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FRAME ANALYSIS DUE TO VERTICAL ACTIONS

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- Structure for multi-story building is actually a connected frame of members, each of which are firmly connected to each other.
- In engineering parlance, these connections are called moment connections, which means that the two members are firmly connected to each other.
- This frame becomes very strong, and must resist the various loads that act on a building during its life.
- The concrete frame rests of foundations, which transfer the forces from the building and on the building to the ground.
- Some other important components of concrete frame structures are shear wall, elevator shaft, masonry/reinforced concrete wall and cladding.







Shear wall	Important structural elements in high-rise buildings. Easily measure 400mm thick by 3m. The function is to help take care of <i>horizontal forces</i> on buildings like wind and earthquake loads. Shear walls also carry vertical loads. It is important to understand that shear wall only work for horizontal loads in one direction (the axis of the long dimension of the wall).
Elevator shaft	Vertical boxes in which the elevators move up and down - normally each elevator is enclosed in its own concrete box. These shafts are also very good structural elements, helping to resist horizontal loads, and also carrying vertical loads.
Masonry/ concrete wall	Concrete frame structures are strong and economical. Hence almost any walling materials can be used with them. Common options include masonry wall of brick, concrete block and light steel partition.
Cladding	Common cladding materials are glass, aluminum panels and ceramic facades. Since these structures can be designed for heavy loading, one could even clad them in solid masonry walls of brick or stone.





Building is 3D frame which consist of slabs, beams and columns





Multi-story building: Low-rise vs High-rise



- The building structure is 3D frame, comprising floor slabs, beams, columns and footings, which monolithically connected and act integrally to resist vertical loads and lateral loads.
- In the design of reinforced concrete structures, it has to analyze the structure subjected to all probable combinations of loads, considering the ultimate limit state.
- Commonly 3D frame analysis is the most accurate method to analyse the frame building. However, 3D frame is complex and need to be carried out using relevant computer software (ESTEEM, STAADPro, ETABS, SAP, finite element software, etc.)
- Once the bending moment, shear force and axial load are obtained, reinforcements can be designed according to the standard.



Frame Analysis

In many cases the slabs are analyzed separately, thus, the analysis may be simplified appropriately consist only beams and columns.





Frame Analysis

In order to simplify the analysis, the 3D structure is generally divided into a series of independent parallel 2D plane frames.





• 2D plane frame can be further simplified into 3 levels sub-frames:

i) Complete sub-frame

The frame consists of all beams at each level with columns top and bottom of beams. Moments at columns and beams are tabulated by analyzing the complete sub-frame.

ii) Simplified sub-frame

The frame consists of a selected beam with columns and neighbouring beams at both sides of selected beam.

iii) Simplified sub-frame at point

The frame consists of a selected point or node with columns at top and bottom, and neighbouring beams coming into the point.



• 2D plane frame >>> complete sub-frame





Complete sub-frame >>> Simplified sub-frame





Simplified sub-frame >>> One point sub-frame:





Type of Frame

1) Braced Framed

Frames that not contribute to the overall stability of the structure.

None of the lateral actions, including wind, are transmitted to the columns and beams but carries by bracing members such as shear wall.

Support vertical actions only.





Type of Frame

2) Unbraced Framed

Frame that contribute to the overall stability of the structure.

All lateral actions, including wind, are transmitted to the columns and beams since there are no bracing members such as shear wall are provided.

Support vertical and lateral actions



Method of Analysis

- Primary objective is to obtain a set of internal forces and moments throughout the structure that are in equilibrium with the design loads for the required loading combinations.
- General provisions to analysis are set out in EN 1992-1-1 Section 5.





- General consideration for sub-frame analysis:
 - i) Method of sub-frame analysis can be conducted using onelevel sub-frame, two-point sub-frame or one-point sub-frame with continuous beam.
 - ii) The column or/and beam ends remote from the beam under consideration may generally be assumed to be fixed unless the assumption of pinned is clearly more reasonable.
 - iii) Stiffness for interior beam is K_B .
 - iv) Stiffness for fixed end (beam elements) posses half their actual stiffness, $0.5K_B$.
 - v) The arrangement of the design ultimate variable loads should be such as to cause the maximum moment the column.



- One-level sub-frame
 - Each sub-frame consist of the beams at one level together with the columns above and below.
 - The ends of the columns remote from the beams may generally be assumed to be fixed unless the assumption of a pinned end is clearly more reasonable



- At least four cases combination of actions: [Max][Min][Max]; [Min][Max][Min] [Max][Max][Min]; [Min][Max][Max]



- Two-point sub-frame
 - The moments and forces in certain individual beam may be found by considering a simplified sub-frame consisting only of the beam, the columns attached to the end of that beam and the beams on either side is any.



- Load at interior beam where stiffness = K_B is always for maximum design load.



- One-point sub-frame with continuous beam
 - The moments and forces in the beams at one level \rightarrow considering the beams as a continuous beam over supports providing no restraint to rotation.
 - The ultimate moment for column → simple moment distribution procedure





- Action on buildings is due to permanent (dead load), variable (imposed, wind, dynamic, seismic loads) and accidental load.
- Mostly multistory buildings for office or residential purpose are design for dead, imposed and wind loads.
- Separate actions must be applied to the structure in appropriate directions and various types of actions combined with partial safety factors selected to cause the most severe design condition.
- Maximum design load = $1.35G_k + 1.5Q_k$
- Minimum design load = 1.35G_k
- Wind load = $1.2W_k$
- Vertical load due to wind = $1.2G_k + 1.2Q_k$



For the combination of dead load and imposed load, the following loading patterns are considered:

Braced frame	1) All spans loaded with maximum dead plus imposed loads									
	2) Alternate spans loaded with maximum dead load and imposed load and all other spans loaded with minimum dead load									
Unbraced frame	1) Three cases loading arrangements as braced sub-frame									
	2) Vertical actions for sub-frame									
	3) Wind load for complete frame									



Combination of Actions

- Load cases for braced frame
- Vertical load

 [Max] [Max] [Max]
 [Min] [Min] [Min]
 [Max] [Min] [Max]
 [Min] [Max] [Min]
 [Max] [Max] [Min]
 [Max] [Max] [Max]
 [Min] [Max] [Max]



With Wisdom We Explore



- Load cases for unbraced frame
- Vertical load from wind loading + lateral load





- Analysis procedure for braced frame:
 - 1. Analyse all actions, maximum and minimum design loads
 - 2. Calculate moment inertia, $I = bh^3 / 12$
 - 3. Calculate stiffness of beams and columns, k = I/L
 - 4. Determine distribution factor, $DF = k_i / \Sigma k$
 - 5. Determine fixed end moment (FEM) of beams
 - 6. Perform moment distribution by cases:
 - a) Case 1 [Max][Max][Min]
 - b) Case 2 [Min][Min][Max]
 - c) Case 3 [Max][Min][Max]
 - d) Case 4 [Min][Max][Min]
 - Calculate actual shear force and bending moment. Draw BMD and SFD diagrams



- Analysis procedure for unbraced frame:
 - 1. Calculate design wind load, $W_d = 1.2W_k$
 - 2. Calculate lateral point load at each level of frame
 - a) Assume contra-flexure point at center of frame
 - Axial loads in column are in its proportion to distances from the centre of gravity of frame
 - c) All columns are equal cross-section area
 - 3. Lateral load analysis using Cantilever Method.
 - Calculate axial force in columns, then shear force in beams and columns from top to ground levels.
 - 4. Vertical load analysis due to wind, 1.2 G_k + 1.2 Q_k
 - analysis of one level sub-frame



The framing plans for a multistory building are shown in the figure. The main dimensions structural features, loads, material, etc. are also set out as at the given data. Analyze sub frame 3/A-D, Level 1 to determine shear forces and bending moments of corresponding beams and columns. Use all the three methods of analysis.

- Permanent office building (Design life = 50 years)
- Location: Near sub-urban (Zone 1 of Malaysia wind speed mapping)
- Topography: Flat area-slope<0.05 (Building around within 1 KM radius)
- Beam in grid line 1,2,3...12 : 250 x 600 mm
- Beam in grid line A, B, C & D : 250 x 500 mm
- Slab thickness = 150 mm
- Columns : 300 x 400 mm
- Imposed load : 4.0 kN/m²
- Finishes, ceiling, services etc : 0.75 kN/m² ; Partitions : 0.5 kN/m²





Figure 2.1



- Step analysis of braced frame:
 - 1. Analysis actions on beam (load transfer from slab, selfweight and wall)
 - 2. Calculate moment of inertia for beams and columns, $I = bh^3/12$
 - 3. Calculate stiffness, k = I/L
 - 4. Calculate distribution factor, $DF = k_i / \Sigma k$
 - 5. Determine carry over factor, CF=0.5 (for pin support)
 - Determine Fixed End Moment, FEM=wl²/12 (for uniform load)
 - 7. Moment distribution by cases:
 - Case 1 [Span 1,2: max / span 3: min]
 - Case 2 [Span 1: min / span 2,3: max]
 - Case 3 [Span 1,3: max / span 2: min]
 - Case 4 [Span 1,3: min / span 2: max]
 - 8. Draw BMD and SFD diagrams



Action on beam:

