Chapter 2

Shear Force and Bending Moment

This chapter begins with a discussion of beam types. It is also important for students to know and understand the reaction from the types of supports holding the beams. In this chapter we will determine the stress in these members caused by bending.

After successfully completing this chapter the students should be able to:

- Draw the shear and moment diagrams.
- Determine the largest shear and moment in a member and specify the location.

2.1 Introduction

This chapter will discuss the changes of shear force and bending moments in beams as reactions to several types of loading or combinations of loading using different support conditions. Consequently the value of shear force and bending moment as well as the location of maximum values can be determined. These maximum values are important because they will be used in the analysis of structural design. Before this discussion can take place, several basic things will have to be discussed first such as the types of beam, support and loading as well as the stability and capability of the beams and the method to determine the support reaction.

In this chapter, changes of shear force and bending moment in beams will be discuss with the reaction of several types of loading or combination of loading using different support conditions. Then, the value of shear force and bending moment and also the location, where maximum value can be determine. These maximum values are important because these values will be used in the analysis of structural design. Before this topic were discuss, few basic thing will be discuss first such as types of beam, support and loading also stability and capability of beam and method to determine the support reaction.

2.2 Types of beam

Beams can be defined as a structural component that has a smaller dimension of cross section than its length. Beams can be classified into two types based on their analyses

- Statically determinate beam
- Statically indeterminate beam

These classifications of beams depend on the unknown total reactions to the static equilibrium equation. There are 3 static equilibrium equations:

$$\sum F_x = 0 \; ; \; \sum F_y = 0 \text{ dan } \sum M = 0$$

if unknown total reaction is more than 3, thus the beam is "Statically indeterminate beam". Below is how the stability and capability of beams are being determined:

$$r < n + 3 \quad \text{- Not stable on static}$$
$$r = n + 3 \quad \text{- Statically determinate (geometrically stable)}$$
$$r > n + 3 \quad \text{- Statically indeterminate}$$

where, r = number of reaction
         n = number of joints (if applicable)

*Generally beams can be recognised according to their supports.

1. Simply supported beam - Figure 2.1
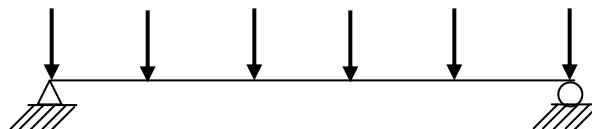
Figure 2.1

- This beam is supported by rollers at one end and by pins at the other end.

- This beam is a statically determinate beam.

2. Cantilever beam- Figure 2.2

Figure 2.2

- This beam has a fixed support at one end and a free support at the other end.
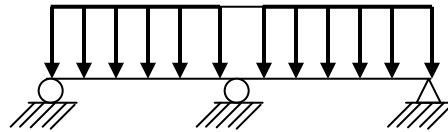- The beam is a statically determinate beam

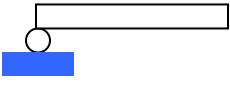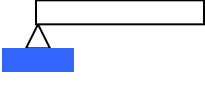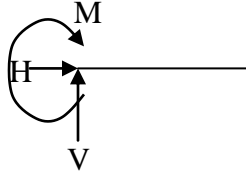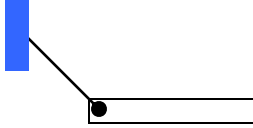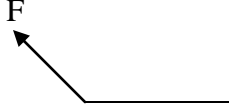3. Continuous beam - Figure 2.3

Figure 2.3
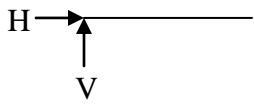
- This beam has several supports.
- This beam is a statically indeterminate beam

2.3 Types of support

Types of support are shown in Table 1. Below is a summary of the table:
:

1. Supports will form horizontal force if they resist horizontal movement (left and right)
2. Supports will form vertical force if they resist vertical movement (up and down)
3. Supports will create moments if they resist the component from circular motion.

Table 2.1: Types of support

No.	Types of support	Reaction Force	Total Force
1	Roller	V	One: V reaction is 90 degrees to the surface
2	Pin	H, V	Two : H and V, support resist structure from moving on vertical direction
3	Fixed	M, H, V	Three : H and V are reactions that resist structure from moving horizontally while M is moment reaction that resists the structure from rotating.
4	Cabel	F	One : F , reaction axis at F direction

2.4 Types of loading

Types of loading:

1. Dead Load
   Loading is fixed and very close to the approximation values (very little change). Some examples are structure self weight, ceiling and fixed equipment such as the piping system services, lighting and air conditioning. For a sample of construction elements please refer to BS 6399.

2. Live Load/Imposed Load

   It is a variable load on a structure for example; human weight, furniture, and other equipment based on structure purpose. Values for the loading can be referred to BS EN 1991.

3. Wind Load

   The design of buildings must account for wind load especially in high-rise buildings. However the effects of this wind load depend on the location, shape, building dimension, and wind speed in the particular region.

2.5 Types of loading distribution

There are 3 methods of loading distribution that are normally used in structure analysis.

1. Point load – Figure 2.4

   This load acts at one position only whether in vertical or inclined direction. The unit for this type of loading is N, kN.
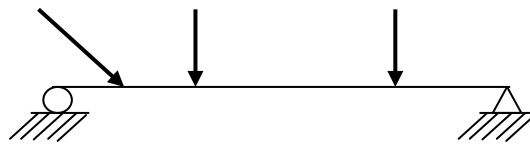
   Figure 2.4

2. Distributed load - Figure 2.5

   This load acts at a certain distance or a distributed load. This loading is measured in unit load per distance which is N/mm, kN/m and so on. The value can be constant or it can increase and decrease along its reaction distance .
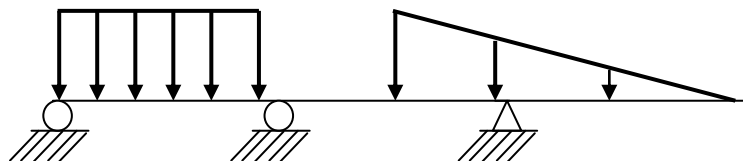
   Figure 2.5

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3. Moment load – Figure 2.6

Resulting load made by a pair of forces and this force will create moment on the point of loading The unit for this load is the unit of load multiple with the perpendicular distance (Nmm, kNm and so on).
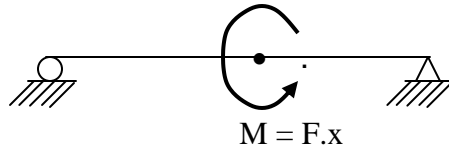
$$M = F.x$$

Figure 2.6

2.6 Reactions on supports

2.6.1 Free Body Diagram

In order to calculate the reaction from any structure, first thing that one needs to do is draw the actual structure again in the form of a Free Body Diagram. Free body diagram is the sketch of the actual structure in lines form that are free from supports as well as from loads acting on it. Draw all reactions that happened at the supports as well as the directions of the reactions and their accurate dimensions. Draw all forces or loads acting on the structure elements. Figure 2.7 shows a a continuous beam and its free body diagram is in Figure 2.8.
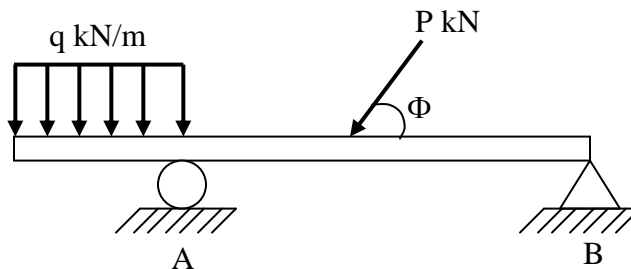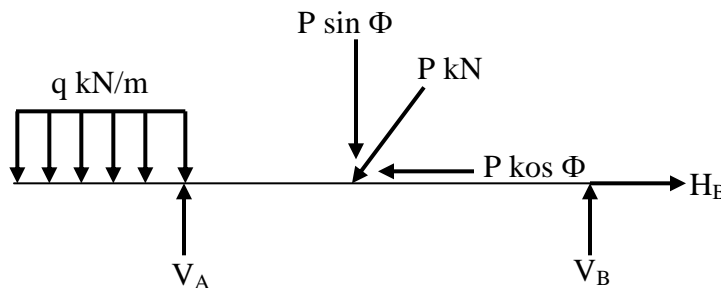
Figure 2.7

Figure 2.8

2.6.2 Equilibrium Equation

This consists of 3 equations, where by using these three equations, reactions of certain structures can be determined. The equation is:

$$\sum F_x = 0 \; ; \; \sum F_y = 0 \text{ dan } \sum M = 0$$

To avoid a simultaneous equation, let's use moment equation, $\sum M = 0$ first on any one support point, followed by other reactions on support point that can be determined using this equation $\sum F_x = 0 \; ; \; \sum F_y = 0$.

