.

If $\frac{M_z}{h'} \geq \frac{M_y}{b'}$, then the increased single axis design moment is

$$M'_z = M_z + \beta \frac{h'}{b'} M_y$$

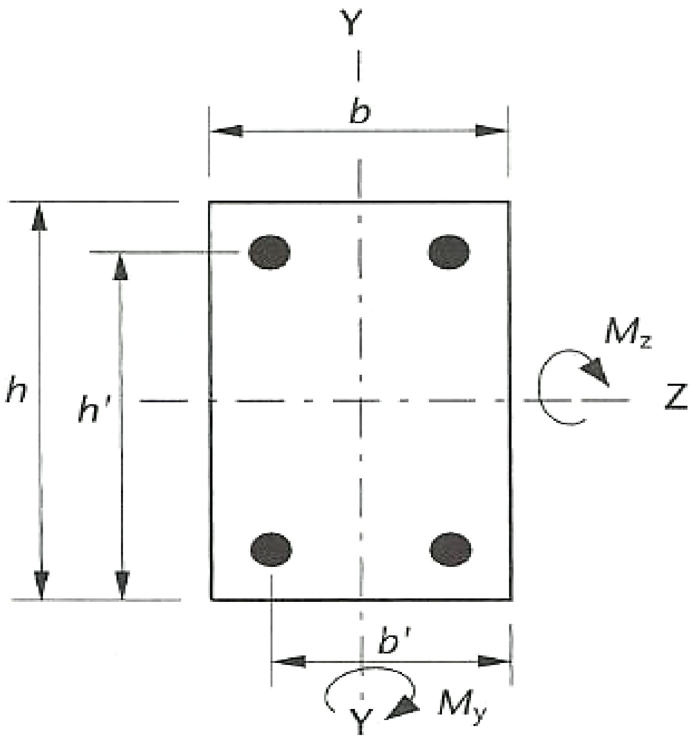
If $\frac{M_z}{h'} < \frac{M_y}{b'}$, then the increased single axis design moment is

$$M'_y = M_y + \beta \frac{h'}{b'} M_z$$

# Design Under Biaxial Bending

- The dimension *h'* and *b'* are defined in the figure below and the coefficient $\beta$ is specified as:

$$\beta = 1 - \frac{N_{Ed}}{bhf_{ck}}$$




Section with biaxial bending

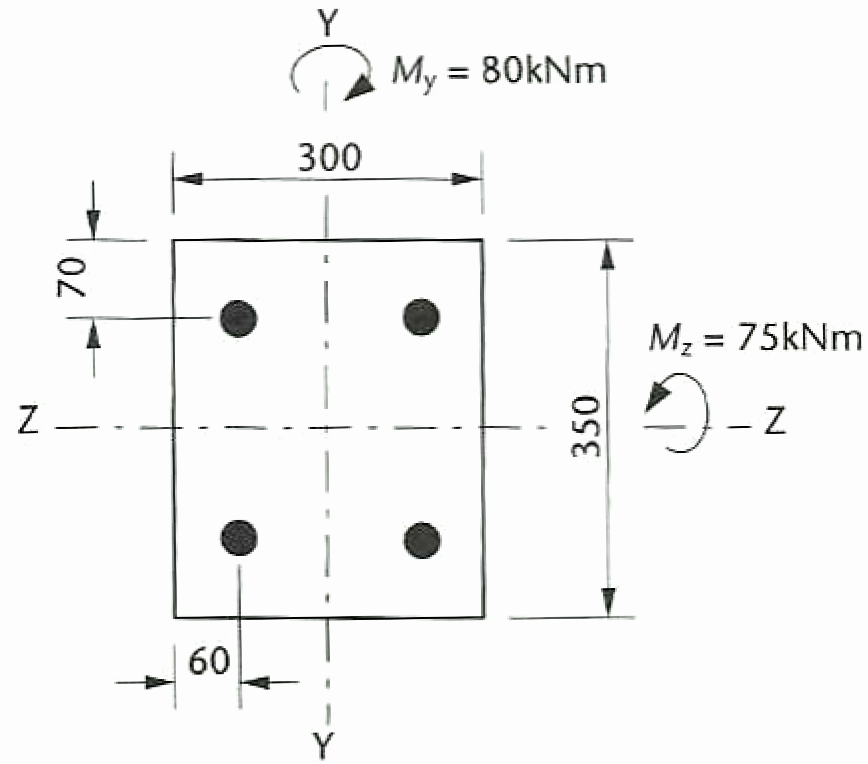
- Value coefficient $\beta$ for biaxial bending

| $\frac{N_{Ed}}{bhf_{ck}}$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | ≥0.7 |
|---|---|---|---|---|---|---|---|---|
| $\beta$ | 1.0 | 0.9 | 0.8 | 0.7 | 0.6 | 0.5 | 0.4 | 0.3 |