Ref.		Calcula	ations			Output
	SPECIFICATION					
	Loading:					
	Finishes, ceiling, serv	vices etc.	=	0.75	kN/m ²	
	Density of concrete		=	25	kN/m ³	
	Imposed load		=	4.00	kN/m ²	
	Partition		=	0.50	kN/m ²	
	Dimension:			_		
	Slab thickness, h		=	150	mm	
	Beam size, $b \ge h$	=	250	x 600	mm	
	Column size, $b \ge h$	=	300	x 400	mm	





Action on beam:





Action on beam:

$$v_{sx} = \beta_{vx} n l_x \qquad v_{sy} = \beta_{vy} n l_x$$

Transformed		β_{vx} for values of $l \mathcal{A}_x$										
Type of panel location	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	Руу			
Four edges continuous												
Continuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33			
One short edge discontinuous												
Continuous edge	0.36	0.39	0.42	0.44	0.45	0.47	0.50	0.52	0.36			
Discontinuous edge	-	-	-	-	-	-	-	-	0.24			
One long edge discontinuous												
Continuous edge	0.36	0.40	0.44	0.47	0.49	0.51	0.55	0.59	0.36			
Discontinuous edge	0.24	0.27	0.29	0.31	0.32	0.34	0.36	0.38	-			
Two adjacent edges discontinuous												
Continuous edge	0.40	0.44	0.47	0.50	0.52	0.54	0.57	0.60	0.40			
Discontinuous edge	0.26	0.29	0.31	0.32	0.34	0.35	0.38	0.40	0.26			
Two short edges discontinuous												
Continuous edge	0.40	0.43	0.45	0.47	0.48	0.49	0.52	0.54	-			
Discontinuous edge	-	-	-	-	-	-	-	-	0.26			
Two long edges discontinuous												
Continuous edge	-	-	-	-	-	-	-	-	0.40			
Discontinuous edge	0.26	0.30	0.33	0.36	0.38	0.40	0.44	0.47	-			
Three edges discontinuous (one long edge continuous)												
Continuous edge	0.45	0.48	0.51	0.53	0.55	0.57	0.60	0.63	-			
Discontinuous edge	0.30	0.32	0.34	0.35	0.36	0.37	0.39	0.41	0.29			
Three edges discontinuous (one short edge continuous)												
Continuous edge	-	-	-	-	-	-	-	-	0.45			
Discontinuous edge	0.29	0.33	0.36	0.38	0.40	0.42	0.45	0.48	0.30			
Four edges discontinuous												
Discontinuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33			







Action on beam:

ACTI	ONS						
Loads	on slab, n l	kN/m^2 :					
Slab se	lfweight =	0.15	x 25	=	3.75	kN/m ²	
Finishe	s, ceiling etc.	=			0.75	kN/m ²	
Chara	acteristic pern	nanent load,	$g_k =$		4.50	kN/m ²	
Impose	ed load =				4.00	kN/m ²	
Partitio	n =				0.50	kN/m ²	
Chara	acteristic varia	able load, q_1	k =		4.50	kN/m ²	
Loads	on beam, w	kN/m :					
		W2					
and the second			COLUMN POINT	W 3		 	
		and a logical state of the state of the	Ť		-		
6.0	m	6.0	m	6.0	m –		
A	 (B)		(c)		C C	シー	
	\sim		-				



Action on beam:

a set a construction of the set o				2 · · · · · · · · · · · · · · · · · · ·		Same and State and State and State and State		
Span 1 (A-B) :	<u>w</u> 1							
Perm. load from	m slab =	0.47 x	4.50 x	4.0 =	8.46	kN/m		
Perm. load from	m slab =	0.42 x	4.50 x	5.0 =	9.45	kN/m		
Beam selfweig	ht =	0.45 x	0.25 x	25 =	2.81	kN/m		
Characteristic	permaner	nt load, (G _k	=	20.72	kN/m		
Variable load f	r.slab =	0.47 x	4.50 x	4.0 =	8.46	kN/m		
Variable load f	r. slab =	0.42 x	4.50 x	5.0 =	9.45	kN/m		
Chacracteristic	variable	load, Q	_k =		17.91	kN/m		
Design load,	$1.35G_k$	$+1.5Q_{k}$		=	54.84	kN/m		
	$1.35G_k$			=	27.98	kN/m		
Span 2 (B-C) :	<u>w</u> 2							
Perm. load from	m slab =	0.45 x	4.50 x	4.0 =	8.10	kN/m		
Perm. load from	Perm. load from slab =			5.0 =	8.78	kN/m	1	
Beam selfweig	Beam selfweight =			25 =	2.81	kN/m		1
Characteristic	permanei	nt load, g	3 k	=	19.69	kN/m		
	Span 1 (A-B) :Perm. load fromPerm. load fromBeam selfweigCharacteristicVariable load fVariable load fChacracteristicDesign load,Span 2 (B-C) :Perm. load fromPerm. load fromBeam selfweigCharacteristic	Span 1 (A-B) : w_1 Perm. load from slab =Perm. load from slab =Beam selfweight =Characteristic permanenVariable load fr.slab =Variable load fr.slab =Chacracteristic variableDesign load, $1.35G_k$ $1.35G_k$ Span 2 (B-C) : w_2 Perm. load from slab =Perm. load from slab =Beam selfweight =Characteristic permanen	Span 1 (A-B) : w_1 Perm. load from slab =0.47 xPerm. load from slab =0.42 xBeam selfweight =0.45 xCharacteristic permanent load, CVariable load fr.slab =0.47 xVariable load fr.slab =0.47 xVariable load fr.slab =0.42 xChacracteristic variable load, fr.slab =0.42 xChacracteristic variable load, fr.slab =0.42 xChacracteristic variable load, fr.slab =0.42 xDesign load, $1.35G_k + 1.5Q_k$ $1.35G_k$ $1.35G_k$ Perm. load from slab =0.45 xPerm. load from slab =0.39 xBeam selfweight =0.45 xCharacteristic permanent load, g	Span 1 (A-B) : w_1 Perm. load from slab = $0.47 \times 4.50 \times$ Perm. load from slab = $0.42 \times 4.50 \times$ Beam selfweight = $0.45 \times 0.25 \times$ Characteristic permanent load, G_k Variable load fr.slab = $0.47 \times 4.50 \times$ Variable load fr.slab = $0.47 \times 4.50 \times$ Chacracteristic variable load, g_k =Design load, $1.35G_k + 1.5Q_k$ 1.35G_kSpan 2 (B-C) : w_2 Perm. load from slab = $0.45 \times 4.50 \times$ Perm. load from slab = $0.45 \times 4.50 \times$ Ream selfweight = $0.45 \times 0.25 \times$ Characteristic permanent load, g_k	Span 1 (A-B) : w_1 Perm. load from slab = $0.47 \times 4.50 \times 4.0 =$ Perm. load from slab = $0.47 \times 4.50 \times 5.0 =$ Beam selfweight = $0.42 \times 4.50 \times 5.0 =$ Characteristic permanent load, G_k =Variable load fr.slab = $0.47 \times 4.50 \times 4.0 =$ Variable load fr.slab = $0.47 \times 4.50 \times 5.0 =$ Chacracteristic variable load, $g_k =$ $0.42 \times 4.50 \times 5.0 =$ Chacracteristic variable load, $Q_k =$ $0.42 \times 4.50 \times 5.0 =$ Chacracteristic variable load, $Q_k =$ $0.42 \times 4.50 \times 5.0 =$ Chacracteristic variable load, $0.45 \times 4.50 \times 5.0 =$ $0.45 \times 4.50 \times 4.0 =$ Perm. load from slab = $0.45 \times 4.50 \times 4.0 =$ Perm. load from slab = $0.39 \times 4.50 \times 5.0 =$ Beam selfweight = $0.45 \times 0.25 \times 25 =$ Characteristic permanent load, g_k =	Span 1 (A-B) : w_1 Perm. load from slab = $0.47 \times 4.50 \times 4.0 =$ 8.46 Perm. load from slab = $0.42 \times 4.50 \times 5.0 =$ 9.45 Beam selfweight = $0.45 \times 0.25 \times 25 =$ 2.81 Characteristic permanent load, G_k = 20.72 Variable load fr.slab = $0.47 \times 4.50 \times 4.0 =$ 8.46 Variable load fr.slab = $0.47 \times 4.50 \times 4.0 =$ 8.46 Variable load fr.slab = $0.47 \times 4.50 \times 5.0 =$ 9.45 Chacracteristic variable load, $Q_k =$ 17.91 Design load, $1.35G_k + 1.5Q_k$ = Span 2 (B-C) : w_2 27.98 Perm. load from slab = $0.45 \times 4.50 \times 4.0 =$ 8.10 Perm. load from slab = $0.39 \times 4.50 \times 5.0 =$ 8.78 Beam selfweight = $0.45 \times 0.25 \times 25 =$ 2.81 Characteristic permanent load, g_k = 19.69	Span 1 (A-B) : w_1 Perm. load from slab = $0.47 \times 4.50 \times 4.0 =$ 8.46 kN/m Perm. load from slab = $0.42 \times 4.50 \times 5.0 =$ 9.45 kN/m Beam selfweight = $0.45 \times 0.25 \times 25 =$ 2.81 kN/m Characteristic permanent load, G_k = 20.72 kN/m Variable load fr.slab = $0.47 \times 4.50 \times 4.0 =$ 8.46 kN/m Variable load fr.slab = $0.47 \times 4.50 \times 4.0 =$ 8.46 kN/m Variable load fr.slab = $0.47 \times 4.50 \times 4.0 =$ 8.46 kN/m Chacracteristic variable load, $g_k =$ 17.91 kN/m Design load, $1.35G_k + 1.5Q_k$ = $1.35G_k$ = 27.98 kN/m Span 2 (B-C) : w_2 27.98 kN/m Perm. load from slab = $0.45 \times 4.50 \times 4.0 =$ 8.10 kN/m Perm. load from slab = $0.39 \times 4.50 \times 5.0 =$ 8.78 kN/m Beam selfweight = $0.45 \times 0.25 \times 25 =$ 2.81 kN/m	Span 1 (A-B) : w_1 Perm. load from slab = $0.47 \times 4.50 \times 4.0 =$ 8.46 kN/m Perm. load from slab = $0.42 \times 4.50 \times 5.0 =$ 9.45 kN/m Beam selfweight = $0.45 \times 0.25 \times 25 =$ 2.81 kN/m Characteristic permanent load, G_k = 20.72 kN/m Variable load fr.slab = $0.47 \times 4.50 \times 4.0 =$ 8.46 kN/m Variable load fr.slab = $0.47 \times 4.50 \times 4.0 =$ 8.46 kN/m Variable load fr.slab = $0.47 \times 4.50 \times 5.0 =$ 9.45 kN/m Chacracteristic variable load, $g_k =$ 17.91 kN/m Design load, $1.35G_k + 1.5Q_k$ = $1.35G_k$ = 27.98 kN/m Perm. load from slab = $0.45 \times 4.50 \times 4.0 =$ 8.10 kN/m Perm. load from slab = $0.39 \times 4.50 \times 5.0 =$ 8.78 kN/m Beam selfweight = $0.45 \times 0.25 \times 25 =$ 2.81 kN/m Characteristic permanent load, g_k = 19.69 kN/m



