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FRAME ANALYSIS DUE TO VERTICAL ACTIONS

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FAKULTI KEJURUTERAAN AWAM DAN ALAM SEKITAR

With Wisdom We Explore

Multi-Story Building

- Structure for multi-story building is actually a connected frame of members, each of which are firmly connected to each other.

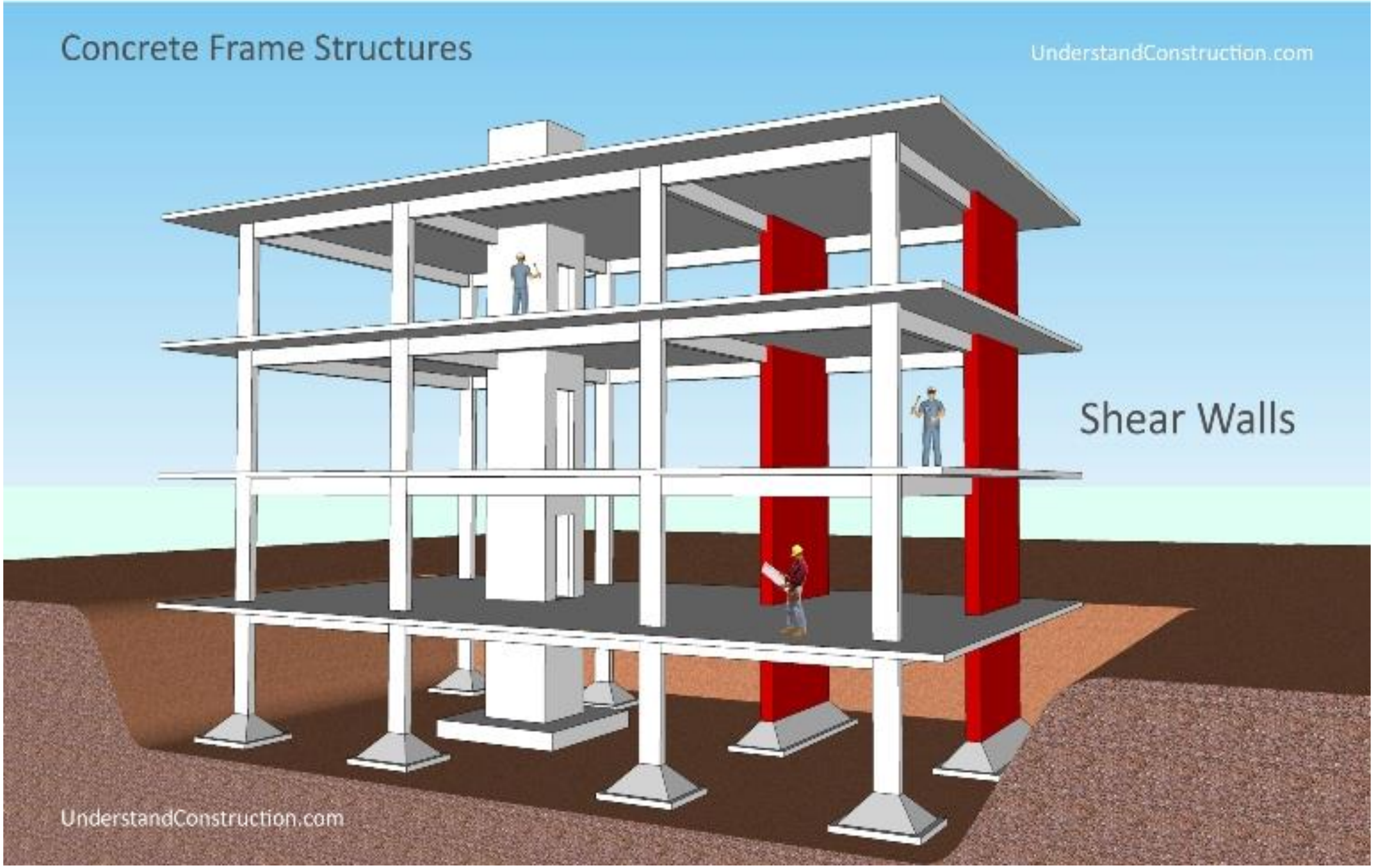
- In engineering parlance, these connections are called moment connections, which means that the two members are firmly connected to each other.

- This frame becomes very strong, and must resist the various loads that act on a building during its life.

- The concrete frame rests of foundations, which transfer the forces from the building and on the building to the ground.

- Some other important components of concrete frame structures are shear wall, elevator shaft, masonry/reinforced concrete wall and cladding.

Multi-Story Building



Multi-Story Building

Shear wall	Important structural elements in high-rise buildings. Easily measure 400mm thick by 3m. The function is to help take care of *horizontal forces* on buildings like wind and earthquake loads. Shear walls also carry vertical loads. It is important to understand that shear wall only work for horizontal loads in one direction (the axis of the long dimension of the wall).
Elevator shaft	Vertical boxes in which the elevators move up and down - normally each elevator is enclosed in its own concrete box. These shafts are also very good structural elements, helping to resist horizontal loads, and also carrying vertical loads.
Masonry/ concrete wall	Concrete frame structures are strong and economical. Hence almost any walling materials can be used with them. Common options include masonry wall of brick, concrete block and light steel partition.
Cladding	Common cladding materials are glass, aluminum panels and ceramic facades. Since these structures can be designed for heavy loading, one could even clad them in solid masonry walls of brick or stone.

Multi-Story Building

Building is 3D frame which consist of slabs, beams and columns

Multi-Story Building

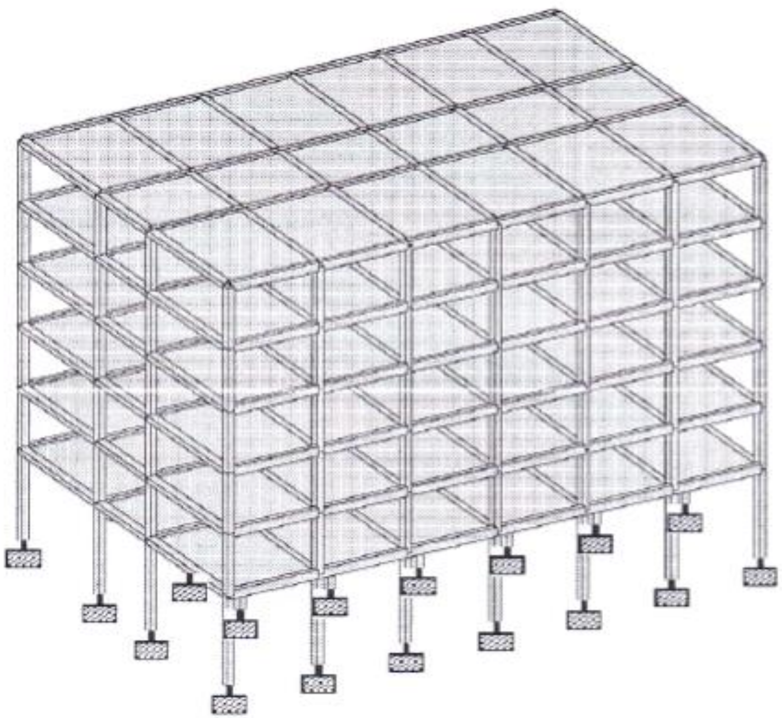
Multi-story building: Low-rise vs High-rise

Frame Analysis

- The building structure is 3D frame, comprising floor slabs, beams, columns and footings, which monolithically connected and act integrally to resist vertical loads and lateral loads.
- In the design of reinforced concrete structures, it has to analyze the structure subjected to all probable combinations of loads, considering the ultimate limit state.
- Commonly 3D frame analysis is the most accurate method to analyse the frame building. However, 3D frame is complex and need to be carried out using relevant computer software (ESTEEM, STAADPro, ETABS, SAP, finite element software, etc.)
- Once the bending moment, shear force and axial load are obtained, reinforcements can be designed according to the standard.