If the solution of this simultaneous equation gives the negative value, it means the earlier assumption of reaction direction is incorrect and the actual direction is the opposite direction.
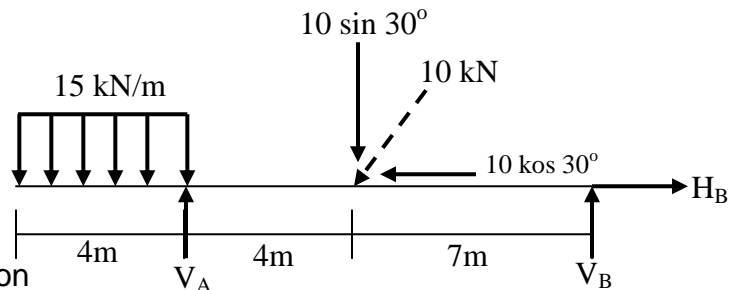
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EXAMPLE 2.1

Determine the reaction of beam stressed with a load as shown in Figure 2.9
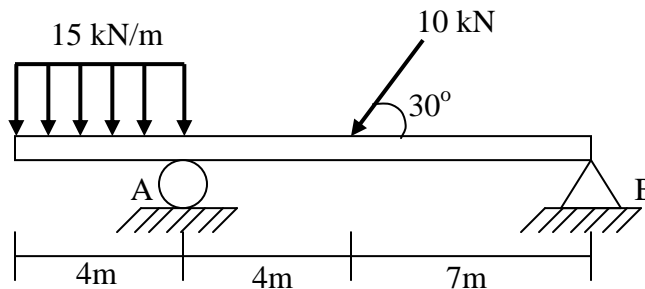
Rajah 2.9

Solution:

Free body diagram

Simultaneous equation

$\circlearrowright_{+} \sum M_A = 0$

$$\frac{15(4)^2}{2} - 10 \sin 30^\circ (4) + V_B(11) = 0$$

$$11V_B = -100$$
$$\mathbf{V_B = -9.1 \ kN \ (\uparrow)}$$

$\uparrow + \sum F_y = 0$

$V_A + V_B - 15(4) - 10 \sin 30^\circ = 0$

$$V_A = -V_B + 15(4) + 10 \sin 30^\circ$$
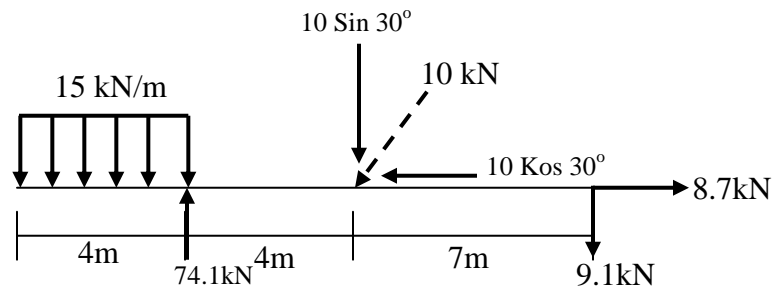$$\mathbf{V_A} = -(-9.1) + 60 + 5$$
$$\mathbf{= 74.1 \ kN \ (\uparrow)}$$

$\xrightarrow{+} \sum F_x = 0$

$H_B - 10 \cos 30^\circ = 0$

$$\mathbf{H_B} = 10 \cos 30^\circ$$
$$\mathbf{= 8.7 \ kN \ (\longrightarrow)}$$

Figure of the complete structure

Example 2.1

Beam with pointed loading
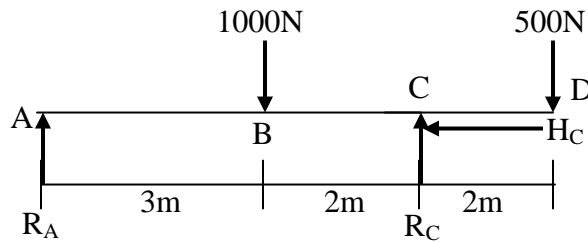
Determine the reaction on beam in Figure 2.10 below

.
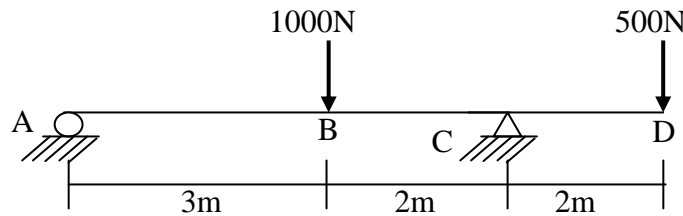
Figure 2.10

Solution :

Simultaneous equation :

To get the reaction, R_A, take the moment at point C.

$$\stackrel{+}{\curvearrowright} \sum M_C = 0$$

$$1000(2) - 500(2) - R_A(5) = 0$$

$$5R_A = 1000$$

$$\mathbf{R_A = \ 200N} \quad (\uparrow)$$

To get the reaction, R_C, use the simultaneous equation at y direction.

$$\uparrow + \sum F_y = 0$$

$$R_A + R_C - 1000 - 5000 = 0$$

$$R_C \ = R_A + 1000 + |5000$$

$$\mathbf{R_C} \ = 200 + 1000 + |5000$$

$$= \mathbf{1300N} \quad (\uparrow)$$

To get the reaction, H$_C$, use the simultaneous equation at x direction.
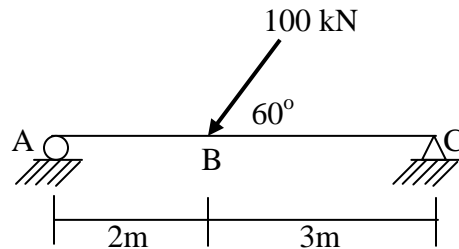
$$\xrightarrow{+}\sum F_x = 0: \qquad\qquad H_B = 0$$
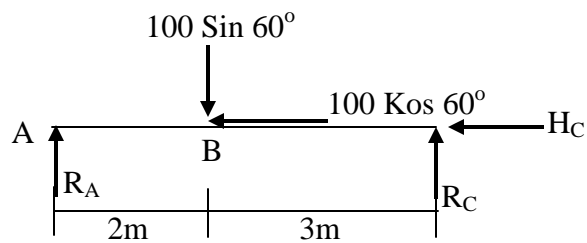
Figure of the complete structure:
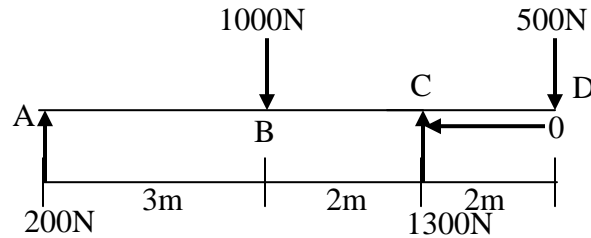
Example 3

Beam with inclined loading

Rajah 2.11

Solution :

Simultaneous equation:

To get the reaction R$_A$, take the moment at point C.

$$\circlearrowright +\sum M_C = 0$$
$$1000(2) - 500(2) - R_A(5) = 0$$

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$$5R_A = 1000$$
$$R_A = 200N \quad (\uparrow)$$

To get the $R_{C \, reaction}$, use the simultaneous equation at y direction.