Action on beam:

Span 2 (B-C) :	<u><i>W</i></u> ₂						
Variable load f	ir. slab =	0.45 x	4.50 x	4.0 =	8.10	kN/m	
Variable load f	ir. slab =	0.39 x	4.50 x	5.0 =	8.78	kN/m	
Chacracteristic	variabk	e load, Q	$P_{\mathbf{k}} = 1$		16.88	kN/m	
 Design load,	$1.35G_k$	$+1.5Q_{k}$		=	51.89	kN/m	
	1.35G _k			=	26.58	kN/m	
Span 3 (C-D)	: w ₃						
Perm. load fro	m slab =	0.47 x	4.50 x	4.0 =	8.46	kN/m	
Perm. load fro	m slab =	0.42 x	4.50 x	5.0 =	9.45	kN/m	
Beam selfweig	ght =	0.45 x	0.25 x	25 =	2.81	kN/m	
Characteristic	permaner	nt load, g	3 k	=	20.72	kN/m	
Variable load f	rom slab	0.47 x	4.50 x	4.0 =	8.46	kN/m	
Variable load f	rom slab	0.42 x	4.50 x	5.0 =	9.45	kN/m	
Chacracteristic	variable	e load, Q	$k_{\mathbf{k}} =$		17.91	kN/m	
Design load,	$1.35G_k$	$+1.5Q_{k}$:	=	54.84	kN/m	
	1.35G _k			=	27.98	kN/m	



Analysis of one level sub-fame

Ref.				Calcu	alculations						Output	t I
	Data:			-	en los contratos							
		-			The second second second					Actions	s:	
									T	w_1 :		kN/m
		w ₁	kN/m	и	2	N/m	w	3 kN/m	3.5 m	1.35Gk	=	28.0
				1 2.25				1.14		1.35Gk	$+1.5Q_{k}=$	54.8
					and the second					w_2 :	i.	
									4.0 m	1.35Gk	=	26.6
		6.0	m	6.	0 n	n	6.0	0 m	-	1.35Gk	$+1.5Q_{k}=$	51.9
sion from top to only writer	A	_		В			с		D	w ₃ :		
	Size :									1.35G _k	=	28.0
	Beam :	b x h	1 =	25	0 x	600	mm			1.35Gk	$+1.5Q_{k}=$	54.8
	Colum	$\mathbf{h}: b \mathbf{x}$	h =	30	0 >	400	mm					
100 1000 100 100 100 1000 1000 1000 10												www.


Analysis of one level sub-fame

Momen of Inerti	$a: I = bh^3/12$			
Beam : $I =$	250 x 600	$^{3}/12 =$	$4.5 \text{ x} 10^9 \text{ mm}^4$	
Column: I =	300 x 400	³ /12 =	$1.6 \text{ x} 10^9 \text{ mm}^4$	
				s
Stiffness : $K =$	= I/L			
Column: $K_{cu} =$	1.6 x $10^9 /$	3500 =	$4.6 \text{ x} 10^5 \text{ mm}^3$	
$K_{\rm cl} =$	$1.6 \text{ x} 10^9 /$	4000 =	$4.0 \text{ x} 10^5 \text{ mm}^3$	
Beam: $K_{AB} =$	$4.5 \text{ x} 10^9 /$	6000 =	$7.5 \text{ x} 10^5 \text{ mm}^3$	
$K_{\rm BC} =$	$4.5 \text{ x} 10^9 /$	6000 =	$7.5 \text{ x} 10^5 \text{ mm}^3$	
$K_{\rm CD} =$	4.5 x 10 ⁹ /	6000 =	7.5 x 10° mm ³	



Analysis of one level sub-fame

Distribu	ation factor : $F = K / \Sigma K$		
Joint A	$F_{\rm AB} = K_{\rm AB} / (K_{\rm AB} + K_{\rm cu} + K_{\rm cl}) =$	0.47	
	$F_{\rm cu} = K_{\rm cu} / (K_{\rm AB} + K_{\rm cu} + K_{\rm cl}) =$	0.28	
	$F_{\rm cl} = K_{\rm cl} / (K_{\rm AB} + K_{\rm cu} + K_{\rm cl}) =$	0.25	
Joint B:	$F_{\rm BA} = K_{\rm AB} / (K_{\rm AB} + K_{\rm BC} + K_{\rm cu} + K_{\rm cl}) =$	0.32	
	$F_{\rm BC} = K_{\rm BC} / (K_{\rm AB} + K_{\rm BC} + K_{\rm cu} + K_{\rm cl}) =$	0.32	
	$F_{\rm cu} = K_{\rm cu}/(K_{\rm AB}+K_{\rm BC}+K_{\rm cu}+K_{\rm cl}) =$	0.19	
	$F_{\rm cl} = K_{\rm cl} / (K_{\rm AB} + K_{\rm BC} + K_{\rm cu} + K_{\rm cl}) =$	0.17	
Joint C:	$F_{\rm CB} = K_{\rm BC} / (K_{\rm BC} + K_{\rm CD} + K_{\rm cu} + K_{\rm cl}) =$	0.32	
	$F_{\rm CD} = K_{\rm CD} / (K_{\rm BC} + K_{\rm CD} + K_{\rm cu} + K_{\rm cl}) =$	0.32	
	$F_{\rm cu} = K_{\rm cu}/(K_{\rm BC} + K_{\rm CD} + K_{\rm cu} + K_{\rm cl}) =$	0.19	
	$F_{\rm cl} = K_{\rm cl} / (K_{\rm BC} + K_{\rm CD} + K_{\rm cu} + K_{\rm cl}) =$	0.17	
Joint D	$F_{\rm DC} = K_{\rm CD} / (K_{\rm CD} + K_{\rm cu} + K_{\rm cl}) =$	0.47	
	$F_{\rm cu} = K_{\rm cu}/(K_{\rm CD} + K_{\rm cu} + K_{\rm cl}) =$	0.28	
	$F_{\rm cl} = K_{\rm cl} / (K_{\rm CD} + K_{\rm cu} + K_{\rm cl}) =$	0.25	



Ref.				С	alculation	s			Ou	tput
	Case 1	: Span	1&20	lesign p	e rmane nt	& variabl	le loads	1.35G _k	+ 1.5Q _k	
		Span	3 design	n permai	nent loads	1.35Gk				
	-	-	_	-				Fixed en	d moment :	
n waken oosana ah in tu			Ma	x		Min		$-M_{AB} =$	$M_{\rm BA} = w$	$L_1^2/12$
a ana an	1	54.8	kN/m	51.9 k	N/m 2	8.0 kN/m	3.5m	=	54.8 x 6	$5.0 ^2/12$
		Sector of	denostania de la	AT CREAT AND	The later	NUMBER OF THE OWN		=	164.5 kNr	n
								$-M_{\rm BC} =$	$M_{\rm CB} = w$	$_{2}L_{2}^{2}/12$
							4.0m	=	51.9 x 6	5.0 ² /12
Parlan <mark>i (mand</mark> aran)		6.0	m	6.0 m	1	5.0 m		=	155.7 kN1	n
waa aageetee waa koor oo k	A	•	B		-	1	D	$-M_{CD} =$	$M_{\rm DC} = w_{\rm c}$	$L_1^2/12$
anna anna anna anna anna	1	******					1	=	28.0 x 6	5.0 ² /12
and water is a serie							1	=	83.9 kN1	n