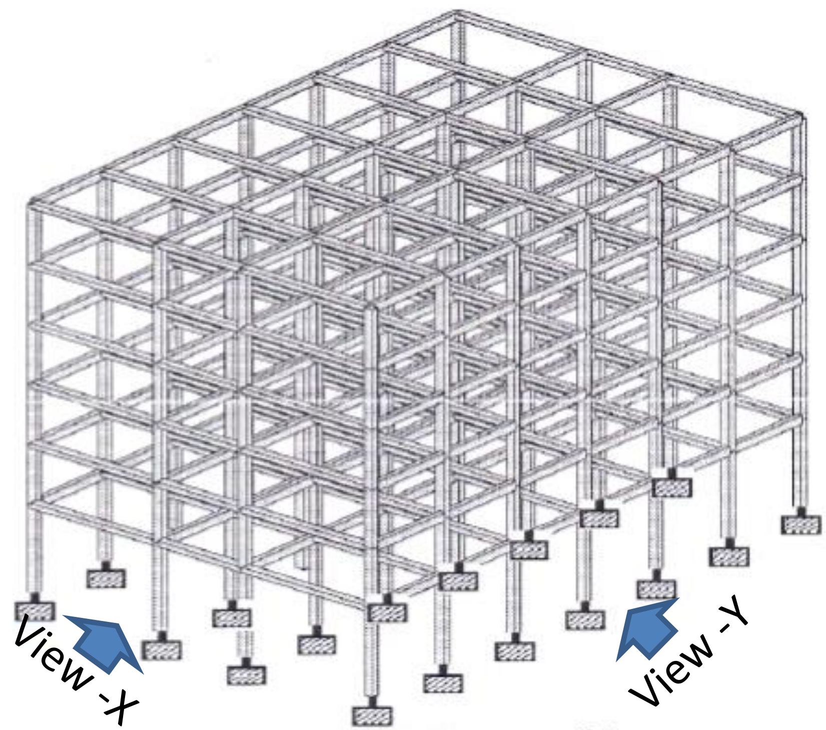
Frame Analysis

- In many cases the slabs are analyzed separately, thus, the analysis may be simplified appropriately consist only beams and columns.



3D frame which consist of slabs, beams and column

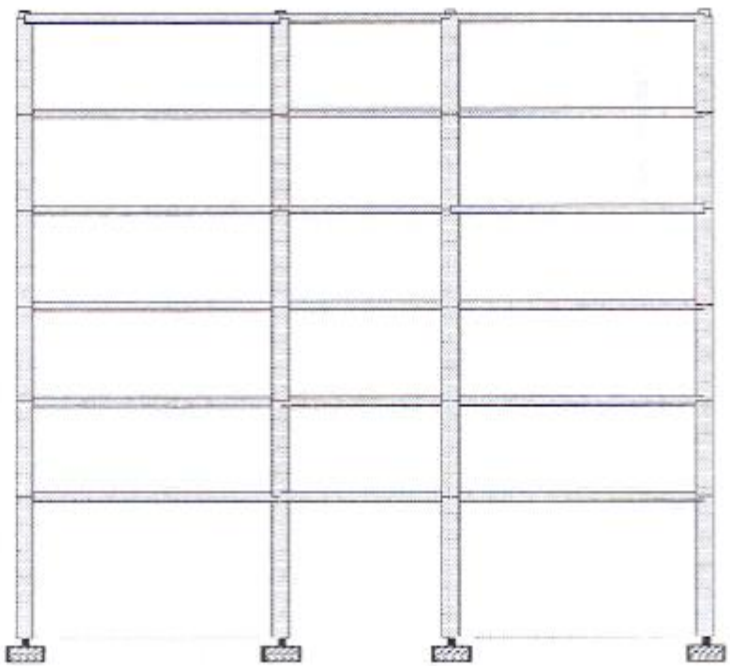
Simplified →



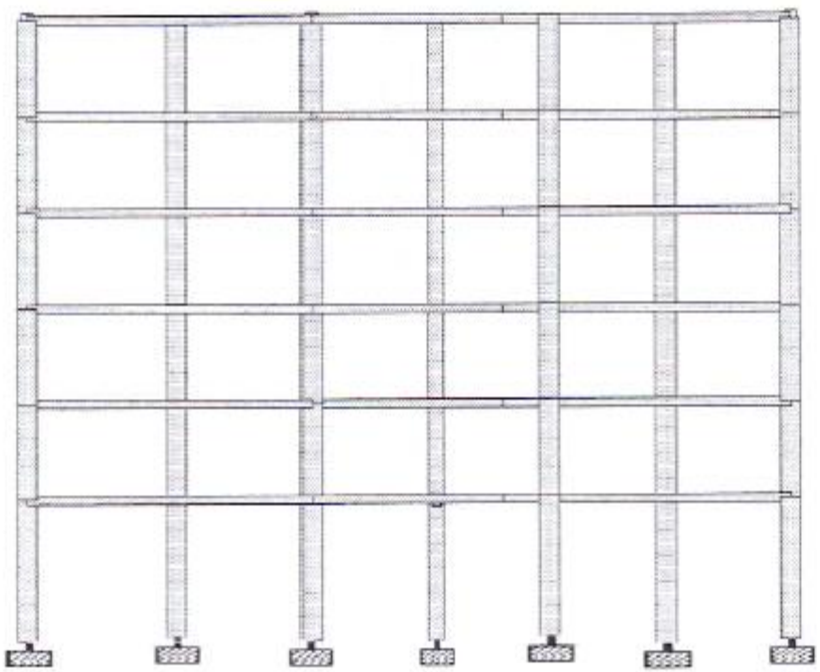
3D frame consist only beams and columns

Frame Analysis

- In order to simplify the analysis, the 3D structure is generally divided into a series of independent parallel 2D plane frames.