# Example 3.3

Design of column for biaxial bending. The column section shown in the figure below is to be designed to resist an ultimate axial load plus moments:

- ultimate axial load = 1200kN
- moment of $M_z$ = 75kNm
- moment of $M_y$ = 80kNm.




The characteristic strength:

- $f_{ck} = 25N/mm^2$
- $f_{yk} = 500N/mm^2$

# Example 3.3

$$e_z = \frac{M_z}{N_{Ed}} = \frac{75 \times 10^6}{1200 \times 10^3} = 62.5mm$$

$$e_y = \frac{M_y}{N_{Ed}} = \frac{80 \times 10^6}{1200 \times 10^3} = 66.7mm$$

Thus,

$$\frac{e_z}{h} \Big/ \frac{e_y}{b} = \frac{62.5}{350} \Big/ \frac{66.7}{300} = 0.8 > 0.2$$

and

$$\frac{e_y}{b} \Big/ \frac{e_z}{h} = \frac{66.7}{300} \Big/ \frac{62.5}{350} = 1.24 > 0.2$$

Hence the column must be designed for biaxial bending

# Example 3.3

$$\frac{M_z}{h'} = \frac{75}{(350 - 70)} = 0.268$$

$$\frac{M_y}{b'} = \frac{80}{(300 - 60)} = 0.333$$

$$\frac{M_z}{h'} < \frac{M_y}{b'}$$

Therefore the increased single axis design moment is

$$M_y' = M_y + \beta \frac{b'}{h'} \times M_z$$

$$\frac{N_{Ed}}{bhf_{ck}} = \frac{1200 \times 10^3}{300 \times 350 \times 25} = 0.46$$

# Example 3.3

From table, $\beta = 0.54$

| $\frac{N_{Ed}}{bhf_{ck}}$ | 0 | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | ≥0.7 |
|---|---|---|---|---|---|---|---|---|
| $\beta$ | 1.0 | 0.9 | 0.8 | 0.7 | 0.6 | 0.5 | 0.4 | 0.3 |

$$M'_y = 80 + 0.54\left(\frac{240}{280}\right) \times 75 = 114.7kN$$

Thus

$$\frac{M_{Ed}}{bh^2 f_{ck}} = \frac{114.7 \times 10^6}{350 \times (300)^2 \times 25} = 0.15$$

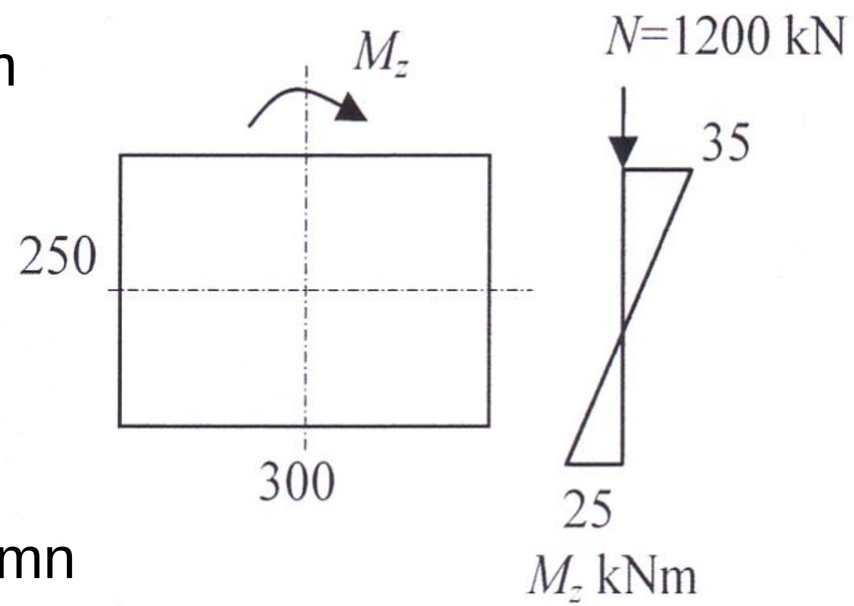
$$\frac{d'}{h} = \frac{70}{350} = 0.2 \Rightarrow \frac{A_s f_{yk}}{bhf_{ck}} = 0.47$$

Therefore required $A_s$=2467mm$^2$. So, provide 4H32

# Example 3.4

Design the longitudinal and transverse reinforcement for the column shown in the figure below. The column is subjected to 50 years working life, 1 hour fire resistance and build inside building. Use grade C25/30 concrete, and grade 500 steel reinforcement.