Mome	nt distribution				\sim				
48.58			-7.90	<mark>-</mark> (B _A	$+B_c$	-9.26			-21.65
0.17			-0.24		مر	0.26			-0.07
1.22	Beam		-0.71	Co Co		0.32	Colum	nn	-0.95
0.40			-5.23			4.07			3.25
46.80			-1.72			-13.91			-23.87
0.28	A	В	0.19		C	0.19		D	0.28
0.25	Ø.4X	0.32	0.17	9.32	0.32	0.17	0.32	0.47	0.25
	-164.5	164.5		-155.7	155.7	7 4	-83.9	83.9	
40.9	76.8	-2.8	-1.5	-2.8	-22.8	-12.2	-22.8	-39.2	-20.9
	-1.4	38.4		-11.4	-1.4		-19.6	-11.4	
0.4	0.7	-8.6	-4.6	-8.6	6.7	3.6	6.7	5.3	2.8
	-4.3	0.3		3.3	-4.3		2.7	3.3	
1.1	2.0	-1.2	-0.6	-1.2	0.5	0.3	0.5	-1.6	-0.8
	-0.6	1.0		0.3	-0.6		-0.8	0.3	
0.1	0.3	-0.4	-0.2	-0.4	0.4	0.2	0.4	-0.1	-0.1
42.5	-91.1	191.8	-6.9	-176.5	134.2	-8.1	-116.8	40.6	-18.9







ΣM (a) C =	0						_	-
6.0	$V_{\rm BC}$ -	(51.9	x 6.0 x	3.0) +	134.2	- 176.5	= 0		
V _B	2 = (934.03	- 134.2	+176.5) / 6.0	=	162.7	kN	
V _{CI}	3 = (51.9 x	6.0) -	162.7		=	148.6	kN	
ΣM (a) D =	0							
6.	$V_{\rm CD}$ -	(28.0	x 6.0 x	3.0) -	116.8	+ 40.6	= 0		
V _{CI}	5 = (503.56	+116.8	- 40.6)/6.0	=	96.6	kN	
V _D	2 = (28.0 x	6.0) -	96.6		=	71.2	kN	
					1000				







Ref.	a pro contactingui a contracting in the second	Calculations						Output						
	Case 2: Spa	Case 2: Span 2 & 3 design permanent & variable loads $1.35G_k + 1.5Q_k$												
	Spa	n 1 desig	n perman	ent loads 1.	$35G_k$									
			_	_	_	1	Fixed en	d moment :						
		Min			Γ	- <i>M</i> _{AB} =	$M_{\rm BA} = w_1 L_1^2 / 12$							
	28	0 kN/m	51.9 kN	V/m 54.8	kN/m	3.5m	=	28.0 x 6.0 $^{2}/12$						
Nakamata kenara ke			SALL				=	83.9 kNm						
							$-M_{BC} =$	$M_{\rm CB} = w_2 L_2^2 / 12$						
						4.0m	=	51.9 x 6.0 $^{2}/12$						
	6	0 m	6.0 m	6.0	m		=	155.7 kNm						
anii Sana Gani Singa		-	5	<u> </u>	•	D	- <i>M</i> _{CD} =	$M_{\rm DC} = w_1 L_1^2 / 12$						
ini-initor -inicitor	· · · · · · · · · · · · · · · · · · ·		•				=	54.8 x 6.0 ² /12						
							=	164.5 kNm						







Ref.				Calcula	tions			Outpu	t
	Case 3:	Span	1&3	design perman	ent & variabl	e loads	1.35G _k	+ 1.5Q _k	
		Span	2 desig	n permanent k	bads $1.35G_k$				
	-		_	-	_		Fixed er	nd moment :	
		N	lax	Min	Max		- <i>M</i> _{AB} =	$=M_{\rm BA}=w_1L_1^2$	2/12
		54.8	kN/m	26.6 kN/m	54.8 kN/m	3.5m	=	54.8 x 6.0	² /12
							=	164.5 kNm	
							$-M_{\rm BC} =$	$M_{\rm CB} = w_2 L_2^2$	/12
						4.0m	=	26.6 x 6.0	² /12
		6.0	m	6.0 m	6.0 m		=	79.7 kNm	
	A		R			D	$-M_{CD} =$	$M_{\rm DC} = w_1 L_1^2$	/12
		-1				Ī	=	54.8 x 6.0	² /12
944						1	=	164.5 kNm	







Ref.			Calcula	tions		Output							
	Case 4:	Case 4: Span 2 design permanent & variable loads $1.35G_k + 1.5Q_k$											
	1	Span 1 & 3	design permane	nt loads 1.35	Gk								
	_	-		-		Fixed end moment :							
		Min	Max	Min		$-M_{AB} = M_{BA} = w_1 L_1^2 / 12$							
		28.0 kN/m	51.9 kN/m	28.0 kN/m	3.5m	$=$ 28.0 x 6.0 $^{2}/12$							
					3	= 83.9 kNm							
		_			in the second	$-M_{\rm BC} = M_{\rm CB} = w_2 L_2^2 / 12$							
					4.0m	$=$ 51.9 x 6.0 $^{2}/12$							
		6.0 m	6.0 m	6.0 m		= 155.7 kNm							
	A				D	$-M_{\rm CD} = M_{\rm DC} = w_1 L_1^2 / 12$							
				·		$=$ 28.0 x 6.0 $^{2}/12$							
			1			= 83.9 kNm							







Analysis of one level sub-fame : ENVELOPE



With Wisdom We Explore



 Beam	A-B (B	eam C-	D simi	lar)					
Case	l : Span	1 desig	n pern	nanent ó	è varia	ble load	s 1.350	G _k +1.5	5Q k
 	: Span	2 desig	gn peri	nanent	& vari	able load	ls 1.350	$G_k + 1$.	5Q k
						Fixed er	nd mome	ent :	
	Ma	x		Max		- <i>M</i> _{AB} =	$=M_{\rm BA}=$	w_1L_1	2/12
 4.6	54.84	kN/m	51.89	kN/m		-	54.8	x 6.0	² /12
					3	=	164.5	kNm	
	7.5			3.75	1	- <i>M</i> _{BC} =	= M _{CB} =	w_2L_2	² /12
4.0						- A	51.9	x 6.0	² /12
	6.0	m		6.0 m		=	155.7	kNm	
A			в						
Distrib	ution fact	or:							
Joint A	$F_{AB} =$	7.5/	(7.5+	4.6 +	4.0)	=	0.47		
	$F_{\rm cu} =$	4.6/	(7.5+	4.6+	4.0)	=	0.28		
	$F_{cl} =$	4.0 /	(7.5 +	4.6 +	4.0)	=	0.25		Į
Joint B	$: F_{BA} =$	7.5/	(7.5 +	3.75 +	4.0 +	4.6) =	0.38		
	$F_{\rm BC} =$	3.8/	(7.5 +	3.75 +	4.0 +	4.6) =	0.19		
	$F_{\rm cu} =$	4.0 /	(7.5 +	3.8 +	4.0 +	4.6) =	0.20		ļ
	$F_{cl} =$	4.6/	(7.5 +	3.8 +	4.0 +	4.6) =	0.23		



	Momer	nt distribution					
	49.36			-9.95			
	0.02			-0.34			
	2.07			-0.08			
	0.48			-7.75			
	46.80			-1.79			
	0.28	A	В	0.20			
	0.25	0.47	0.38	0.23	0.19		
		-164.5	164.5		-155.7		
	40.9	76.8	-3.3	-2.0	-1.7	 _	
		-1.7	38.4	2	0.0		
	0.4	0.8	-14.5	-8.9	-7.3		
		-7.3	0.4		0.0		
	1.8	3.4	-0.1	-0.1	-0.1		
		-0.1	1.7		0.0		
1	0.0	0.0	-0.6	-0.4	-0.3		
	43.2	-92.6	186.3	-11.4	-165.0		



		: Span	2 desig	n pern	anent l	oads 1.	.35G _k				
		Ma	X	ST. Sette	Min		Fixed en	d mom	ent :		
			1	Г			$-M_{AB} =$	$M_{BA} =$	$= w_1 L_1^2$	2/12	
	4.6	54.84	kN/m	26.58	kN/m		=	54.8	x 6.0	2/12	
		The sease					=	164.5	kNm		
anne anne t		7.5			3.75	1	$-M_{\rm BC} =$	M _{CB} =	w_2L_2	² /12	
	4.0						=	26.6	x 6.0	² /12	
		6.0	m		6.0 m		-	79.7	kNm	10	
	A			в							
	Distribu	tion fact	tor:								
	Joint A:	$F_{AB} =$	7.5 /	(7.5 +	4.6+	4.0)	=	0.47			
		$F_{cu} =$	4.6 /	(7.5 +	4.6+	4.0)	=	0.28	-		
		$F_{cl} =$	4.0 /	(7.5 +	4.6+	4.0)	=	0.25			
	Joint B:	$F_{BA} =$	7.5 /	(7.5 +	3.75 +	4.0+	4.6) =	0.38			
		$F_{\rm BC} =$	3.8 /	(7.5 +	3.75 +	4.0+	4.6) =	0.19			
		$F_{cu} =$	4.0 /	(7.5 +	3.8+	4.0+	4.6) =	0.20			
		$F_{\rm cl} =$	4.6 /	(7.5 +	3.8+	4.0+	4.6) =	0.23		1	