View-X



View Y

Frame Analysis

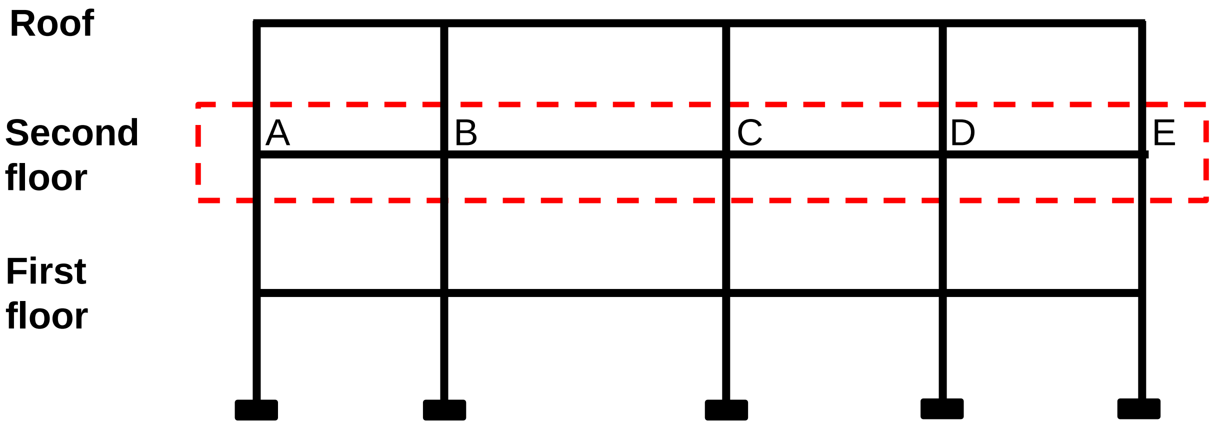
- 2D plane frame can be further simplified into 3 levels sub-frames:

i) **Complete sub-frame**
The frame consists of all beams at each level with columns top and bottom of beams. Moments at columns and beams are tabulated by analyzing the complete sub-frame.

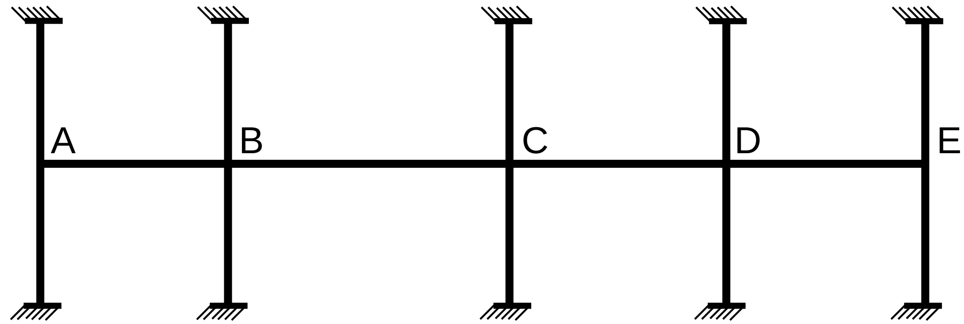
ii) **Simplified sub-frame**
The frame consists of a selected beam with columns and neighbouring beams at both sides of selected beam.

iii) **Simplified sub-frame at point**
The frame consists of a selected point or node with columns at top and bottom, and neighbouring beams coming into the point.