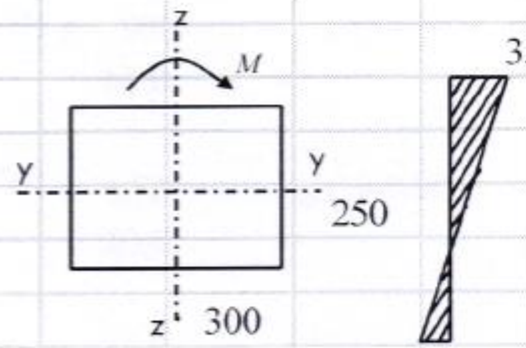
- Effective height, $l_o$ = 3.8m




- Braced non-slender column
- Short column bend about major axis

# Example 3.4

| Ref. | Calculation | Output |
|---|---|---|
| | **SPECIFICATION** Classification: Braced non-slender column | |
| | Material: Axial force, $N_{Ed}$ = | 1200 kN |
| | Concrete, $f_{ck}$ = 25 $N/mm^2$ | |
| | Reinforcement, $f_{yk}$ = 500 $N/mm^2$ | |
| | Exposure class XC1 | |
| | Fire resistance 1.0 hours | |
| | Design life 50 years | |
| | Size, $b \times h$ = 250 x 300 mm | |
| | Effective length, $l_o$ = 4.2 m | |
| | Assumed : $\phi_{link}$ = 6 mm | |
| | $\phi_{bar}$ = 20 mm | |




| Ref. | Calculation | Output |
|---|---|---|
| | **DURABILITY, BOND & FIRE RESISTANCE** | |
| Table 4.2 | Min. cover with regard to bond, $C_{min,b}$ = 20 mm | |
| Table 4.4 | Min. cover with regard to durability, $C_{min,dur}$ = 15 mm | |
| | Min. required axis distance for R60 fire resistance | EN 1992-1-2 |
| Table 5.2a. | $a_{sd}$ = 36 mm | |
| | Min. concrete cover with regard to fire, | |
| | $C_{min} = a_{sd} - \phi_{link} - \phi_{bar}/2$ = 36 - 6 - 20/2= 20.0 mm | |
| | Allowance in design for deviation, $\Delta C_{dev}$ = 10 mm | |
| 4.4.1.1(2) | Nominal cover, | Use: |
| | $C_{nom} = C_{min} + \Delta C_{dev}$ = 20 + 10 = 30 mm | $C_{nom}$ = 30 mm |

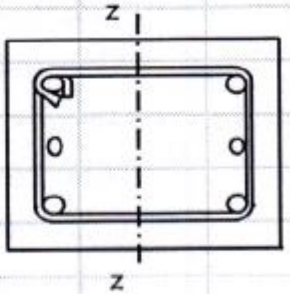
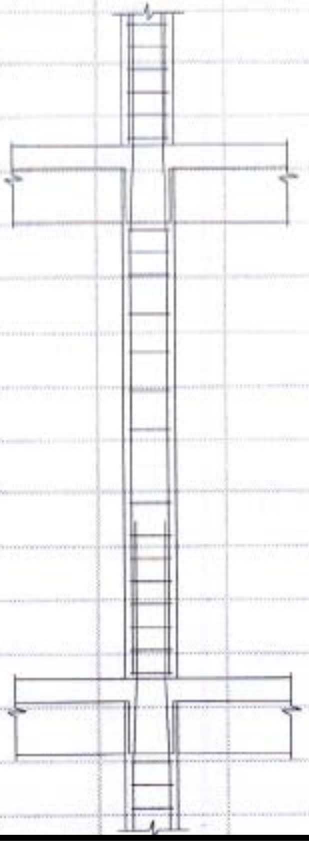
# Example 3.4