	Momer	nt distribution				and and			
	52 (2		_	25.05					
	53.03			-25.95			1	 	
	0.20			-0.34					
	2.07			-0.76	2011			 	
_	4.56			-7.75					
	46.80			-17.11					
	0.28	A	В	0.20					
	0.25	0.47	0.38	0.23	0.19				
		-164.5	164.5	1.2	-79.7			+	
	40.9	76.8	-32.1	-19.6	-16.0				
		-16.0	38.4		0.0		-		
	4.0	7.5	-14.5	-8.9	-7.3				
		-7.3	3.7	II.	0.0				
	1.8	3.4	-1.4	-0.9	-0.7				
		-0.7	1.7	1	0.0				
	0.2	0.3	-0.6	-0.4	-0.3				
	46.9	-100.6	159.7	-29.7	-104.1				
-									
								1	











	Beam B-C									
	Case 1: Span	1 and 2 c	design	perma	nent &	variab	le loads	1.35G	+ 1.5	2ĸ
	: Span	3 design	n pe rn	nane nt l	oad 1.3	35G _k				
	_	_					Fixed er	nd mome	ent :	
	4.6						- <i>M</i> _{AB} =	$= M_{\rm BA} =$	w_1L_1	2/12
	54.84 kN/m	51.89 k	N/m		27.98	kN/m	=	54.8	x 6.0	² /12
*******							=	164.5	kNm	
	3.75	7.	.50	1.10	3.75		- <i>M</i> _{BC} =	$= M_{\rm CB} =$	w_2L_2	2/12
	4.0					D	=	51.9	x 6.0	² /12
							=	155.7	kNm	
	6.0 m	3 6	.0 m	c	6.0 m		-M _{CD} =	$= M_{\rm DC} =$	w_3L_3	²/12
							=	28.0	x 6.0	² /12
			-				=	83.9	kNm	
	Distribution fact	tor:					10.0.ps	Sec 1		
	Joint B: $F_{BA} =$	3.75 / (3	3.75 +	4.57 +	4.0 +	7.5) =	0.19			
	$F_{\rm BC} =$	7.50 / (3	3.75 +	4.57 +	4.0 +	7.5) =	0.38			
	$F_{cu} =$	4.57 / (3	3.75 +	4.57 +	4.0 +	7.5) =	0.23			
	$F_{\rm d} =$	4.00 / (3	3.75 +	4.57 +	4.0 +	7.5) =	0.20			
	Joint C: $F_{CB} =$	7.50 / (3	3.75 +	7.50 +	4.0 +	4.6) =	0.38			
	$F_{\rm CD} =$	3.75 / (3	3.75 +	7.50 +	4.0 +	4.6) =	0.19	10.000		
	$F_{cu} =$	4.57 / (3	3.75 +	7.50 +	4.0 +	4.6) =	0.23			
	$F_{d} =$	4.00 / (3	3.75 +	7.50 +	4.0 +	4.6) =	0.20			



Momen	t distrib	ution					2 Sure	1	
	1.13				-16.74				
	0.11			_	0.01				
	-0.07				-0.59			1	
	3.13				0.39				
	-2.04				-16.55				
В	0.23			C	0.23				
0.19	0.20	0.38		0.38	0.20	0.19			
164.5		-155.7	-	155.7		-83.9			
-1.7	-1.8	-3.3		-27.1	-14.5	-13.6			
 0.0		-13.6		-1.7		0.0			
2.6	2.7	5.1		0.6	0.3	0.3			
0.0		0.3		2.6		0.0			
-0.1	-0.1	-0.1		-1.0	-0.5	-0.5			
 0.0		-0.5		-0.1	1	0.0			
0.1	0.1	0.2		0.0	0.0	0.0			
165.4	1.0	-167.6		129.0	-14.6	-97.7			



Case 2	: Span 2	2 desiş	ls 1.35 <i>G</i>	_k +1.5	Q _k					
	: Span	1 and 3	3 desig	n perma	nent l	oad 1.3	5G _k			
	-	-					Fixed en	d mom	ent :	
	4.6						$-M_{AB} =$	$M_{\rm BA} =$	$= w_1 L_1$	² /12
27.98	kN/m	51.89	kN/m		27.98	kN/m	=	28.0	x 6.0	² /12
							=	83.9	kNm	
I	3.75		7.50		3.75	1	$-M_{\rm BC} =$	$M_{\rm CB} =$	$= w_2 L_2$	²/12
^	4.0					- Б	=	51.9	x 6.0	² /12
							=	155.7	kNm	
6.0	m B		6.0 m	c	6.0 m		-M _{CD} =	$M_{\rm DC} =$	$= w_3 L_3$	² /12
							=	28.0	x 6.0	² /12
							=	83.9	kNm	
Distribut	tion facto	or:								_
Joint B:	$F_{BA} =$	3.75/	(3.75 +	4.57 +	4.0+	7.5) =	0.19			
	$F_{\rm BC} =$	7.50/	(3.75 +	4.57 +	4.0+	7.5) =	0.38			
	$F_{cu} =$	4.57/	(3.75 +	4.57 +	4.0+	7.5) =	0.23			
	$F_{\rm cl} =$	4.00 /	(3.75 +	4.57 +	4.0+	7.5) =	0.20			
Joint C:	$F_{\rm CB} =$	7.50/	(3.75 +	7.50 +	4.0+	4.6) =	0.38			
	$F_{\rm CD} =$	3.75 /	(3.75 +	7.50 +	4.0+	4.6) =	0.19			
	$F_{\rm cu} =$	4.57/	(3.75 +	7.50 +	4.0+	4.6) =	0.23			
	$F_{\rm cl} =$	4.00 /	(3.75 +	7.50 +	4.0+	4.6) =	0.20			



Momen	t distrib	ution							
	20.38			-20.38					
	0.11			-0.11					
	0.59			-0.59					
	3.13			-3.13					
	16.55			-16.55					
В	0.23		C	0.23					
 0.19	0.20	0.38	0.38	0.20	0.19				
83.9		-155.7	155.7		-83.9				
13.6	14.5	27.1	-27.1	-14.5	-13.6				
0.0		-13.6	13.6		0.0				
 2.6	2.7	5.1	-5.1	-2.7	-2.6				
0.0		-2.6	2.6		0.0				
0.5	0.5	1.0	-1.0	-0.5	-0.5				
0.0		-0.5	0.5		0.0	29		_	
0.1	0.1	0.2	-0.2	-0.1	-0.1		-		
100.6	17.8	-138.9	138.9	-17.8	-100.6				



SI	near f	orce :			0.301-			- 5			
C	ase 1	- T		Sec. 1.				-			
		167.6	51.9	× 129.0						1	
		· /	6.0	11 44							
		V _{BC}		V _{CB}							
				1943							
Σ	М @	C = 0	0	11				1			
	6.0	$V_{\rm BC}$ -	(51.9	x 6.0 x	3.0) +	129.0	- 167.6	= 0			
	VBC	= (934.03	- 129.0	+167.6) / 6.0	=	162.1	kN		
	V _{CB}	= (51.9 x	6.0) -	162.1		=	149.3	kN		
C	ase 2										
		138.9	51.9	138.9							
								1			
		ľ	6.0		0.00						
		V _{BC}	<u>.</u>	V _{CB}	1	S					
			- 1971 - B.								
Σ	M @	$\mathbf{C} = 0$	0		6F						
	6.0	$V_{\rm BC}$ -	(51.9	x 6.0 x	3.0) +	138.9	- 138.9	= 0		1	
	$V_{\rm AB}$	= (934.03	- 138.9	+138.9) / 6.0		155.7	kN		
	$V_{\rm BA}$	= (51.9 x	6.0) -	155.7		=	155.7	kN	1	







Example: One Point Sub-Frame

Continuous Be	am :							
Bending moment	t and sh	ear for	ce in bea	ım.				
	1		1.1					
				2.52 2.52				
		1000000	CONTRACTOR NOT	100 03 0 2				
1	1		1		1			
I 6.0 I	n I	6.0	m	6.0	m		1.1	
A	В		L		U			
Moment of Inert	ia :				1	100		
$I = bh^3/12 =$	250	x 600	$^{3}/12 =$	4.50	x 10 ⁹	mm ⁴		
Stiffnass :								
$AD \cdot k = 0.75$	111 -	2.20	× 10 ⁹ /	(000		105 mm3		
$AB: k_{AB} = 0.75$	I/L =	5.58	x 10 /	6000 =	5.63 X			
$BC: k_{BC} = I/I$. =	4.50	x 10° /	6000 =	7.5 x	10° mm ^o		
CD: $k_{CD} = 0.75$	I/L =	3.38	$x 10^{9} /$	6000 =	5.63 x	$10^5 \mathrm{mm}^3$		



Example: One Point Sub-Frame

 Distribution factor:			
Distribution factor:			
 Joint A: $F_{AB} = k_{BA}/(k_{AB}+0)$			
 = 5.63 / (5.63 + 0.00)	=	1.00	
 Joint B: $F_{BA} = k_{AB}/(k_{AB}+k_{BC})$			
= 5.63 / (5.63 + 7.50)		0.43	
$F_{\rm BC} = k_{\rm BC}/(k_{\rm AB}+k_{\rm BC})$			
 = 7.50 / (5.63 + 7.50)	=	0.57	
Joint C: $F_{BC} = k_{BC}/(k_{BC}+k_{CD})$			
= 7.50 / (7.5 + 5.63)	=	0.57	
$F_{\rm CD} = k_{\rm CD}/(k_{\rm BC}+k_{\rm CD})$			
= 5.63 / (7.5 + 5.63)	=	0.43	
Joint D: $F_{DC} = k_{CD}/(k_{CD}+0)$			
= 5.63 / (5.63 + 0.00)	=	1.00	