- 2D plane frame >>> complete sub-frame

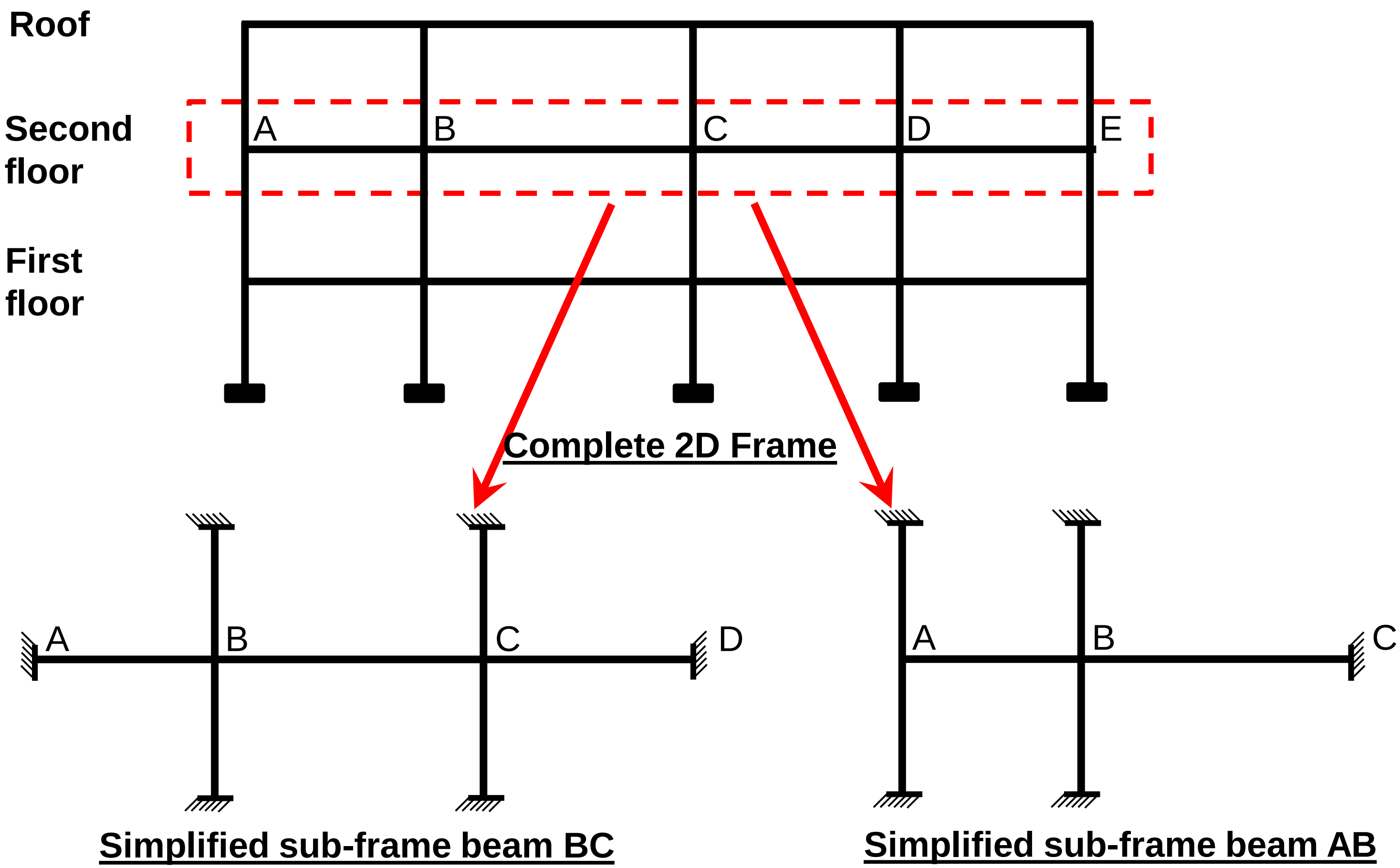


Complete 2D Frame

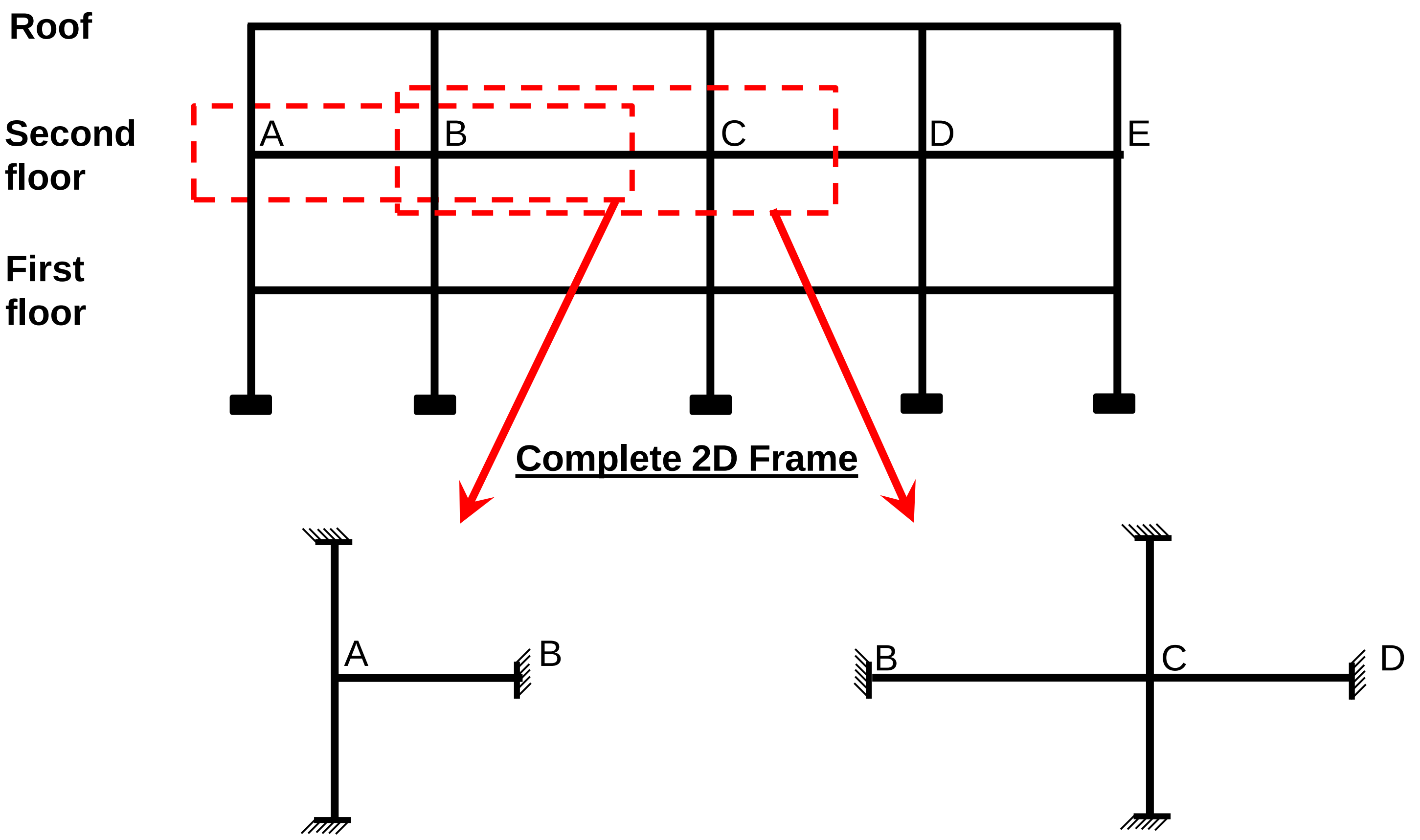


Sub-frame at second floor

- Complete sub-frame >>> Simplified sub-frame



- Simplified sub-frame >>> One point sub-frame:



Simplified sub-frame at point A

Simplified sub-frame at point C

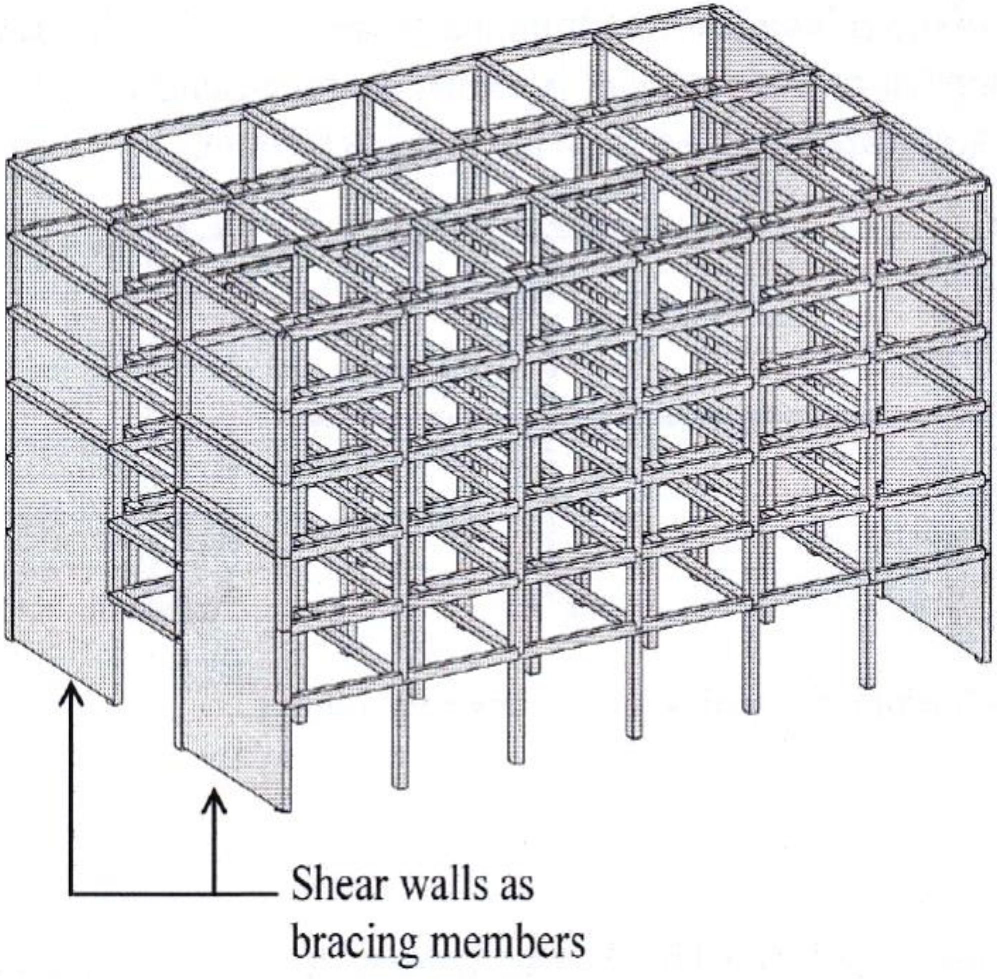
Type of Frame

1) **Braced Framed**

Frames that not contribute to the overall stability of the structure.

None of the lateral actions, including wind, are transmitted to the columns and beams but carries by bracing members such as shear wall.

Support vertical actions only.