$$\uparrow + \sum F_y = 0$$
$$R_A + R_C - 1000 - 5000 = 0$$
$$R_C = R_A + 1000 + |5000$$
$$R_C = 200 + 1000 + |5000$$
$$= 1300N \quad (\uparrow)$$

To get the $H_{C \, reaction}$, use the simultaneous equation at x direction.

$$\xrightarrow{+} \sum F_x = 0 : \qquad H_B = 0$$

SHEAR FORCE AND BENDING MOMENT

Shear force at a section is the total algebra force taken only at one side of the section . Any section can be taken to be totaled provided that both sections must have the same values and they follow the marking direction. Bending moment is the total algebra of moments force acting on the section. Figure 2.12 below explains the situation.
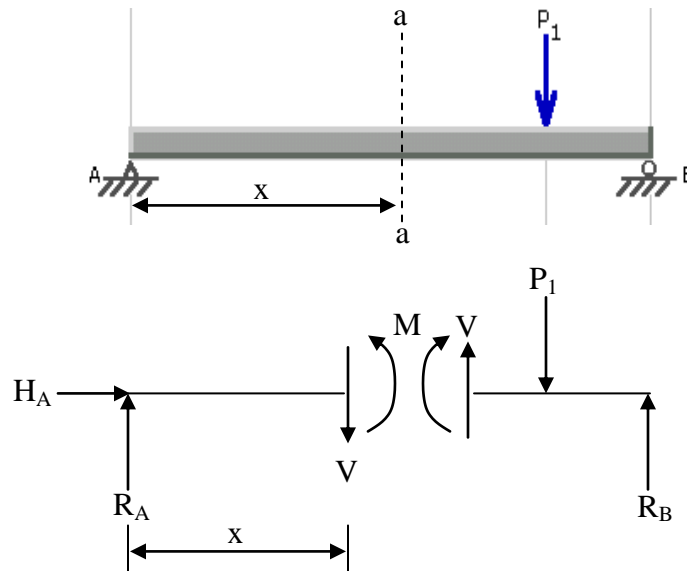

Figure 2.12

Simply supported beam is cut at a-a. Therefore both sections that have been cut need to be at the same equilibrium condition as before they were cut, otherwise the section that was cut will move.

 On the left section of the beam, to make sure that the beam is in a balanced vertical direction, there should exist a V force that balances the RA force and one moment M with an anti clock wise movement to prevent the beam from rotating. The same thing applies with the right side of the beam. Both the force and moment must be balanced on the section that has been ct. . The total force and moment at this section equals to zero.

 The V force is known as a shear force, the force that causes the member to slide and separate into two sections . Meanwhile moment M is known as the bending moment, which is a reaction moment at a certain point for all forces as well as combinations reacting on the left or right side of the point.

NATURAL SIGN FOR SHEAR FORCE AND BENDING MOMENT

 Markings are used to determine the values of the internal axis force, internal shear force and internal bending moment. A positive marking is normally used as shown in Figure 2.13.
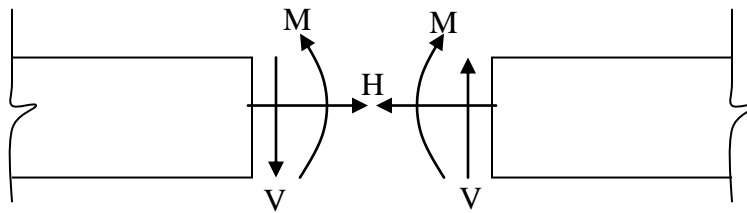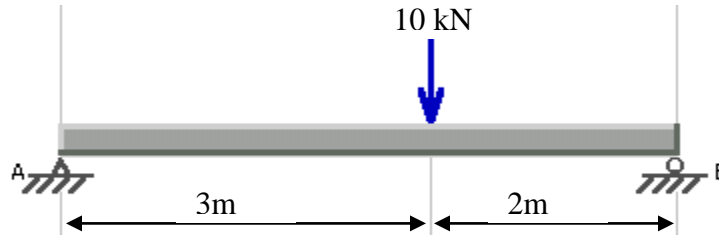

Figure 2.13

positive axis force, H +ve   : will try to elongate the element.
Positive shear force, V +ve   : will try to turn the element
                                        clockwise.
Positive bending moment, M +ve : will try to bend the element sagging
                                        downward.

EQUATION AND DIAGRAM OF SHEAR FORCE AND BENDING MOMENT

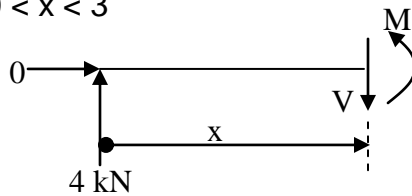1. Simply supported beam with point load acting.

reaction:

$$\circlearrowright_{+} \sum M_A = 0 \; ; \qquad R_B(5) - 10(3) = 0$$
$$\mathbf{R_B = 6 \ kN} \quad (\uparrow)$$

$$\uparrow_{+} \sum F_y = 0 \; ; \qquad R_A + R_B - 10 = 0$$
$$\mathbf{R_A} = 10 - 6 = \mathbf{4 \ kN} \ (\uparrow)$$

$$\xrightarrow{+} \sum F_x = 0 : \qquad \mathbf{H_A = 0}$$

Take x as the distance of the section from the end of the left side, A:

(a) 0 < x < 3

$$\uparrow +\sum F_y = 0 \; ; \qquad V = 4 \ kN$$

$$\circlearrowright_{+} \sum M = 0 \; ; \qquad M = 4.x \ kNm$$

For x = 0 ; V = 4 kN
            M = 4(0) = 0 kNm

     x = 3 ; V = 4 kN
             M = 4(3) = 12 kNm

(b) $3 < x < 5$

10 kN

0

3m

x

V

M

4 kN
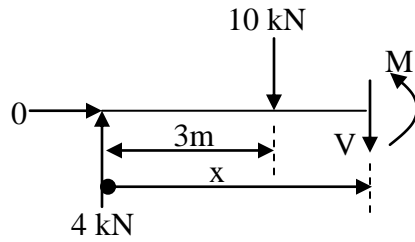
$\uparrow + \sum F_y = 0$; $\qquad$ V = 4 − 10 = -6 kN

$\circlearrowright + \sum M = 0$; $\qquad$ M = 4.x − 10(x-3)

$\qquad\qquad\qquad\qquad$ = 30 − 6x kNm

for x = 3 ; $\quad$ V = -6 kN

$\qquad\qquad$ M = 30 − 6(3) = 12 kNm

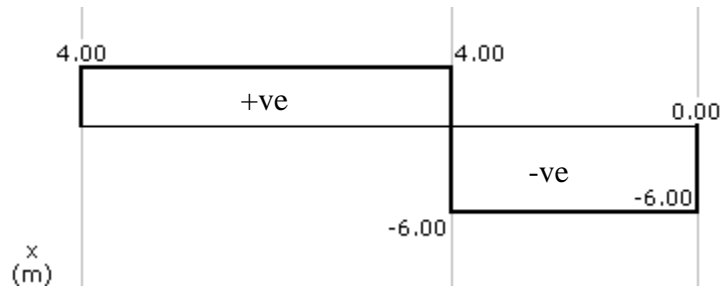$\quad$ x = 5 ; V = -6 kN

$\qquad\qquad$ M = 30 − 6(5) = 0 kNm

 The above equation shows that shear force caused by point load is constant or fixed and the bending moment changes linearly with x.
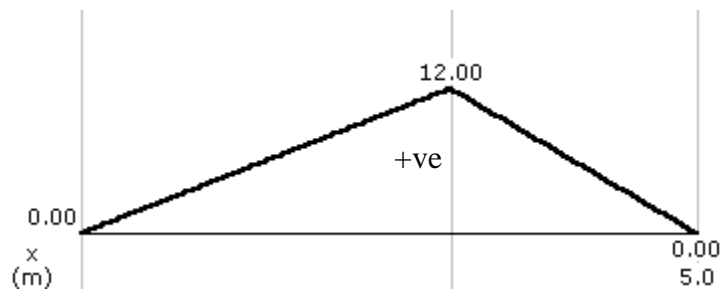
Diagram:

$\qquad$ Free body diagram (FBD)

10 kN

3m

2m

4 kN

6 kN
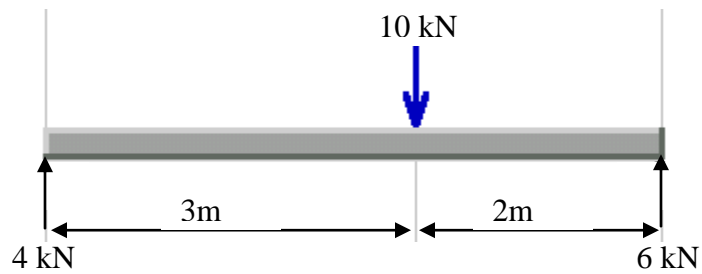
Shear force diagram (SFD)

Bending moment diagram (BMD)

Note:

- SFD positive, +ve is drawn on top of the beam.
- BMD positive, +ve is drawn on top of the beam.
- To get the V and M equations, make sections on each part between the point load,
    distributed load and support.
- To draw SFD and BMD, it is enough to determine V and M value on the
    important points only such as at the support, point under the point load and at both
    ends and in the middle of the distributed load.