Span 3 design permanent loads $1.35G_k$ 54.8 kN/m 51.9 kN/m 28.0 kN/m 6.0 m 6.0 m 6.0 m A B C D Fixed End Moment : - - -M _{AB} = $M_{BA} = w_1 L_1^2/12 = 54.84$ x $6.0^2 / 12 = 164.5$ kNm -M _{BC} = $M_{CB} = w_2 L_2^2/12 = 51.89$ x $6.0^2 / 12 = 155.7$ kNm -M _{CD} = $M_{DC} = w_3 L_3^2/12 = 27.98$ x $6.0^2 / 12 = 83.9$ kNm Moment Distribution : - 0.00 1.00 0.43 0.57 0.57 0.43 1.00 -164.5 164.5 -155.7 155.7 -83.9 83.9 164.5 -3.8 -5.1 -41.0 -30.7 -83.9 82.3 -20.5 -2.5 -42.0 -26.5 -35.3 25.4 19.1	k	+1.5Q	$.35G_k$	loads 1	variable	ent & v	pe rmane	design	1 & 2	: Span	Case 1	
54.8 kN/m 51.9 kN/m 28.0 kN/m 6.0 m 6.0 m 6.0 m A B C Fixed End Moment : -MAB = MBA = w1L1^2/12 = 54.84 x 6.0 2/12 = 164.5 kNm -MBC = MCB = w2L2^2/12 = 51.89 x 6.0 2/12 = 155.7 kNm -MCD = MDC = w3L3^2/12 = 27.98 x 6.0 2/12 = 83.9 kNm Moment Distribution : -164.5 164.5 164.5 164.5 -3.8 -164.5 -3.8 82.3 -20.5 -26.5 -35.3					35 <i>G</i> _k	ads 1.3	ane nt lo	n perm	3 desig	Span		
6.0 m 6.0 m 6.0 m 6.0 m A B C D Fixed End Moment : $-M_{AB} = M_{BA} = w_1 L_1^2/12 = 54.84$ x $6.0^2/12$ = 164.5 kNm $-M_{BC} = M_{CB} = w_2 L_2^2/12 = 51.89$ x $6.0^2/12$ = 155.7 kNm $-M_{CD} = M_{DC} = w_3 L_3^2/12 = 27.98$ x $6.0^2/12$ = 83.9 kNm Moment Distribution : 0.00 1.00 0.43 0.57 0.57 0.43 1.00 -164.5 164.5 -155.7 155.7 -83.9 83.9 164.5 -3.8 -5.1 -41.0 -30.7 -83.9 82.3 -20.5 -2.5 -42.0 -26.5 -35.3 25.4 19.1		. 2 . 1			kN/m	28.0	kN/m	51.9	kN/m	54.8		E.
Image: Constraint of the constrain							<u> </u>		11. 11. 11. 11. 11. 11. 11. 11. 11. 11.			
A B C D Fixed End Moment : - $M_{AB} = M_{BA} = w_1 L_1^2/12 = 54.84 \times 6.0^2/12 = 164.5 \text{ kNm}$ - $M_{AB} = M_{BA} = w_2 L_2^2/12 = 51.89 \times 6.0^2/12 = 155.7 \text{ kNm}$ - $M_{BC} = M_{CB} = w_2 L_2^2/12 = 27.98 \times 6.0^2/12 = 83.9 \text{ kNm}$ - $M_{CD} = M_{DC} = w_3 L_3^2/12 = 27.98 \times 6.0^2/12 = 83.9 \text{ kNm}$ Moment Distribution : -164.5 164.5 -155.7 155.7 -83.9 83.9 164.5 -155.7 155.7 -83.9 83.9 164.5 -3.8 -5.1 -41.0 -30.7 -83.9 83.9 164.5 -3.8 -5.1 -41.0 -30.7 -83.9 83.9 164.5 -26.5 -35.3 25.4 19.1					m 1	6.0	m T	6.0	m T	6.0		
Fixed End Moment : Image: matrix index with the matrix index withe matrix index withe matrix index with the matrix index with the				5			С		В	4	4	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $									ent :	nd Mom	Fixed E	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		kNm	164.5	=	² / 12	x 6.0	54.84	$\frac{2}{12} = 12$	$= w_{I}L$	$= M_{BA}$	- <i>M</i> _{AB}	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $		kNm	155.7		² / 12	x 6.0	51.89	$2^{2}/12 =$	$= w_2 L$	$= M_{CB}$	- <i>M</i> _{BC}	
Moment Distribution : 0.00 1.00 0.43 0.57 0.57 0.43 1.00 -164.5 164.5 -155.7 155.7 -83.9 83.9 164.5 -3.8 -5.1 -41.0 -30.7 -83.9 82.3 -20.5 -2.5 -42.0 -83.9 -26.5 -35.3 25.4 19.1		kNm	83.9	i = i	² / 12	x 6.0	27.98	$\frac{3^2}{12} =$	$= w_3 L_3$	$= M_{\rm DC}$	-M _{CD} =	
0.00 1.00 0.43 0.57 0.57 0.43 1.00 -164.5 164.5 -155.7 155.7 -83.9 83.9 164.5 -3.8 -5.1 -41.0 -30.7 -83.9 82.3 -20.5 -2.5 -42.0 -83.9 -26.5 -35.3 25.4 19.1									ution :	t Distribu	Momen	
-164.5 164.5 -155.7 155.7 -83.9 83.9 164.5 -3.8 -5.1 -41.0 -30.7 -83.9 82.3 -20.5 -2.5 -42.0 -83.9 -26.5 -35.3 25.4 19.1	0.00	1.00		0.43	0.57		0.57	0.43		1.00	0.00	
164.5 -3.8 -5.1 -41.0 -30.7 -83.9 82.3 -20.5 -2.5 -42.0 -26.5 -35.3 25.4 19.1		83.9	1.1	-83.9	155.7		-155.7	164.5		-164.5		
82.3 -20.5 -2.5 -42.0 -26.5 -35.3 25.4 19.1		-83.9		-30.7	-41.0		-5.1	-3.8		164.5		
-26.5 -35.3 25.4 19.1	-		-	-42.0	-2.5		-20.5	82.3				
				19.1	25.4		-35.3	-26.5				
12.7 -17.6					-17.6		12.7					
-5.4 -7.3 10.1 7.6		-		7.6	10.1		-7.3	-5.4				1
5.0 -3.6					-3.6		5.0					2
-2.2 -2.9 2.1 1.6				1.6	2.1		-2.9	-2.2				second second
1.0 -1.4			_		-1.4		1.0		1	1		
-0.4 -0.6 0.8 0.6				0.6	0.8		-0.6	-0.4				
0.0 208.5 -208.5 127.8 -127.8 0.0		0.0		-127.8	127.8		-208.5	208.5		0.0		



 Shear fo	orce :									
		54.8	×208.5	208.5	51.9	≠ 127.8	127.8	28.0		
						(\mathbf{I})				
		6.0	`	. /	6.0		1	6.0		
	VAB		V _{BA}	V _{BC}		V _{CB}	V _{CD}		V _{DC}	
Σ M @	$\mathbf{B} = 0$)								
6.0	V_{AB} -	(54.8	x 6.0 x	3.0) +	208.5	= 0				
V _{AB}	= (987.13	- 208.5)/6.0	=	129.8	kN			
V _{BA}	= (54.8 x	6.0) -	129.8	=	199.3	kN			
Σ M @	$\mathbf{C} = 0$)						122.034		
6.0	V _{BC} -	(51.9	x 6.0 x	3.0) +	127.8	- 208.5	= 0			
V _{BC}	= (934.03	- 127.8	+208.5)/6.0 =	169.1	kN			
V _{CB}	= (51.9 x	6.0) -	169.1	=	142.2	kN			
Σ M @	$\mathbf{D} = 0$)								
6.0	V _{CD} -	(28.0	x 6.0 x	3.0) -	127.8	= 0				
V _{CD}	= (503.56	+127.8)/6.0	=	105.2	kN			
 V _{DC}	= (28.0 x	6.0) -	105.2	=	62.6	kN			