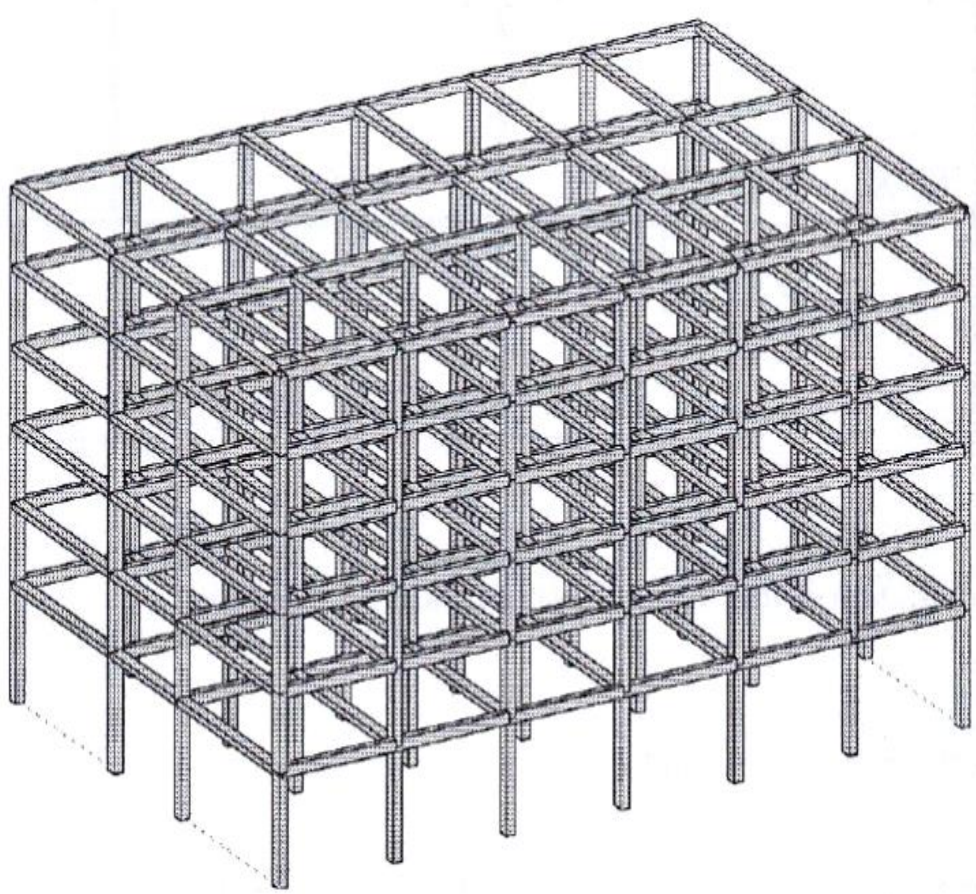
Type of Frame

2) Unbraced Framed

Frame that contribute to the overall stability of the structure.

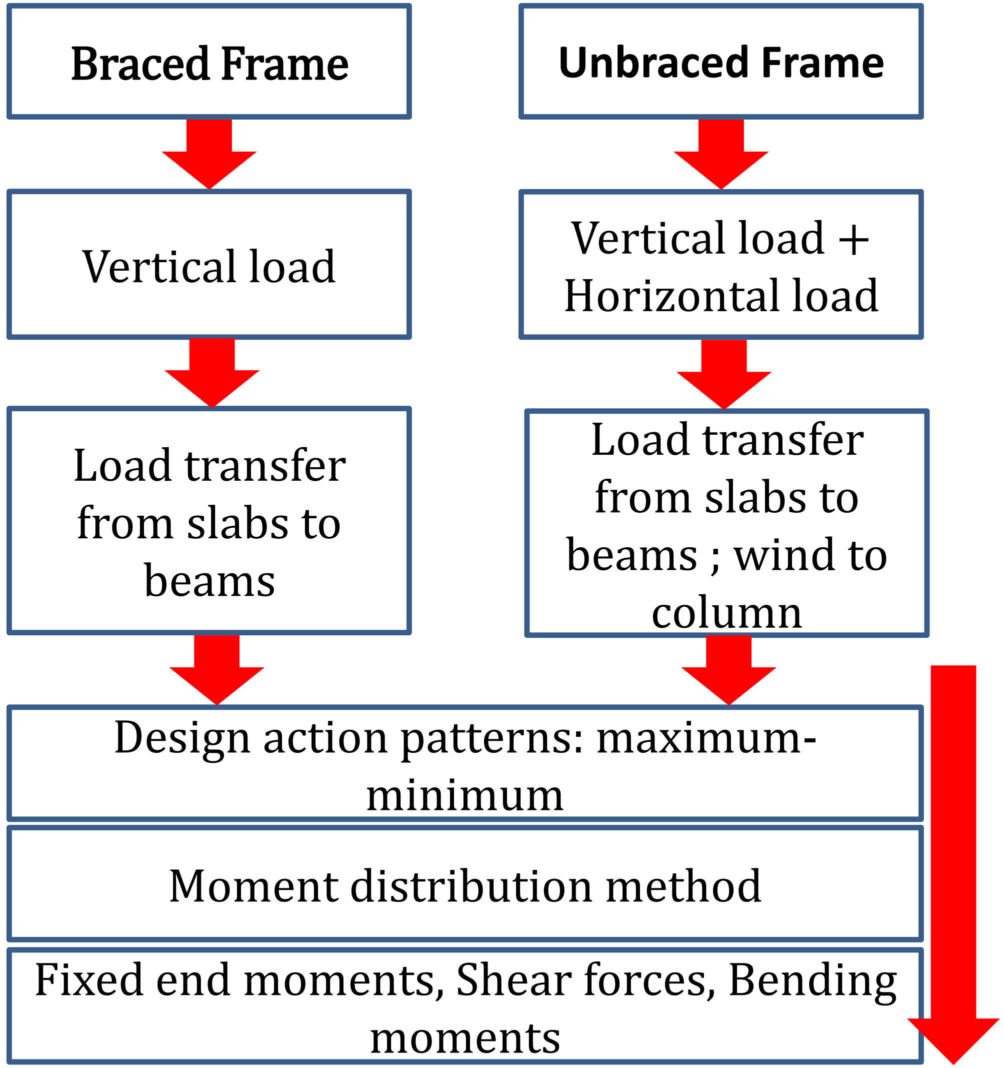
All lateral actions, including wind, are transmitted to the columns and beams since there are no bracing members such as shear wall are provided.

Support vertical and lateral actions



Method of Analysis

- Primary objective is to obtain a set of internal forces and moments throughout the structure that are in equilibrium with the design loads for the required loading combinations.
- General provisions to analysis are set out in EN 1992-1-1 Section 5.



General Consideration

- General consideration for sub-frame analysis:

i) Method of sub-frame analysis can be conducted using one-level sub-frame, two-point sub-frame or one-point sub-frame with continuous beam.

ii) The column or/and beam ends remote from the beam under consideration may generally be assumed to be fixed unless the assumption of pinned is clearly more reasonable.

iii) Stiffness for interior beam is K_B.