2. Distributed uniform load acting on simply supported beam.
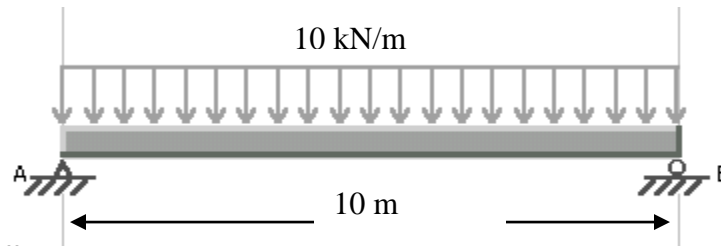
Reaction:

$$\circlearrowleft \sum M_A = 0 \; ; \qquad R_B(10) - \frac{10(10)^2}{2} = 0$$

$$R_B = 50 \text{ kN} \qquad (\uparrow)$$

$$\uparrow + \sum F_y = 0 \; ; \qquad R_A + R_B - 10(10) = 0$$

$$R_A = 100 - 50 = \textbf{50 kN} \; (\uparrow)$$

$$\xrightarrow{+} \sum F_x = 0 : \qquad H_A = 0$$

Take x as the distance of the section at the end of the left side, A:

0 < x < 10
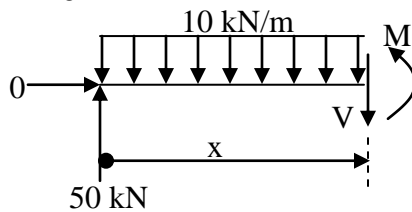
$$\uparrow + \sum F_y = 0 \; ; \qquad V = 50 - 10x$$

$$\circlearrowleft \sum M = 0 \; ; \quad M = 50x - \frac{10x^2}{2}$$

For x = 0 ; V = 50 − 10(0) = 50 kN

$$M = 50(0) - \frac{10(0)^2}{2} = 0 \text{ kNm}$$

x = 10 ; V = 50 − 10(10) = -50 kN

$$M = 50(10) - \frac{10(10)^2}{2} = 0 \text{ kNm}$$

The aboe equation shows that shear force caused by the distributed uniform load will change linearly with x. SFD needs to drawn first to see the relation with BMD.

SFD
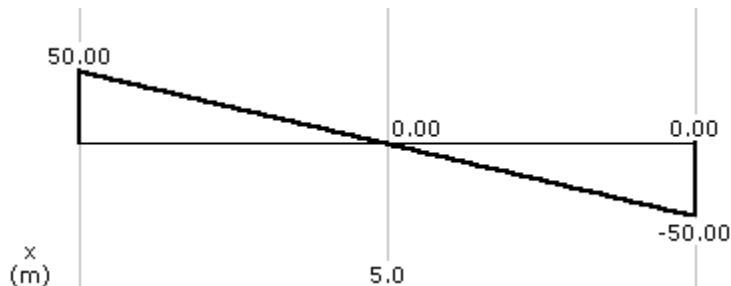
From SFD above, there is one contra point, where the SFD line cuts across the beam axis (shear force = 0). The distance of the point from point A can be obtained as:

$$\text{Distance, } x = \frac{ShearForceAtA}{UniformLoadingLoad} = \frac{50}{10} = 5m$$

See the relation between this point with the equation of bending moment.
.

At x = 5m ; $M = 50(5) - \frac{10(5)^2}{2} = 125 \text{ kNm}$

Put the bending moment value into BMD. When the distance values, x from 0 to 10m is inserted in the equation of bending moment above and each point is plotted on top of BMD, the BMD will appear as below:
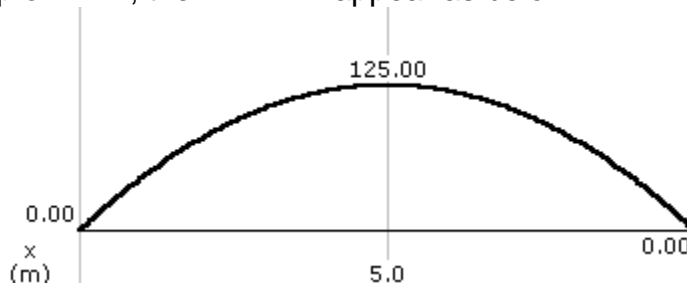
When x = 5m, the value of the bending moment is maximum.

Therefore, maximum bending moment happens when the shear force is zero (SFD line across beam axis / at shear contra point). The value of bending moment changes like a curve as shown above.
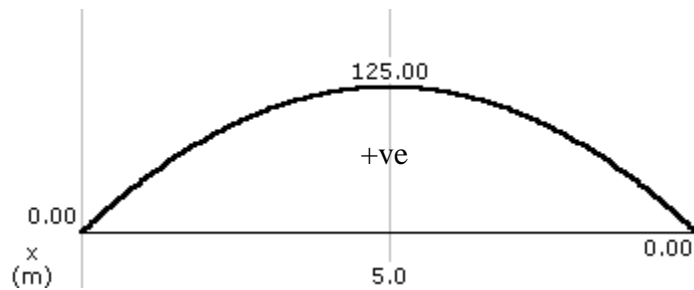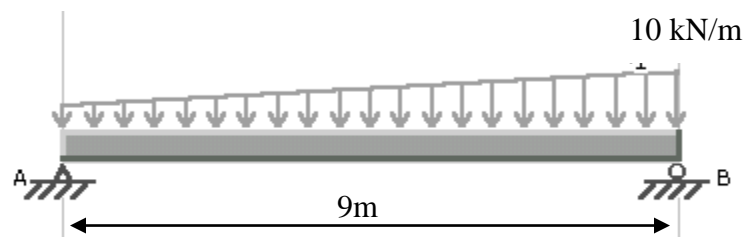
Diagram:

FBD

10 kN/m

50 kN   50 kN

x
(m)   0   10.