Span 1 design permanent loads $1.35 G_k$ 28.0 kN/m 51.9 kN/m 54.8 kN/m 6.0 m 6.0 m 6.0 m 6.0 m 6.0 m $-M_{AB} = M_{BA} = w_1 L_1^2 / 12 = 27.98 \times 6.0^2 / 12 = 83.9$ kN $-M_{BC} = M_{CB} = w_2 L_2^2 / 12 = 51.89 \times 6.0^2 / 12 = 155.7$ kN $-M_{CD} = M_{DC} = w_3 L_3^2 / 12 = 54.84 \times 6.0^2 / 12 = 164.5$ kN 164.5 kN Moment Distribution : 0.43 0.57 0.43 1	
28.0 kN/m 51.9 kN/m 54.8 kN/m 6.0 m 6.0 m 6.0 m 6.0 m 6.0 m 6.0 m A B C D Fixed End Moment : -MAB = MBA = W1L12/12 = 27.98 x 6.0 2/12 = 83.9 kN -MBC = MCB = W2L22/12 = 51.89 x 6.0 2/12 = 155.7 kN -MBC = MCB = W2L22/12 = 51.89 x 6.0 2/12 = 155.7 kN -MCD = MDC = W3L32/12 = 54.84 x 6.0 2/12 = 164.5 kN Moment Distribution : 0.00 1.00 0.43 0.57 0.43 11	
6.0 m 6.0 m 6.0 m 6.0 m 6.0 m A B C D Fixed End Moment : D $-M_{AB} = M_{BA} = w_1 L_1^2 / 12 = 27.98 \times 6.0^2 / 12 = 83.9 \text{ km}$ $-M_{BC} = M_{CB} = w_2 L_2^2 / 12 = 51.89 \times 6.0^2 / 12 = 155.7 \text{ km}$ $-M_{CD} = M_{DC} = w_3 L_3^2 / 12 = 54.84 \times 6.0^2 / 12 = 164.5 \text{ km}$ Moment Distribution : 0.00 1.00 0.43 0.57 0.57 0.43	
A B C 6.0 m C 6.0 m D Fixed End Moment : -MAB = MBA = WILI2/12 = 27.98 C D D -MAB = MBA = WIL12/12 = 27.98 X 6.0 2/12 = 83.9 kM -MBC = MCB = WIL22/12 = 51.89 X 6.0 2/12 = 155.7 kM -MCD = MDC = WIL22/12 = 54.84 X 6.0 2/12 = 164.5 kM Moment Distribution : 0.00 0.43 0.57 0.43 1	
A B C D Fixed End Moment : -MAB = MBA = WIL12/12 = 27.98 x 6.0 2/12 = 83.9 km -MBC = MCB = W2L22/12 = 51.89 x 6.0 2/12 = 155.7 km -MCD = MDC = W3L32/12 = 54.84 x 6.0 2/12 = 164.5 km Moment Distribution : 0.00 1.00 0.43 0.57	
Fixed End Moment : $-M_{AB} = M_{BA} = w_1 L_1^2 / 12 = 27.98 \times 6.0^2 / 12 = 83.9 \text{ kM}$ $-M_{BC} = M_{CB} = w_2 L_2^2 / 12 = 51.89 \times 6.0^2 / 12 = 155.7 \text{ kM}$ $-M_{CD} = M_{DC} = w_3 L_3^2 / 12 = 54.84 \times 6.0^2 / 12 = 164.5 \text{ kM}$ Moment Distribution : $0.00 1.00 0.43 0.57 0.57 0.43 10^{-10}$	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	
$-M_{BC} = M_{CB} = w_2 L_2^2 / 12 = 51.89 \times 6.0^2 / 12 = 155.7 \text{ kN}$ $-M_{CD} = M_{DC} = w_3 L_3^2 / 12 = 54.84 \times 6.0^2 / 12 = 164.5 \text{ kN}$ Moment Distribution : 0.00 1.00 0.43 0.57 0.57 0.43 1	Im
$-M_{CD} = M_{DC} = w_3 L_3^2 / 12 = 54.84 \times 6.0^2 / 12 = 164.5 \text{ kN}$ Moment Distribution : $0.00 1.00 \qquad 0.43 0.57 \qquad 0.57 0.43 \qquad 1$	lm
Moment Distribution : 0.00 0.43 0.57 0.43 1	Im
0.00 1.00 0.43 0.57 0.57 0.43 1	a a parte
	.00 0.00
-83.9 83.9 -155.7 155.7 -164.5 1	64.5
83.9 30.7 41.0 5.1 3.8 -1	64.5
42.0 2.5 20.5 -82.3	
-19.1 -25.4 35.3 26.5	
17.6 -12.7	
-7.6 -10.1 7.3 5.4	
3.6 -5.0	1.1
-1.6 -2.1 2.9 2.2	
1.4 -1.0	
-0.6 -0.8 0.6 0.4	
0.0 127.8 -127.8 208.5 -208.5	0.0



	Shear force :									
		28.0	127.8	127.8	51.9	208.5	208.5	54.8		
				17						
		6.0			6.0			6.0		1
	V _{AB}		$V_{\rm BA}$	V _{BC}		V _{CB}	V _{CD}		V _{DC}	
	$\Sigma M @ B = 0$	0								
	6.0 V _{AB} -	(28.0	x 6.0 x	3.0) +	127.8	= 0			-	_
	$V_{AB} = ($	503.56	- 127.8)/6.0	=	62.6	kN			
	$V_{\rm BA} = ($	28.0 x	6.0) -	62.6	=	105.2	kN	÷.	= [
	$\Sigma M @ C = 0$	0								
	6.0 V _{BC} -	(51.9	x 6.0 x	3.0) +	208.5	- 127.8	= 0			
_	$V_{\rm BC} = ($	934.03	- 208.5	+127.8)/6.0 =	142.2	kN	, ==		
	$V_{\rm CB} = ($	51.9 x	6.0) -	142.2	=	169.1	kN			
	$\Sigma M @ D =$	0								
	6.0 V _{CD} -	(54.8	x 6.0 x	3.0) -	208.5	= 0				
	$V_{\rm CD} = ($	987.13	+208.5)/6.0	=	199.3	kN			
	$V_{\rm DC} = ($	54.8 x	6.0) -	199.3	=	129.8	kN			



	Case 3	: Span 1 &	3 design	pe rman	ent & v	variable	e loads 1	$.35G_k$	$+1.5Q_{1}$	c .
P		Span 2 de	sign perm	ane nt lo	ads 1.3	B5Gk				
		54.8 kN/n	n 26.6	kN/m	54.8	kN/m				
		6.0 m	T 6.0	m T	6.0	m	T I			
		A	В	С			D			
	Fixed E	End Moment :		1			La selar			
	- <i>M</i> _{AB}	$=M_{\rm BA} = w$	$L_1^2/12 =$	54.84	x 6.0	² / 12	=	164.5	kNm	
	- <i>M</i> _{BC}	$= M_{\rm CB} = w$	$v_2 L_2^2 / 12 =$	26.58	x 6.0	² / 12	=	79.7	kNm	
	-M _{CD}	$= M_{\rm DC} = w$	$_{3}L_{3}^{2}/12 =$	54.84	x 6.0	² / 12	=	164.5	kNm	
	Momen	t Distribution	:							=
	0.00	1.00	0.43	0.57		0.57	0.43		1.00	0.00
		-164.5	164.5	-79.7		79.7	-164.5		164.5	
		164.5	-36.3	-48.4	- i	48.4	36.3		-164.5	
			82.3	24.2		-24.2	-82.3			
			-45.6	-60.8		60.8	45.6			
				30.4		-30.4				
			-13.0	-17.4		17.4	13.0			
			7	8.7		-8.7				
			-3.7	-5.0		5.0	3.7			
				2.5		-2.5			-	
			-1.1	-1.4		1.4	1.1			
		0.0	147.0	-147.0		147.0	-147.0		0.0	
1-00				-						



Shear fo	orce :									
		54.8	<u>+147.0</u>	147.0	26.6	147.0	147.0	54.8		
							17			
	-	6.0			6.0			6.0		
	V _{AB}		$V_{\rm BA}$	$V_{\rm BC}$		V _{CB}	V _{CD}		V _{DC}	
$\Sigma M @$	$\mathbf{B} = 0$)	-							
6.0	$V_{\rm AB}$ -	(54.8	x 6.0 x	3.0) +	147.0	= 0				
V _{AB}	= (987.13	- 147.0)/6.0	=	140.0	kN		-	
V _{BA}	= (54.8 x	6.0) -	140.0	=	189.0	kN			
$\Sigma M @$	$\mathbf{C} = 0$)								
 6.0	$V_{\rm BC}$ -	(26.6	x 6.0 x	3.0) +	147.0	- 147.0	= 0			
 V _{BC}	= (478.41	- 147.0	+147.0)/6.0 =	79.7	kN			
V _{CB}	= (26.6 x	6.0) -	79.7	=	79.7	kN			
$\Sigma M @$	D = 0	0						-		
 6.0	$V_{\rm CD}$ -	(54.8	x 6.0 x	3.0) -	147.0	= 0				
V _{CD}	= (987.13	+147.0) / 6.0	=	189.0	kN			
V _{DC}	= (54.8 x	6.0) -	189.0	=	140.0	kN			



 Case 4	Case 4: Span 2 design permanent & variable loads 1.35G _k +1.5Q _k									
	Span 1 & 3	design j	pe rmane	nt load	ls 1.35	Gk				
	28.0 kN/m	51.9	kN/m 28.0		kN/m					
			-			2				
	6.0 m	T 6.0	m T	6.0	m	I				
/	A 1	В	C			Б				
 Fixed E	and Moment :		1							
- <i>M</i> _{AB}	$= M_{BA} = w_I L$	$\frac{1}{1}^{2}/12 =$	27.98	x 6.0	² / 12	=	83.9	kNm		
- <i>M</i> _{BC}	$= M_{\rm CB} = w_2$	$L_2^2/12 =$	51.89	x 6.0	² / 12	=	155.7	kNm		
-M _{CD} =	$= M_{\rm DC} = w_3 L$	$\frac{3^2}{12} =$	27.98	x 6.0	² / 12	=	83.9	kNm		
Momen	t Distribution :									
 0.00	1.00	0.43	0.57		0.57	0.43		1.00	0.00	
	-83.9	83.9	-155.7		155.7	-83.9		83.9		
	83.9	30.7	41.0		-41.0	-30.7		-83.9		
	1	42.0	-20.5		20.5	-42.0				
		-9.2	-12.3		12.3	9.2				
			6.1		-6.1					
		-2.6	-3.5		3.5	2.6				
			1.8		-1.8					
		-0.8	-1.0		1.0	0.8				
			0.5		-0.5					
		-0.2	-0.3		0.3	0.2				
	0.0	143.8	-143.8	11	143.8	-143.8		0.0		