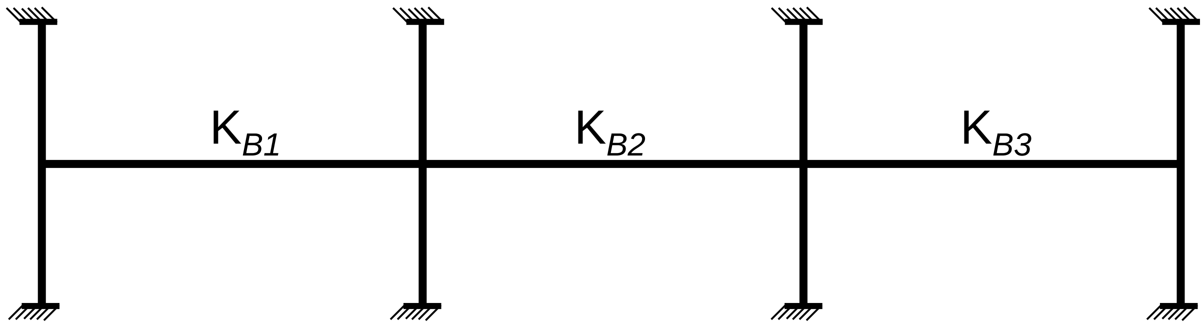
iv) Stiffness for fixed end (beam elements) posses half their actual stiffness, $0.5K_B$.

v) The arrangement of the design ultimate variable loads should be such as to cause the maximum moment the column.

Sub-Frame Analysis

- One-level sub-frame

  - Each sub-frame consist of the beams at one level together with the columns above and below.
  - The ends of the columns remote from the beams may generally be assumed to be fixed unless the assumption of a pinned end is clearly more reasonable



  - At least four cases combination of actions:

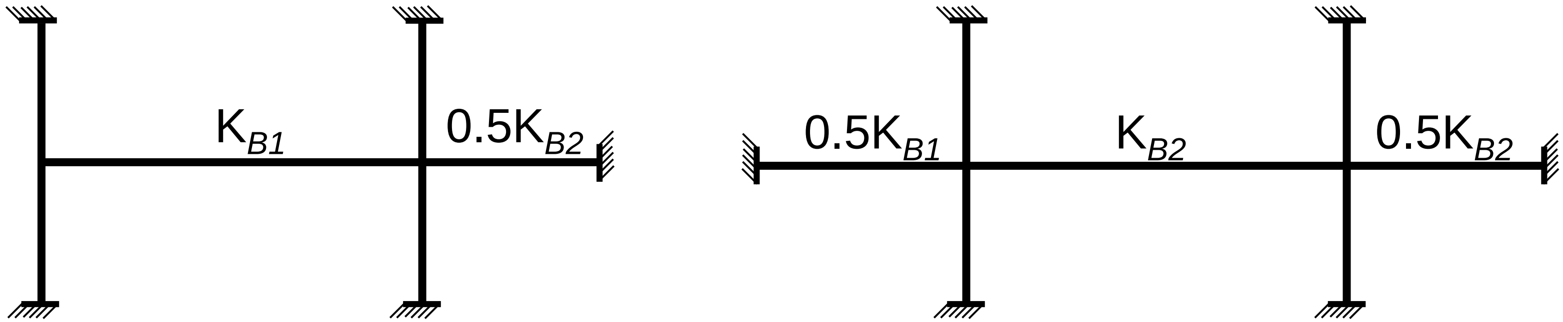
    [Max][Min][Max]; [Min][Max][Min]

    [Max][Max][Min]; [Min][Max][Max]

Sub-Frame Analysis

- Two-point sub-frame

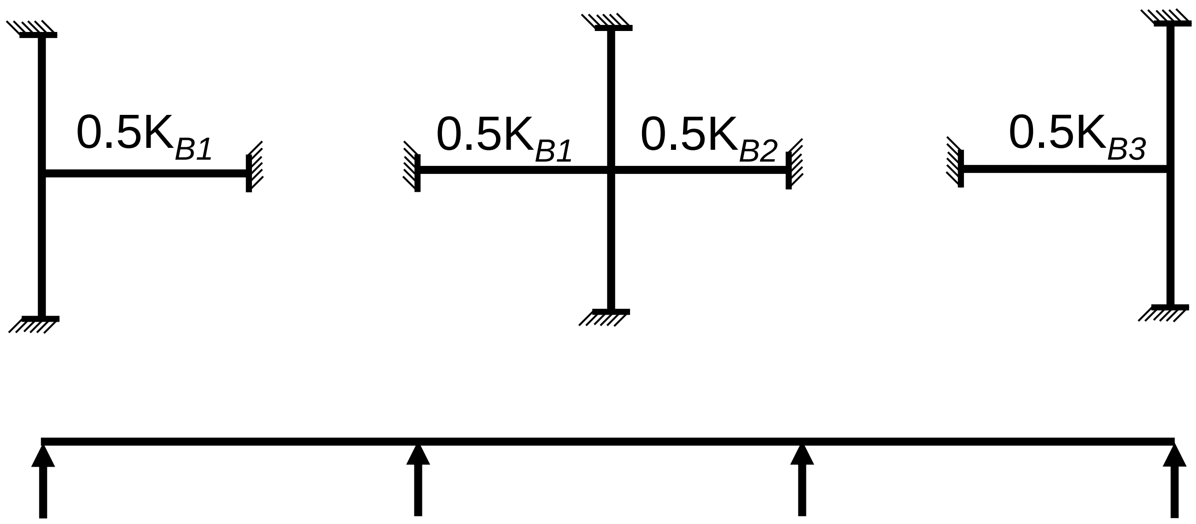
  - The moments and forces in certain individual beam may be found by considering a simplified sub-frame consisting only of the beam, the columns attached to the end of that beam and the beams on either side is any.



  - Load at interior beam where stiffness = K_B is always for maximum design load.

Sub-Frame Analysis

- One-point sub-frame with continuous beam
  - The moments and forces in the beams at one level → considering the beams as a continuous beam over supports providing no restraint to rotation.
  - The ultimate moment for column → simple moment distribution procedure



Combination of Actions

- Action on buildings is due to permanent (dead load), variable (imposed, wind, dynamic, seismic loads) and accidental load.
- Mostly multistory buildings for office or residential purpose are design for dead, imposed and wind loads.
- Separate actions must be applied to the structure in appropriate directions and various types of actions combined with partial safety factors selected to cause the most severe design condition.
- Maximum design load = $1.35G_k + 1.5Q_k$
- Minimum design load = $1.35G_k$
- Wind load = $1.2W_k$
- Vertical load due to wind = $1.2G_k + 1.2Q_k$

Combination of Actions

- For the combination of dead load and imposed load, the following loading patterns are considered:

Braced frame	1) All spans loaded with maximum dead plus imposed loads 2) Alternate spans loaded with maximum dead load and imposed load and all other spans loaded with minimum dead load
Unbraced frame	1) Three cases loading arrangements as braced sub-frame 2) Vertical actions for sub-frame 3) Wind load for complete frame