SFD

50.00

+ve   0.00   0.00

-ve

5.0   -50.00

x
(m)

BMD

125.00

+ve

0.00   0.00

x
(m)   5.0
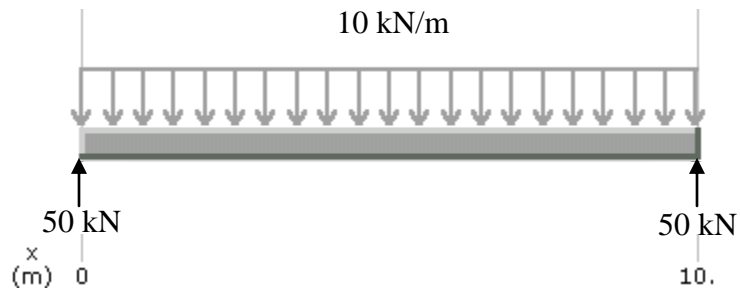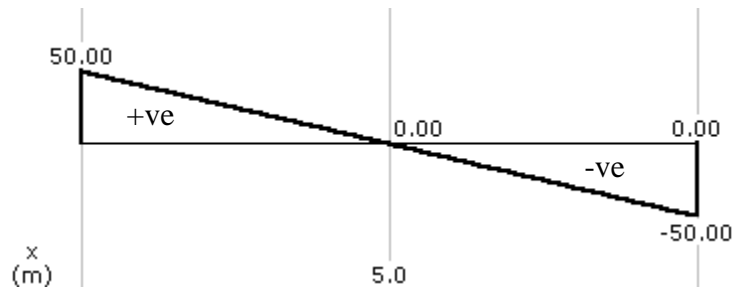
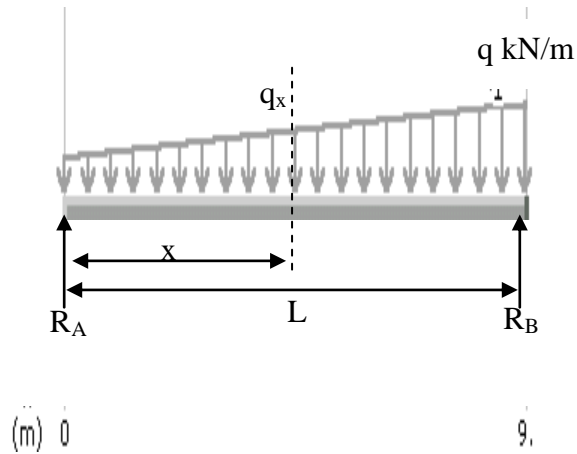3. Uniform increase load acting on simply supported beam.
.

10 kN/m

9m

Reaction:

$$\circlearrowright \sum M_A = 0 \; ; \qquad R_B(9) - \left[\frac{10(9)}{2} x \frac{2(9)}{3} \right] = 0$$

$$R_B = 30 \text{ kN} \quad (\uparrow)$$

$$\uparrow_+ \sum F_y = 0 \; ; \qquad R_A + R_B - \frac{10(9)}{2} = 0$$

$$R_A = 45 - 30 = \textbf{15 kN} (\uparrow)$$

$$\xrightarrow{+} \sum F_x = 0 : \qquad H_A = 0$$

Take x as the distance of the section from the left end of A :

To determine the value of g_x which is the value of uniform increase load when the distance x is from A, the intensity of the load on the section needs to obtained. From the figure above;

$$\frac{q_x}{q} = \frac{x}{L}$$

$$q_x = q\frac{x}{L}$$

therefore,　　$q_x = 10\frac{x}{9} = 1.11x$
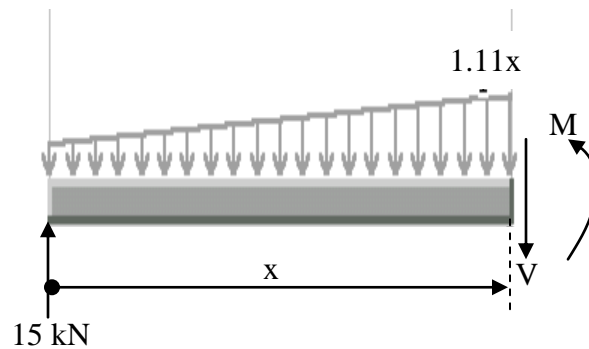
15 kN

$$\uparrow + \sum F_y = 0 \; ; \qquad V = 15 - (1.11x)(\frac{x}{2})$$

$$= 15 - 0.56x^2$$

$$\circlearrowright + \sum M = 0 \; ; \quad M = 15x - \left(\frac{1.11x^2}{2}\right)\left(\frac{x}{3}\right)$$

$$= 15x - 0.185x^3$$

At   x = 0 ; V = 15 kN   ,   M = 0
        x = 2 ; V = 12.8 kN   ,   M = 28.5 kNm
        x = 4 ; V = 6.04 kN   ,   M = 48.2 kNm
        x = 6 ; V = -5.16 kN,   M = 50.04 kNm
        x = 8 ; V = -20.8 kN ,   M = 25.3 kNm
        x = 9 ; V = -30.4 kN ,   M = 0.1 $\cong$ 0 kNm

plot every point on SFD and BMD, thus SFD and BMD will appear as shown below.

SFD
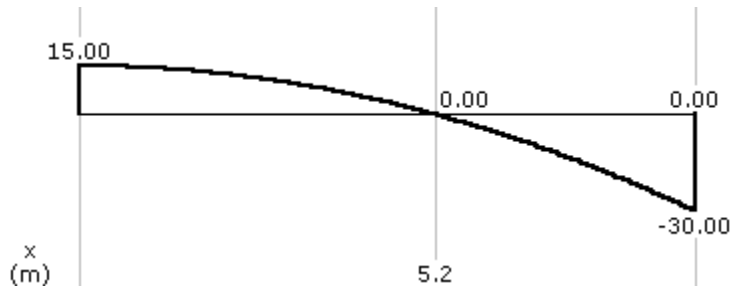
When shear force, V = 0 ; bending moment is at the maximum and the point distance from A is:

$$15 - 0.56x^2 = 0$$
$$x = 5.2m$$

so the maximum bending moment value, M

$$, M = 15x - 0.185x^3$$
$$= 15(5.2) - 0.185(5.2)^3$$
$$= 51.96 \text{ kNm}$$

        BMD

Diagram

FBD

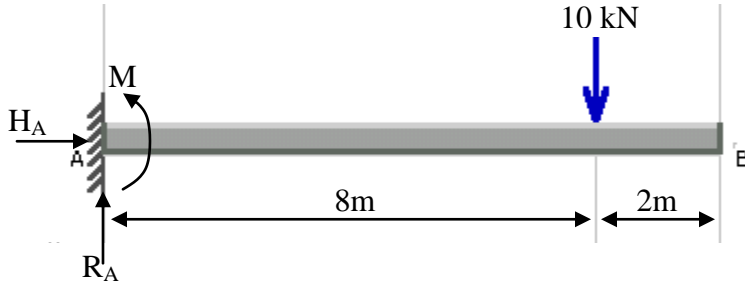10kN/m

15 kN   30 kN

x
(m) 0   9.
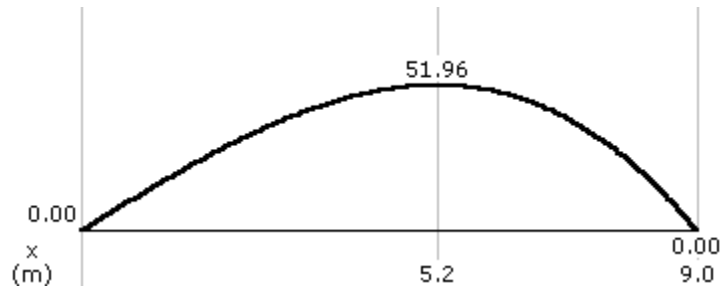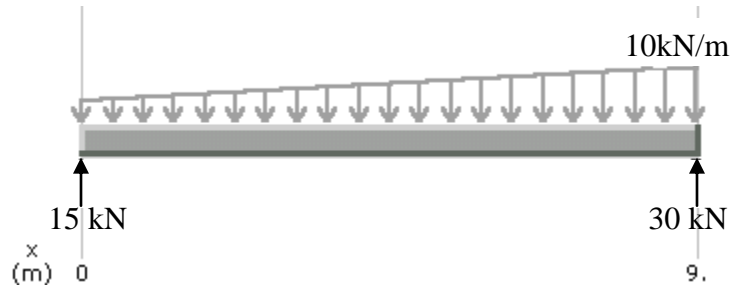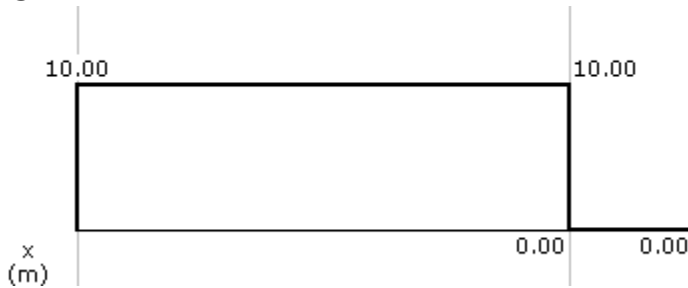
SFD
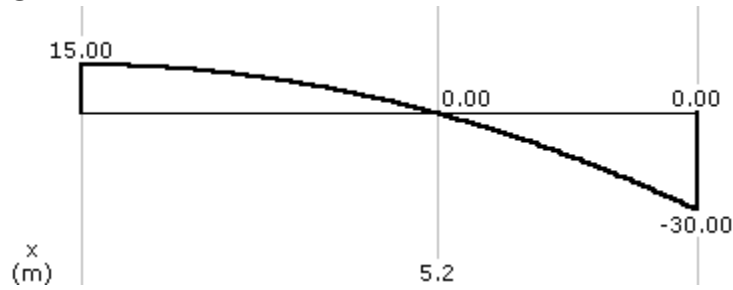
BMD