	Shear fo	orce :					1			
			28.0	143.8 م	143.8	51.9	143.8	143.8	28.0	
				F				17		
			6.0			6.0	Ì		6.0	
		VAB		$V_{\rm BA}$	V _{BC}		V _{CB}	V _{CD}		V _{DC}
	Σ M @	B = (0							
	6.0	V _{AB} -	(28.0	x 6.0 x	3.0) +	143.8	= 0		1	
	V _{AB}	= (503.56	- 143.8) / 6.0	=	60.0	kN		
	V _{BA}	= (28.0 x	6.0) -	60.0	=	107.9	kN		
	Σ Μ @	C = (0							
	6.0	V _{BC} -	(51.9	x 6.0 x	3.0) +	143.8	- 143.8	= 0		
	V _{BC}	= (934.03	- 143.8	+143.8)/6.0 =	155.7	kN		
	V _{CB}	= (51.9 x	6.0) -	155.7	=	155.7	kN		
	Σ Μ @	D = (0							
	6.0	V _{CD} -	(28.0	x 6.0 x	3.0) -	143.8	= 0			
	V _{CD}	= (503.56	+143.8) / 6.0	=	107.9	kN		
here the state of the second second	Vnc	= (28.0 x	6.0) -	107.9	=	60.0	kN		


Example: One Point Sub-Frame

	Shear F	orce Diag	gram								
	140.0			169.1			199.3				
	129.8			155.7			189.0				
	62.6			142.2			107.9				
	60.0		61	79.7			105.2		-		
		r,	1	1.1.1	105.2	1		79.7	1.2		60.0
		× x2		1	107.9			142.2			62.6
		× x3 >	7	1	189.0		1	155.7		1	129.8
		< <u>x</u> ₄ →			199.3			169.1		1000	140.0
	Distanc	e				Mid-spa	an mome	ent			
	Span 1					Span 1					
	$x_1 =$	129.8 /	54.8 =	2.4 m		$M_1 =$	129.8x	2.4/2	=	153.6	kNm
	$x_{2} =$	62.6/	28.0 =	2.2 m		$M_{2} =$	62.6x	2.2/2	=	70.1	kNm
	$x_{3} =$	140.0 /	54.8 =	2.6 m		$M_{3} =$	140.0x	2.6/2	=	178.8	kNm
10	$x_4 =$	60.0 /	28.0 =	2.1 m		$M_{4} =$	60.0x	2.1/2	=	64.2	kNm
	Span 2					Span 2					
	$x_1 =$	169.1 /	51.9 =	3.3 m		$M_1 =$	(169.1	x3.3/2)	-208.5	= 67.1	kNm
	$x_{2} =$	142.2 /	51.9 =	2.7 m		$M_{2} =$	(142.2	x2.7/2)	-127.8	= 67.1	kNm
	$x_3 =$	79.7/	26.6 =	3.0 m		$M_{3} =$	(79.7	x3.0/2)	-147.0	-= 27.4	kNm
-	$x_4 =$	155.7/	51.9 =	3.0 m		$M_{4} =$	(155.7	x3.0/2)	-143.8	= 89.7	kNm







	Sub-frame: Be							
	Momen of Iner	tia: $I = i$	<i>bh</i> ³ /12		- FR		1.0.4	
	Column: $I =$	300	x 400	³ /12 =	1.6 x	10 ⁹ mm ⁴	-	
	Beam : $I =$	250	x 600	3 /12 =	4.5 x	10° mm ⁴	24000	
	Stiffness :	K = I/L					1.0000	
	Column _{lower} : /	$K_{cl} =$	1.6 x	10^{9} /	3500 =	4.6 x 10 ⁵ mm ³		
	Column _{upper} : I	$K_{cu} =$	1.6 x	10 ⁹ /	4000 =	4.0 x 10 ⁵ mm ³		
-41001-000-00000100-000	Beam:	$K_{AB} =$	4.5 x	10^{9} /	6000 =	$7.5 \times 10^5 \text{ mm}^3$		
		$K_{\rm BC} =$	4.5 x	10^{9} /	6000 =	7.5 x 10 ⁵ mm ³	1.1	
		$K_{\rm CD} =$	4.5 x	10 ⁹ /	6000 =	7.5 x 10 ⁵ mm ³		
					7			



Example: One Point Sub-Frame

	Column A	
	$FEM = wL^2/12$	
	54.8 kN/m = 54.8 x 6.0 2 /	12
	= 164.52 kNm	
	К _с 0.5К _{АВ}	
	6.0m	T
	Moment in upper column,	Δ
	$M = FEM \ge K_{cu} / (K_{cu} + K_{cl} + K_{b}/2)$	A
Ť	$= 164.5 \times 4.0 / (4.0 + 4.6 + 3.8)$	53.4
	= 53.4 kNm	61.0
	Moment in lowercColumn,	
	$M = FEM \ge K_{cl} / (K_{cu} + K_{cl} + K_{b}/2)$	
	$= 164.5 \times 4.6 / (4.0 + 4.6 + 3.8)$	
1	= 61.0 kNm	



Column	в _	_								
		Kcu		$FEM_1 =$	$= wL^2/1$	12				
54.8 k	N/m	26.6	kN/m	=	54.8	x 6.0	² /12			
infection and	(\mathbf{i}		=	164.5	kNm				
0.5KA	B	0	.5KBC	FEM ₂ =	$= wL^2/1$	12				
		K _{cl}		=	26.6	x 6.0	² /12			
6.0 n	n	6.0	m	=	79.7	kNm				
ΔΜ	= FEN	1 - FE	$M_2 =$	164.5 -	79.7 =	84.8	kNm		2 2 8 1	
		1.1.1							X	1
Moment	in uppe	r colum	n,				1		1	
M =	$\Delta M \mathbf{x}$	$K_{\rm cu}$ /(K	$C_{\rm cu} + K_{\rm c}$	1 + 0.5K	AB + 0.5	$5K_{\rm BC}$)			A	
=	84.8	x 4.0 /	(4.0 +	4.6 +	3.75 +	3.75)			A	
=	21.1	kNm						21.1	A.	<u></u>
									N	24.1
Moment	in lowe	r colum	n,						3	
M =	$\Delta M \mathbf{x}$	$K_{\rm cl}/(K$	$_{\rm cu} + K_{\rm cl}$	+0.5K	AB + 0.5	$K_{\rm BC}$)	100.301			
	84.8	x 4.6 /	(4.0 +	4.6 +	3.75 +	3.75)			A	
-	24.1	kNm								



Column C								
 	K	FEM 1	$= wL^{2}/1$	2				
 26.6 kN/m	54.8 kN/m	=	26.6	x 6.0	² /12			
	>	=	79.7	kNm				
0.5KAB	0.5KBC	FEM ₂	$= wL^2/1$	2				-
	K _{cl}	=	54.8	x 6.0	² /12			
 6.0 m	6.0 m	=	164.5	kNm				
$\Delta M = FE$	$M_1 - FEM_2 =$	164.5 -	79.7 =	84.8	kNm			
 Momont in upp	ar column						म	
 $M = \Lambda M \gamma$	K / (K + K)	+0.5K	$n_{0} \pm 0.5$	K _c p)			1	
 = 84.8	x 4.0 / (4.0 +	4.6 +	3.75 +	3.75)			A	
 = 21.1	kNm						A	21.1
						24.1	E.	
 Moment in low	er column,						E	
$M = \Delta M x$	$K_{\rm cl} / (K_{\rm cu} + K_{\rm cl})$	+0.5K	$_{\rm BC} + 0.5$	$K_{\rm CD}$)			1	
= 84.8	x 4.6 / (4.0 +	4.6 +	3.75 +	3.75)			A	
= 24.1	kNm							



Example: One Point Sub-Frame

Column D									
	Kcu	FEM =	$= wL^2/1$	12					
54.8 kN/m	-		54.8	x 6.0	² /12				
		=	164.52	kNm					
 0.5K _{CD}	K _{cl}					=1			
 6.0 m	1	-			8-1	-	a segretarità	T	
Moment in upp	er colur	nn,						Å	
M = FEM	$ K_{cu} / (I$	$K_{\rm cu} + K_{\rm cl}$	+ 0.5K	ср)				A	
= 164.5	5 x 4.0	/ (4.0+	4.6 +	3.8)			53.4	A.	
 = 53.4	4 kNm							Į	61.0
Moment in low	ercColu	ımn,						Ĭ	
M = FEM	$ K_{cl} / (k$	$K_{cu} + K_{cl}$	$+0.5K_{0}$	CD)				A	
= 164.5	5 x 4.6	/ (4.0+	4.6 +	3.8)					
= 61.0) kNm					-			
	1								