Combination of Actions

- Load cases for braced frame
- Vertical load

  ~~[Max] [Max] [Max]~~

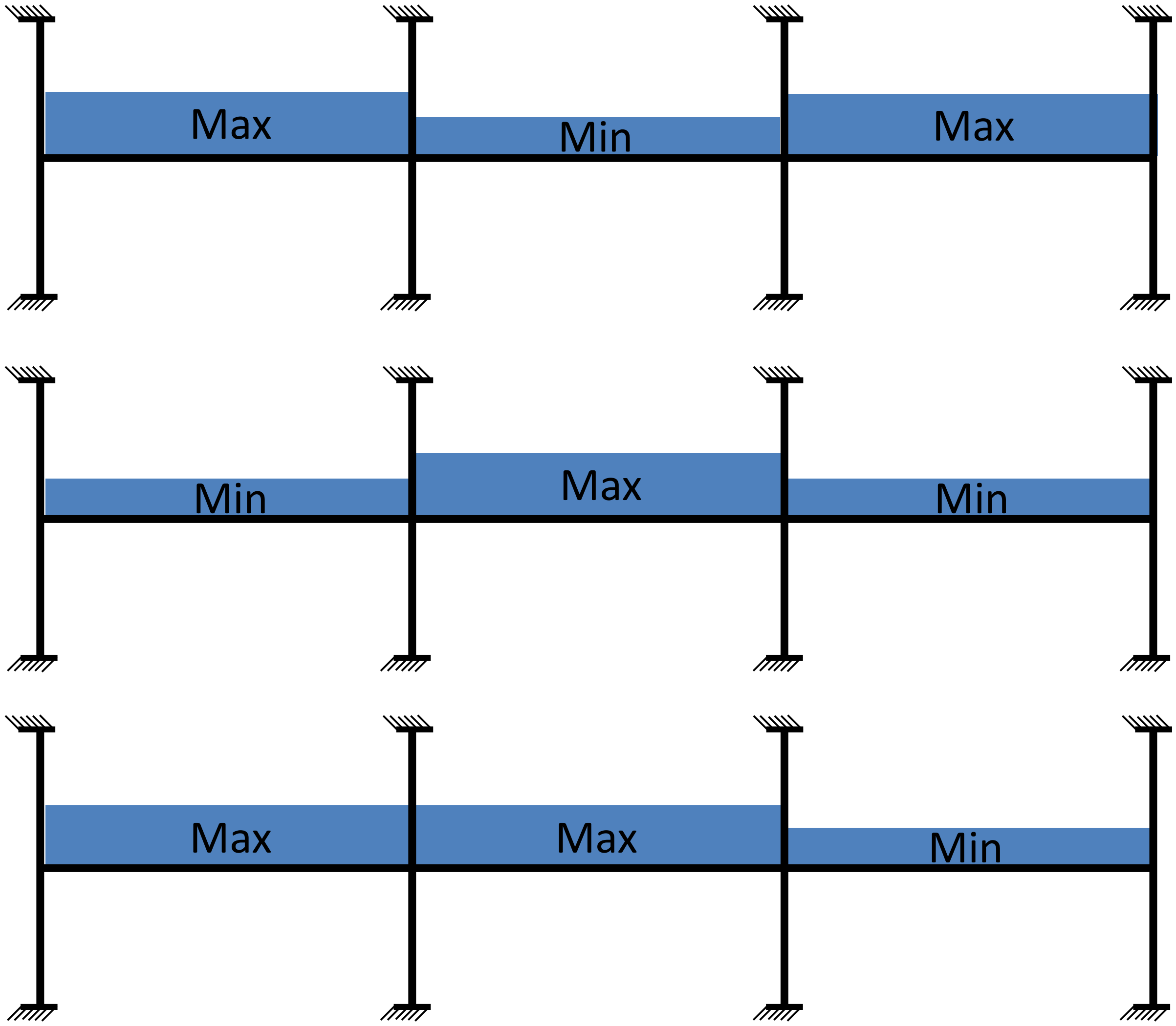
  ~~[Min] [Min] [Min]~~

  [Max] [Min] [Max]

  [Min] [Max] [Min]

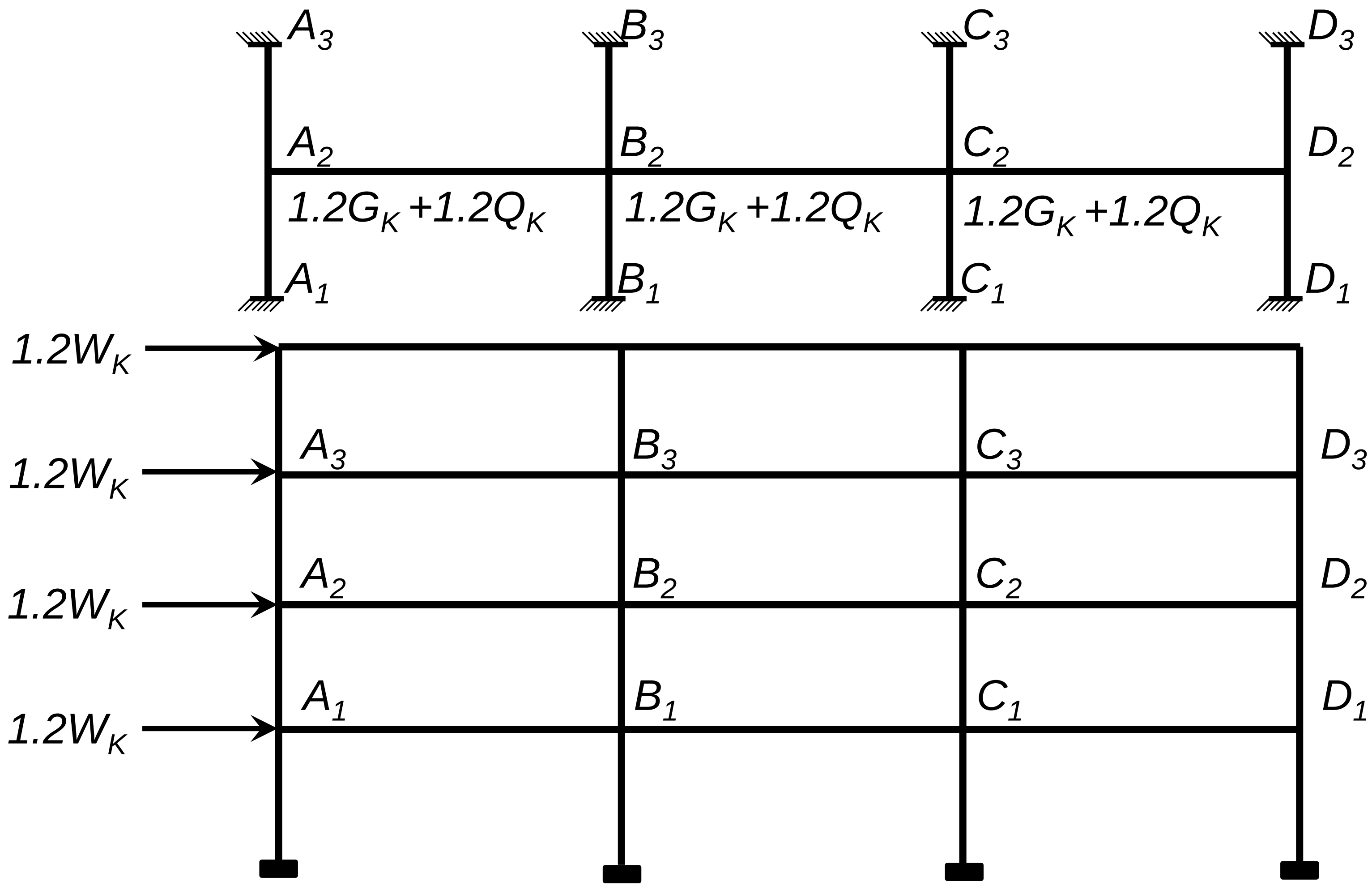
  [Max] [Max] [Min]