| | |
|---|---|
| 5.8.8.2 | **DESIGN MOMENT** |
| | For non-slender column the design moment, |
| | $M_{Ed} = \text{Max}\{M_{o2}, M_{min}\}$ |
| | where |
| | $M_{o2} = M + N_{Ed}.e_i$ |
| | $M = \text{Max}\{M_{bot}, M_{top}\} = 35.0$ kNm |
| 5.2(7) | $e_i = (l_o/400) = 4200 / 400 = 10.5$ mm |
| | $M_{o2} = 35.0 + (1200 \text{x } 0.0105) = 47.6$ kNm |
| | |
| 6.1(4) | $M_{min} = N_{Ed}.e_o$ |
| | $e_o = h/30 \geq 20$ |
| | $= 300 / 30 = 10 \text{ mm} \geq 20$ mm |
| | $M_{min} = 1200 \text{ x } 0.020 = 24.0$ kNm |
| | |
| | ==> $M_{Ed} = 47.6$ kNm |

# Example 3.4

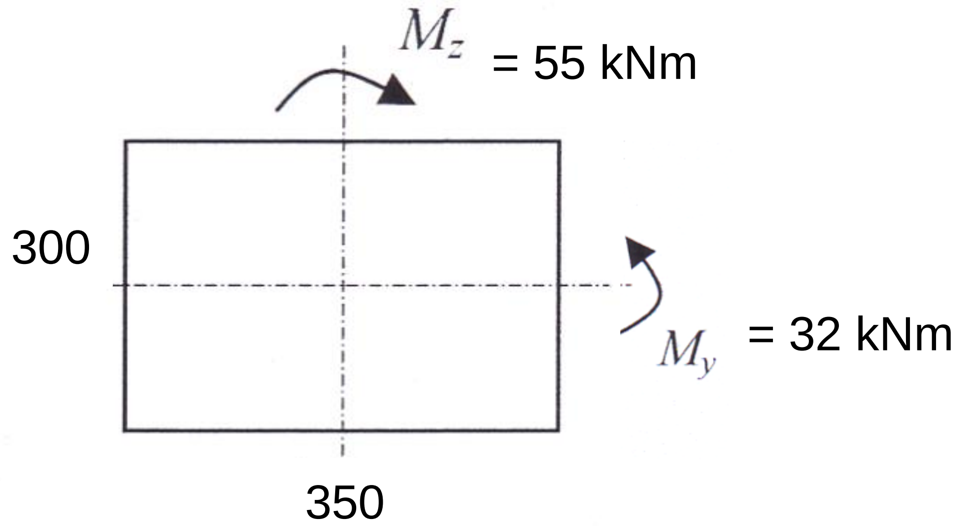
| | | |
|---|---|---|
| | **REINFORCEMENT** | |
| | $d_2 = C + \phi_{link} + \phi_{bar}/2 = 30 + 6 + 20/2 = 46$ mm | |
| | $d_2/h = 46 / 300 = 0.15$ | |
| | $N/bhf_{ck} = 1200 \times 10^3 / (250 \times 300 \times 25)$ | |
| | $= 0.64$ | |
| | $M/bh^2f_{ck} = 47.6 \times 10^6 / (250 \times 300^2 \times 25)$ | |
| | $= 0.08$ | |
| Design Chart | $A_sf_{yk} / bhf_{ck} = 0.35$ | |
| | $A_s = 0.35\ bhf_{ck} / f_{yk}$ | Use : 4H 20 |
| | $= 0.35 (250 \times 300 \times 25) / 500$ | 2H 12 |
| | $= 1313$ mm$^2$ | (1483 mm$^2$) |

# Example 3.4

| | | |
|---|---|---|
| 9.5.2(2) | $A_{s,min} = 0.1N_{Ed}/f_{yd} = 0.1N_{Ed} / (0.87f_{yk})$<br>$= 0.1 \times 1200 \times 10^3 / (0.87 \times 500)$<br>$= 276\ mm^2$ or $0.002A_c = 150\ mm^2$ |  |
| 9.5.2(3) | $A_{s,max} = 0.04A_c = 0.04\ (250 \times 300) = 3000\ mm^2$ | |
| 9.5.3 | Links, $\phi_{min}$ = the larger of<br>$= 0.25 \times (20) = 5.0\ mm$<br>or 6 mm<br>$S_{v\ max}$ = the lesser of<br>$= 20 \times (12) = 240\ mm$<br>or 250 mm<br>or 400 mm — Use : H6 - 240<br><br>At section 300 mm below and above beam and<br>at lapped joints, $S_{v\ max} = 0.6 \times 240 = 144\ mm$<br><br>Use : H6 - 140 |  |