4. Point load acting on cantilever beam

Reaction

$$\uparrow + \sum F_y = 0 \; ; \qquad R_A = 10 \text{ kN}$$

$$\xrightarrow{+} \sum F_x = 0 \; ; \qquad H_A = 0 \text{ kN}$$

$$\circlearrowleft + \sum M_A = 0 \; ; \quad M = 10(8) = 80 \text{kNm}$$
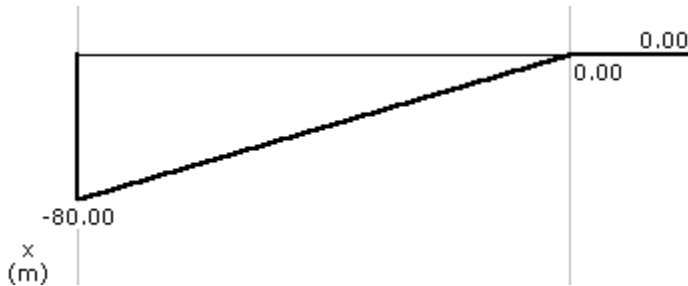
SFD :

Shear force and Bending moment Equation

0 < x < 8

$$V = R_A = 10 \text{ kN}$$
$$M = R_A x - 80 = 10x - 80$$

Di x = 0 ; V = 10 kN, M = - 80 kNm
   x = 8 ; V = 10 kN, M = 0

BMD :

8 < x < 10

$$V = R_A - 10 = 0 \text{ kN}$$
$$M = R_A x - 80 - 10(x - 8)$$
$$= 10x - 80 - 10x + 80$$
$$= 0 \text{ kNm}$$

Di x = 8 ; V = 0 kN, M = 0 kNm
   x = 10 ; V = 0 kN, M = 0 kNm

6. Distributed uniform load acting on cantilever beam.

20 kN/m

M

H_A

A

B

8m

2m

R_A
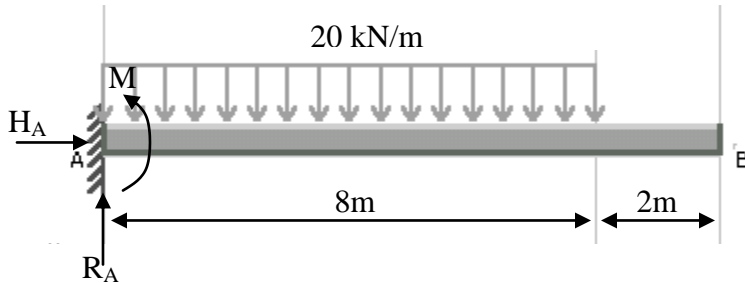
Reaction :

$\uparrow + \sum F_y = 0$; $R_A = 20(8) = 160$ kN

$\xrightarrow{+} \sum F_x = 0$; $H_A = 0$ kN

$\circlearrowright \sum M_A = 0$; $M = \dfrac{20(8)^2}{2} = 640$ kNm

Shear force and bending moment Equation

SFD :

160.00

0.00

0.00   0.00

9.6

x
(m)
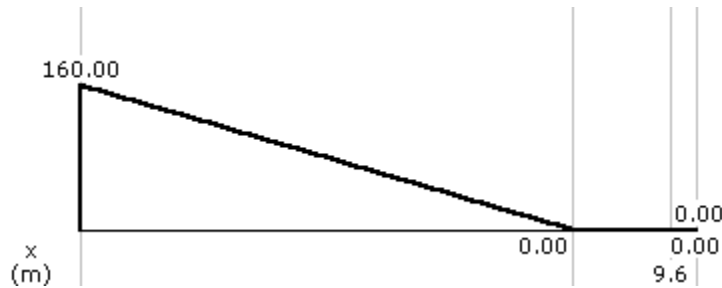
$0 < x < 8$

$V = R_A - 20x = 160 - 20x$

$M = R_Ax - 640 - \dfrac{20x^2}{2}$

$= 160x - 640 - 10x^2$

Di x = 0 ; V = 160 kN, M = - 640 kNm

x = 8 ; V = 0 kN, M = 0

BMD :

0,10.00

0.00

-640.00

x
(m)
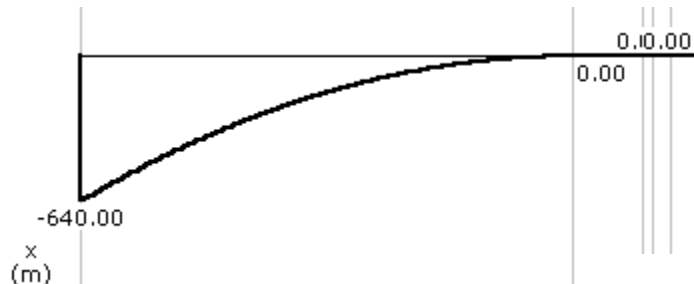
$8 < x < 10$
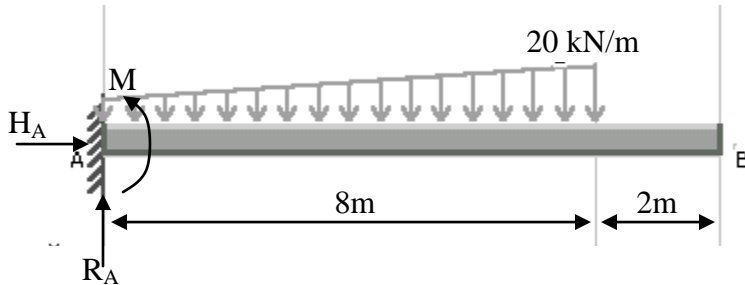
$V = R_A - 20(80)$

$= 160 - 160 = 0$ kN

$M = R_Ax - 640 - [20(8)]\left(x - \dfrac{8}{2}\right)$

$= 160x - 640 - 160x + 640$

$= 0$ kNm

Di x = 8 ; V = 0 kN, M = 0 kNm

x = 10 ; V = 0 kN, M = 0 kNm

7. Uniform increase load acting on cantilever beam.

Reaction

$$\uparrow + \sum F_y = 0; \; R_A = 20(8)\left(\frac{1}{2}\right) = 80 \text{ kN}$$

$$\xrightarrow{+} \sum F_x = 0 \; ; \qquad H_A = 0 \text{ kN}$$

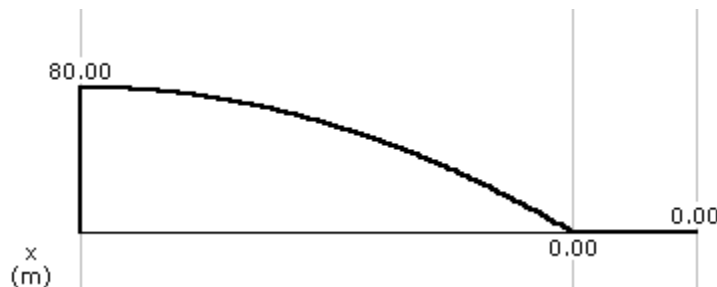$$\circlearrowright \sum M_A = 0 \; ; \; M = 20(8)\left(\frac{1}{2}\right)\left(\frac{2}{3}(8)\right)$$

$$= 426.67 \text{ kNm}$$

Equation shear force and bending moment
Total load at distance x from A :

$$q_x = q \frac{x}{L}$$

$$= 20 \frac{x}{8} = 2.5\text{x}$$

0 < x < 8

$$V = 80 - (2.5\text{x})(\frac{x}{2})$$

$$= 80 - 1.25\text{x}^2$$

$$M = 80\text{x} - \left(\frac{2.5x^2}{2}\right)\left(\frac{x}{3}\right) - 426.67$$

$$= 80\text{x} - 0.42\text{x}^3 - 426.67$$

SFD:

At x = 0 ;
V = 80 kN, M = - 426.67 kNm

x = 8 ;
V = 0 kN, M = 0

8 < x < 10

BMD:

$$V = 80 - (20)(\frac{8}{2}) \quad = 0 \text{ kN}$$

$$M = 80\text{x} - \left[20(\frac{8}{2})\right]\left(x - \frac{2(8)}{3}\right) - 426.67$$
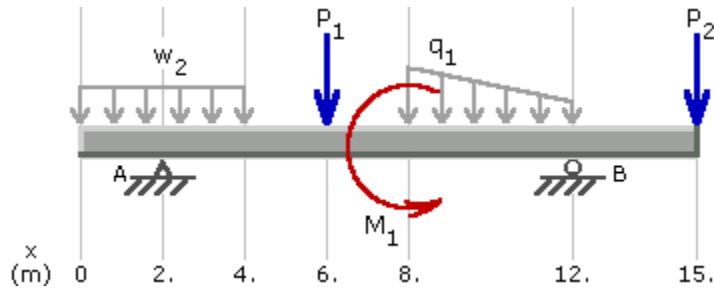
$$= 80\text{x} - 80\text{x} + 426.67 - 426.67$$

$$= 0 \text{ kNm}$$

At x = 8 ; V = 0 kN, M = 0 kNm
x = 10 ; V = 0 kN, M = 0 kNm

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8. Several types of distribution load acting on beam.

$P_1 = 75$ kN
$P_2 = 50$ kN
$w_2 = 20$ kN/m
$q_1 = 30$ kN/m
$M_1 = 30$ kNm

Reaction :

$\overset{+}{\curvearrowright} \sum M_B = 0$; $- R_A(10) + 20(4)(10) + 75(6) + 30 + \left(\dfrac{30(4)}{2}\right)\left(\dfrac{2(4)}{3}\right) - 50(3) =$
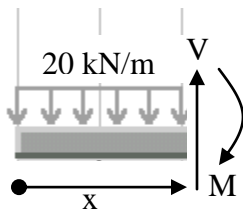0

$$R_A = \frac{800 + 450 + 30 + 160 - 150}{10}$$
$$= 129 \text{ kN} \quad (\uparrow)$$

$\uparrow + \sum F_y = 0$; $R_A + R_B - 20(4) + 75 + \dfrac{30(4)}{2} + 50 = 0$
$R_B = 265 - 129 = \textbf{136 kN} \ (\uparrow)$

$\underset{\rightarrow}{+} \sum F_x = 0$: $H_A = \textbf{0}$

Take x as the distance from the left end of the section:
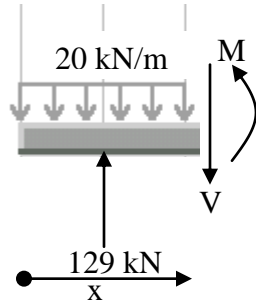**The equation for shear force and bending moment;**

(a) $0 < x < 2$

$V = -20x$
$M = -\dfrac{20x^2}{2}$

Di $x = 0$; $V = 0$, $M = 0$
$x = 2$; $V = -40$ kN , $M = -40$ kNm

(b) 2 < x < 4

$V = 129 - 20x$

$M = 129(x-2) - \dfrac{20x^2}{2}$

Di x = 2; V = 89 kN, M = - 40 kNm

  x = 2; V = 49 kN , M = 98 kNm

(c) 4 < x < 6
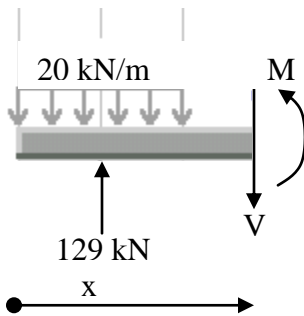
$V = 129 - 20(4) = 49$ kN

$M = 129(x-2) - 20(4)\left(x-\dfrac{4}{2}\right)$

  $= 129x - 258 - 80x + 160$

  $= 49x - 98$

Di x = 4; V = 49 kN,   M = 98 kNm

  x = 6; V = 49 kN , M = 196 kNm

(d) 6 < x < 8
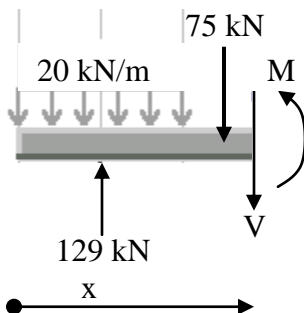
$V = 129 - 20(4) - 75 = -26$ kN

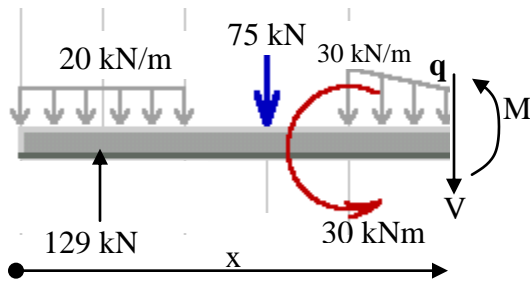$M = 129(x-2) - 20(4)\left(x-\dfrac{4}{2}\right) - 75(x-6)$

  $= 129x - 258 - 80x + 160 - 75x + 450$

  $= -26x + 352$

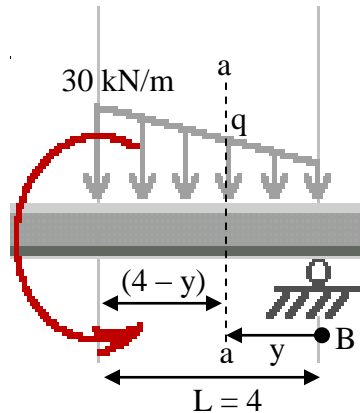Di x = 6; V = -26 kN,   M = 196 kNm

  x = 8; V = -26 kN , M = 144 kNm

(e) 8 < x < 12

To determine the value of load q, the intensity of the load on the section needs to obtained.

From the above figure, distance y is from point B. thus: $q = 30\dfrac{y}{L} = 30\dfrac{y}{4} = 7.5y$

From the figure of shear force and bending moment section :

$$V = 129 - 20(4) - 75 - (60 - 30y + 3.75y^2) - (30y - 7.5y^2)$$
$$= -86 + 3.75y^2$$

$$M = 129(x - 2) - 20(4)\left(x - \frac{4}{2}\right) - 75(x - 6) - 30 - (60 - 30y$$

$$+ 3.75y^2)\left[x - 8 + \left(\frac{4 - y}{3}\right)\right] - (30y - 7.5y^2)\left[x - 8 + \left(\frac{4 - y}{2}\right)\right]$$

$$= 129x - 258 - 80x + 160 - 75x + 450 - 30$$

$$- (60 - 30y + 3.75y^2)\left[x - 8 + \left(\frac{4 - y}{3}\right)\right]$$

$$- (30y - 7.5y^2)\left[x - 8 + \left(\frac{4-y}{2}\right)\right]$$

At x = 8; Distance y = 4

$V = - 86 + 3.75(4)^2 = - 26$ kN

$M = 129(8) - 258 - 80(8) + 160 - 75(8) + 450 - 30$
$\quad - (60 - 30(4) + 3.75(4)^2)\left[8 - 8 + \left(\frac{4-4}{3}\right)\right]$