# Example 3.5

Design the longitudinal and transverse reinforcement for a rectangular column size 300 x 350 mm. This column is classified as non-slender and subjected to ultimate axial load of 1800kN and bending moments of 55 kNm and 32 kNm about major and minor axis respectively. Use grade C25/30 concrete, grade 500 steel reinforcement and nominal cover of 30mm.




- Short column under biaxial moments

# Example 3.5

**SPECIFICATION**

Classification: Short braced column

Axial force, $N_{Ed}$ = 1800 kN

Material:

Concrete, $f_{ck}$ = 25 $N/mm^2$

Reinforcement, $f_{yk}$ = 500 $N/mm^2$

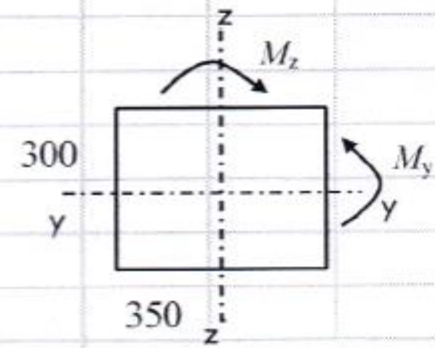
Size, $b \times h$ = 300 x 350 mm

Effective length, $l_{oz}$ = 3.70 m

$l_{oy}$ = 3.00 m

Slenderness ratio, $\lambda_z$ = 27.7

$\lambda_y$ = 34.2




Assumed : $\phi_{link}$ = 6 mm

$\phi_{bar}$ = 25 mm

Nominal cover, $C_{nom}$ = 30 mm

Bending moment:

$M_z$ = 55 kNm

$M_y$ = 32 kNm

5.8.8.2 **DESIGN MOMENT**

5.2(7) The imperfection moment,

$M_{imp} = N_{Ed}.e_i = N_{Ed}.(l_o/400)$

$M_{imp,z}$ = 1800 x (3.70 / 400) = 16.7 kNm

$M_{imp,y}$ = 1800 x (3.00 / 400) = 13.5 kNm

The design moment including the effect of imperfection,

$M_{Edz}$ = 55 + 16.7 = 71.7 kNm

$M_{Edy}$ = 32 + 13.5 = 45.5 kNm

# Example 3.5

| | | |
|---|---|---|
| 5.8.9 | **CHECK BIAXIAL BENDING** | |
| | $e_z = M_{edy} / N_{Ed} =$ 71.7 x $10^6$ / 1800 x $10^3$ = 40 mm | |
| | $e_y = M_{edz} / N_{Ed} =$ 45.5 x $10^6$ / 1800 x $10^3$ = 25 mm | |
| | $(e_y/h)/(e_z/b) =$ (25 / 350) / (40 / 300) = 0.54 > 0.2 | |
| | $(e_z/b)/(e_y/h) =$ (40 / 300) / (25 / 350) = 1.84 > 0.2 | |
| | ==> Check biaxial bending | 5.8.9(4) |
| | $\lambda_y/\lambda_z =$ 34.2 / 27.7 = 1.2 < 2 | ==> Check |
| | $\lambda_z/\lambda_y =$ 27.7 / 34.2 = 0.8 < 2 | biaxial bending |
| | ==> Ignore biaxial bending | |
| | **REINFORCEMENT DESIGN** | |
| | Effective depth, $d = h - C_{nom} - \phi_{link} - 0.5\phi_{bar}$ | |
| | $h'$ = 350 - 30 - 6 -(0.5 x 25) = 301.5 mm | |
| | $b'$ = 300 - 30 - 6 -(0.5 x 25) = 251.5 mm | |

# Example 3.5

$M_z/h'$ = 71.7 $\times 10^6$ / 301.5 = 238 kN

$M_y/b'$ = 45.5 $\times 10^6$ / 251.5 = 181 kN

$M_z/h'$ > $M_y/b'$

Use ==> $M'_z = M_z + \beta(h'/b')\,M_y$

$M'_y = M_y + \beta(b'/h')\,M_z$

$N/bhf_{ck}$ = 1800 $\times 10^3$ / (300 x 350 x 25) = 0.69

$\beta = 1 - N/bhf_{ck}$ = 1 - 0.69 = 0.31 ≥ 0.3

$M'z$ = 71.7 + 0.31 ( 302 / 252) x 45.5

= 88.8 kNm

$d_2 = C + \phi_{link} + 0.5\phi_{bar}$ = 30 + 6 + 25/2 = 49 mm

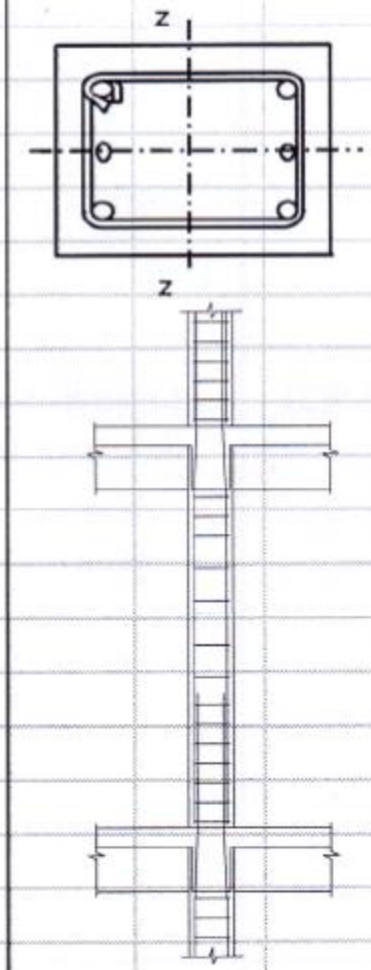
$d_2/h$ = 49 / 350 = 0.14

$N/bhf_{ck}$ = 1800 x $10^3$ / (300 x 350 x 25) = 0.69

$M/bh^2f_{ck}$ = 88.8 x $10^6$ / (300 x $350^2$ x 25) = 0.10

# Example 3.5

| | | |
|---|---|---|
| Design Chart | $A_s f_{yk} / bhf_{ck} =$ 0.48 | Use : 4H 25 |
| | $A_s = 0.48\ bhf_{ck} / f_{yk}$ | 2H 20 |
| | $= 0.48\ (300 \times 350 \times 25) / 500$ | (2592 $mm^2$) |
| | $= 2520\ mm^2$ | |
| 9.5.2(2) | $A_{s,min} = 0.1 N_{Ed}/f_{yd} = 0.1 N_{Ed} / (0.87 f_{yk})$ | |
| | $= 0.1 \times 1800 \times 10^3 / (0.87 \times 500)$ | |
| | $= 414\ mm^2$ or $0.002 A_c = 210\ mm^2$ | |
| 9.5.2(3) | $A_{s,max} = 0.04 A_c = 0.04\ (300 \times 350) = 4200\ mm^2$ | |
| 9.5.3 | Links, $\phi_{min} = 0.25 \times (25) = 6.3\ mm \geq 6\ mm$ | |
| | $S_{v\ max}$ = the lesser of | |
| | $= 20 \times (20) = 400$ mm | |
| | or 300 mm | |
| | or 400 mm Use : H8 - 300 | |
| | At section 350 mm below and above beam and | |
| | at lapped joints, $S_{v\ max} = 0.6 \times 300 = 180$ mm | |
| | Use : H8 - 175 | |



# Example 3.5

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 5.8.9(4) | **CHECK BIAXIAL BENDING** | | | | | | | | | |
| | Steel area, | | | | | | | | | |
| | All: | 4H 25 | + | 2H 20 | | $A_s =$ | 2592 | $mm^2$ | | |
| | z-z : | 4H 25 | + | 2H 20 | | $A_{sz} =$ | 2592 | $mm^2$ | | |
| | y-y : | 4H 25 | + | 0H 20 | | $A_{sy} =$ | 1964 | $mm^2$ | | |
| | $d_{2z}/h$ | = 49 | / 350 | = | 0.14 | | | | | |
| | $d_{2y}/b$ | = 49 | / 300 | = | 0.16 | | | | | |
| | $N/bhf_{ck}$ = | | 1800 | x $10^3$ / | (300 x | 350 x | 25) | | | |
| | | = | 0.69 | | | | | | | |
| | $A_{sz}f_{yk}/bhf_{ck}$ = | | 2592 | x 500 / | (300 x | 350 x | 25) = | 0.49 | | |
| | $M/bh^2f_{ck}$ = | | 0.10 | | | | | | | |
| | | $M_{Rdz}$ = | 0.10 | x 300 | x $350^2$ | x 25 | | | | |
| | | = | 91.9 | kNm | | | | | | |
| | $A_{sy}f_{yk}/bhf_{ck}$ = | | 1964 | x 500 / | (350 x | 300 x | 25) = | 0.37 | | |
| | $M/bh^2f_{ck}$ = | | 0.07 | | | | | | | |
| | | $M_{Rdy}$ = | 0.07 | x 350 | x $300^2$ | x 25 | | | | |
| | | = | 55.1 | kNm | | | | | | |