$\quad - (30(4) - 7.5(4)^2)\left[8 - 8 + \left(\frac{4-4}{2}\right)\right]$

$\quad = 774 - 640 + 160 - 600 + 450 - 30 - 0 - 0$
$\quad = 114$ kNm

At x = 12m; Distance y = 0

$V = - 86 + 3.75(4)^2 = - 26$ kN

$M = 129(12) - 258 - 80(12) + 160 - 75(12) + 450 - 30$
$\quad - (60 - 30(0) + 3.75(0)^2)\left[8 - 8 + \left(\frac{4-0}{3}\right)\right]$

$\quad - (30(0) - 7.5(0)^2)\left[8 - 8 + \left(\frac{4-0}{2}\right)\right]$

$\quad = 1548 - 258 - 960 + 160 - 900 + 450 - 30 - 160 - 0$
$\quad = - 150$ kNm

(f) 12 < x < 15

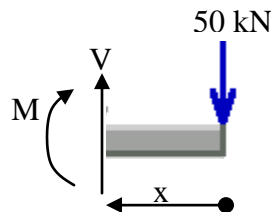To simplify the calculation, take the distance from the end of the right side of the section.
Thus, the range is 0 < x < 3

$V = 50$ kN
$M = - 50x$

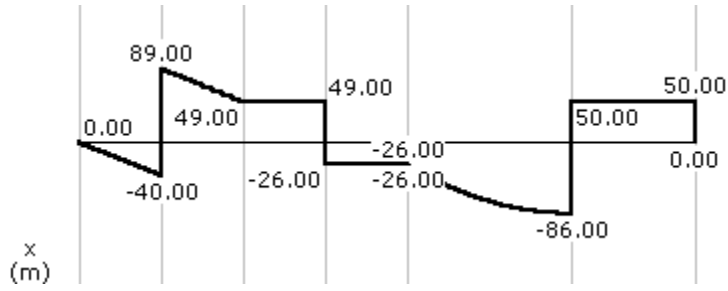At x = 0; $V = 50$ kN , $M = 0$ kNm
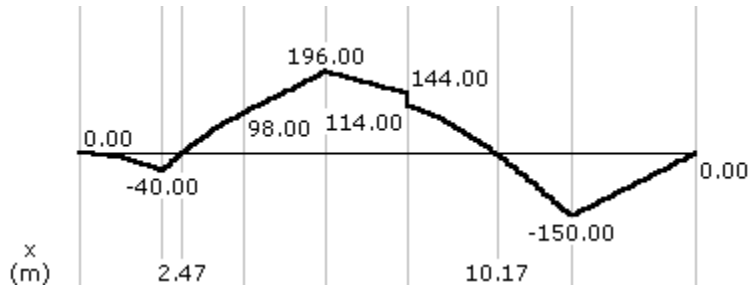x = 3; $V = 50$ kN , $M = - 150$ kNm

50 kN

V

M

x

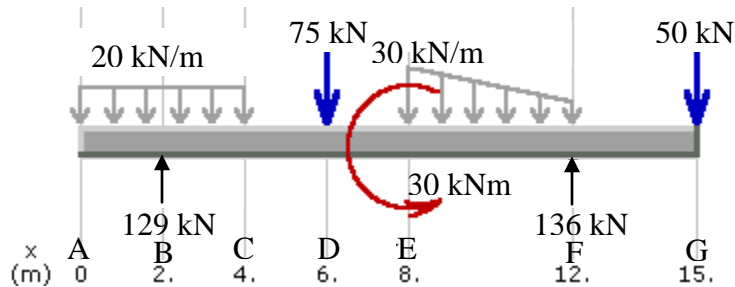Plot all the values of V and M in SFD and BMD. SFD and BMD will appear as follows
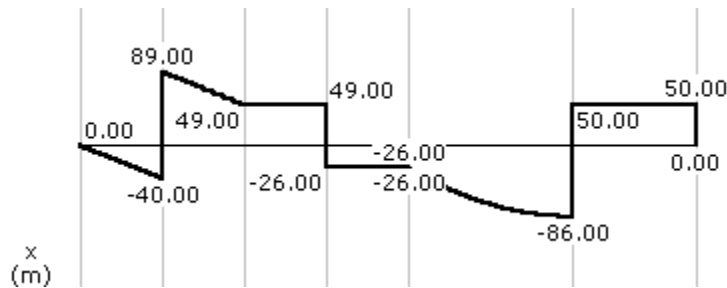
SFD :

BMD :

THE RELATIONSHIP BETWEEN SHEAR FORCE AND BENDING MOMENT

Let's take case 7 as an example to see the relationship between shear force and bending moment.

SFD

$V_A = 0$ (No load at A)

$V_{B(left)} = V_A + $ (Load area)$_{AB} = 0 + (-20 \times 2) = -40$ kN (shear force on the left B)

$V_{B(right)} = V_{B(left)} + $ reaction at B $= -40 + 129 = 89$ kN

$V_C = V_{B(right)} + $ (load area)$_{BC} = 89 + (-20 \times 2) = 49$ kN

$V_{D(left)} = V_C + $ (load area)$_{CD} = 49 + 0$ (No load) $= 49$ kN

$V_{D(right)} = V_{D(left)} + $ (load point)$_D = 49 + (-75) = -26$ kN

$V_E = V_{D(right)} + $ (load area)$_{DE} = -26 + 0$ (No load) $= -26$ kN

$V_{F(left)} = V_E + $ (load area)$_{EF} = -26 + \left(\dfrac{-30 \times 4}{2} \right) = -86$ kN

$V_{F(right)} = V_{F(left)} + $ reaction at F $= -86 + 136 = 50$ kN

$V_G = V_{F(right)}$ + (Load point)$_G$ = 50 + (-50) = 0 kN

SFD :
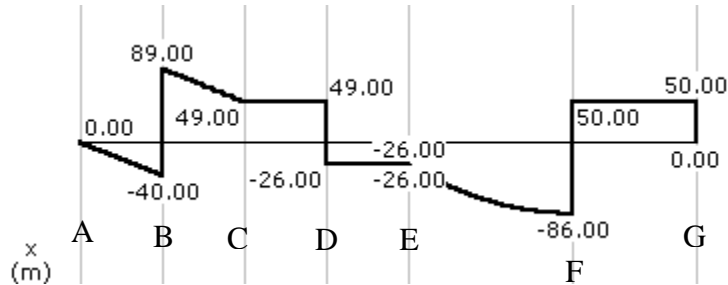
BMD :
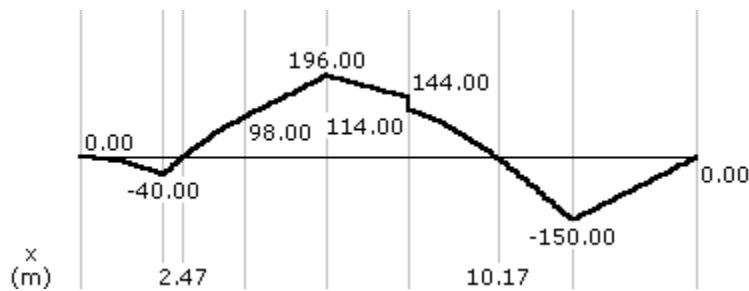
$M_A = 0$

$M_B = M_A$ + (GDR area)$_{AB}$ = 0 + $\left[\dfrac{1}{2}(2)(-40)\right]$ = - 40 kNm

$M_C = M_B$ + (GDR area)$_{BC}$ = - 40 + $\left[\dfrac{1}{2}(2)(89+49)\right]$ = 98 kNm

$M_D = M_C$ + (GDR area)$_{CD}$ = 98 + (49)(2) = 196 kNm

$M_E = M_D$ + (GDR area)$_{DE}$ = 196 + (-26)(2) = 144 kNm (Bending moment on the E)

$M_E = M_{E(left)}$ + Momen di E = 144 + (-30) = 114 kNm (Bending moment on the E)

$M_F = M_E$ + (GDR area)$_{EF}$ = 114 + [(-26)(4)] + $\left[(-60)(4)-[\dfrac{1}{3}(-60)(4)]\right]$ = - 150 kNm

$M_G = M_F + (GDR\ area)_{FG} = -150 + (50)(3) = 0\ kNm$