# Example 3.5

| | | |
|---|---|---|
| | $N_{Rd} = 0.567 f_{ck} A_c + 0.87 f_{yk} A_s$ | |
| | $= (0.567 \times 25 \times 300 \times 350) + (0.87 \times 500 \times 2592)$ | |
| | $= 2616$ kN | |
| | $N_{Ed} / N_{Rd} = 1800 / 2616 = 0.69$ | |
| | $a = 1.49$ | |
| 5.8.9(2) | Imperfections need only be taken in one direction - where they have the most unfavourable effect. | |
| | $M_{Edz} = 71.7$ kNm | |
| | $M_{Edy} = 32.0$ kNm | |
| | $(M_{Edz}/M_{Rdz})^a + (M_{Edy}/M_{RDy})^a \leq 1.0$ | |
| | $(71.7 / 91.9)^{1.49} + (32.0 / 55.1)^{1.49}$ | |
| | $= 0.69 + 0.44$ | |
| | $= 1.14 > 1.0$ | Fail |

# Example 3.5

**New arrangement of reinforcement**

Steel area,

| | | | | | |
|---|---|---|---|---|---|
| All: | 4H 25 + | 4H 20 | $A_s$ = | 3221 | $mm^2$ |
| z-z : | 4H 25 + | 2H 20 | $A_{sz}$ = | 2592 | $mm^2$ |
| y-y : | 4H 25 + | 2H 20 | $A_{sy}$ = | 2592 | $mm^2$ |

$d_{2z}/h$ = 49 / 350 = 0.14

$d_{2y}/b$ = 49 / 300 = 0.16

$N/bhf_{ck}$ = 1800 x $10^3$ / (300 x 350 x 25)
= 0.69

$A_{sz}f_{yk}/bhf_{ck}$ = 2592 x 500 / (300 x 350 x 25) = 0.49

$M/bh^2f_{ck}$ = 0.10

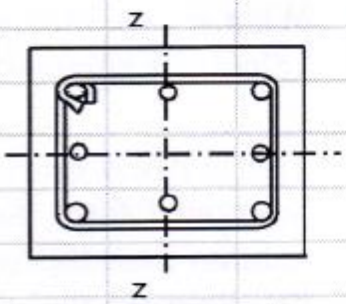
$M_{Rdz}$ = 0.10 x 300 x $350^2$ x 25
= 91.9 kNm

$A_{sy}f_{yk}/bhf_{ck}$ = 2592 x 500 / (350 x 300 x 25) = 0.49

$M/bh^2f_{ck}$ = 0.10

$M_{Rdy}$ = 0.10 x 350 x $300^2$ x 25
= 78.8 kNm

Use : 4H 25
4H 20
(3221 $mm^2$)



# Example 3.5

$N_{Rd} = 0.567 f_{ck} A_c + 0.87 f_{yk} A_s$

$= (0.567 \times 25 \times 300 \times 350) + (0.87 \times 500 \times 3221)$

$= 2889$ kN

$N_{Ed} / N_{Rd} = 1800 / 2889 = 0.62$

$a = 1.44$

5.8.9(2) Imperfections need only be taken in one direction - where they have the most unfavourable effect.

$M_{Edz} = 71.7$ kNm

$M_{Edy} = 32.0$ kNm

$(M_{Edz}/M_{Rdz})^a + (M_{Edy}/M_{RDy})^a \leq 1.0$

$(71.7 / 91.9)^{1.44} + (32.0 / 78.8)^{1.44}$

$= 0.70 + 0.27$

$= 0.97 < 1.0$ Ok