  [Min] [Max] [Max]



Combination of Actions

- Load cases for unbraced frame
- Vertical load from wind loading + lateral load



Analysis Procedure

- Analysis procedure for braced frame:
  1. Analyse all actions, maximum and minimum design loads
  2. Calculate moment inertia, $I = bh^3 /12$
  3. Calculate stiffness of beams and columns, $k = I/L$
  4. Determine distribution factor, $DF = k_i / \Sigma k$
  5. Determine fixed end moment (FEM) of beams
  6. Perform moment distribution by cases:
     a) Case 1 [Max][Max][Min]
     b) Case 2 [Min][Min][Max]
     c) Case 3 [Max][Min][Max]
     d) Case 4 [Min][Max][Min]
  7. Calculate actual shear force and bending moment. Draw BMD and SFD diagrams

Analysis Procedure

- Analysis procedure for unbraced frame:

1. Calculate design wind load, W_d=1.2W_k
2. Calculate lateral point load at each level of frame
   a) Assume contra-flexure point at center of frame
   b) Axial loads in column are in its proportion to distances from the centre of gravity of frame
   c) All columns are equal cross-section area
3. Lateral load analysis using Cantilever Method.
   - Calculate axial force in columns, then shear force in beams and columns from top to ground levels.
4. Vertical load analysis due to wind, 1.2 G_k + 1.2Q_k
   - analysis of one level sub-frame

Example

The framing plans for a multistory building are shown in the figure. The main dimensions structural features, loads, material, etc. are also set out as at the given data. Analyze sub frame 3/A-D, Level 1 to determine shear forces and bending moments of corresponding beams and columns. Use all the three methods of analysis.

- Permanent office building (Design life = 50 years)
- Location: Near sub-urban (Zone 1 of Malaysia wind speed mapping)
- Topography: Flat area–slope<0.05 (Building around within 1 KM radius)
- Beam in grid line 1,2,3…12 : 250 x 600 mm
- Beam in grid line A, B, C & D : 250 x 500 mm
- Slab thickness = 150 mm
- Columns : 300 x 400 mm
- Imposed load : 4.0 kN/m^2
- Finishes, ceiling, services etc : 0.75 kN/m^2 ; Partitions : 0.5 kN/m^2

Example

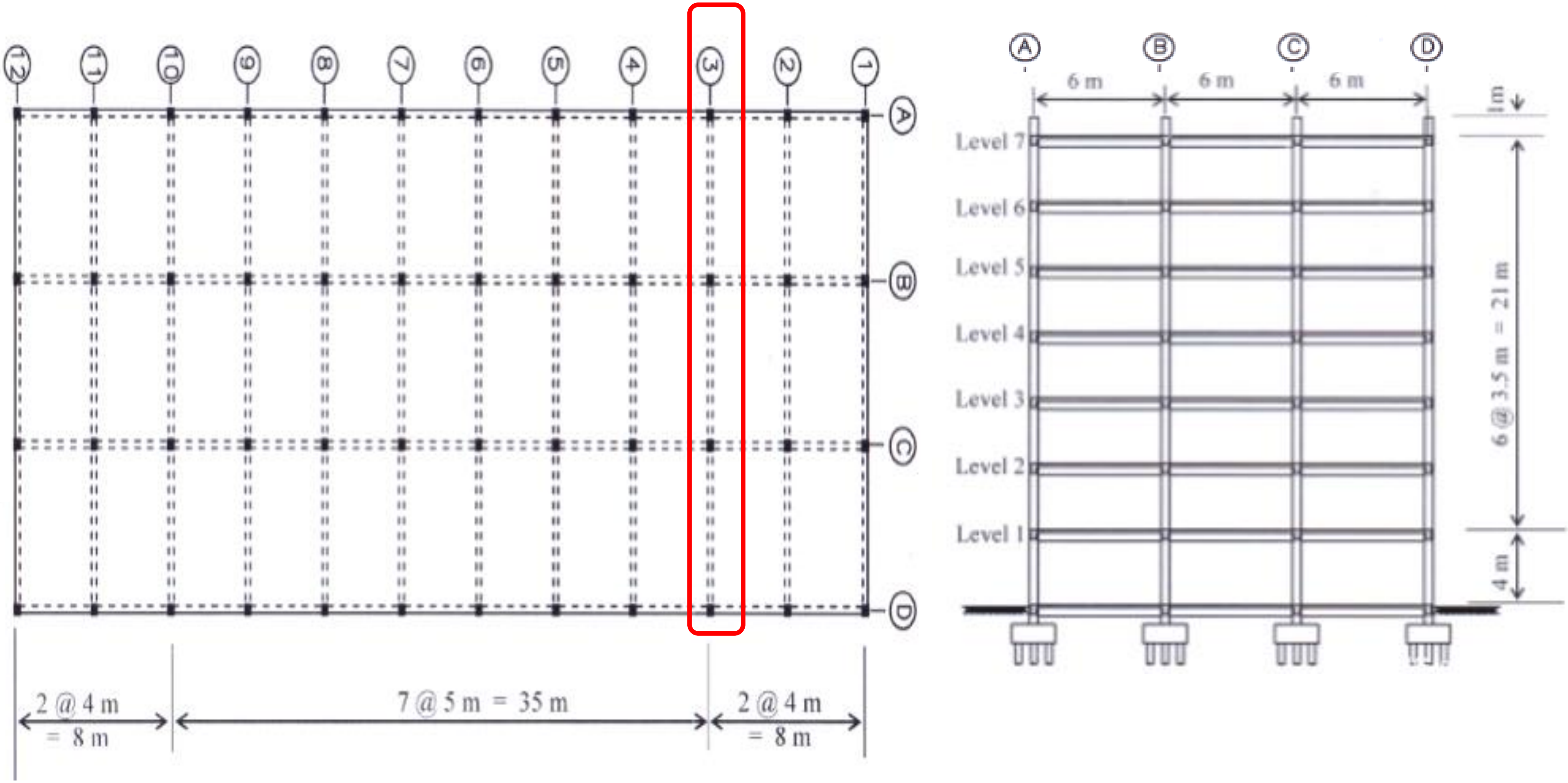


Figure 2.1

Example

- Step analysis of braced frame:
  1. Analysis actions on beam (load transfer from slab, selfweight and wall)
  2. Calculate moment of inertia for beams and columns, $I = bh^3/12$
  3. Calculate stiffness, $k = I/L$
  4. Calculate distribution factor, $DF = k_i / \Sigma k$
  5. Determine carry over factor, CF=0.5 (for pin support)
  6. Determine Fixed End Moment, $FEM = wl^2/12$ (for uniform load)
  7. Moment distribution by cases:
     - Case 1 [Span 1,2: max / span 3: min]
     - Case 2 [Span 1: min / span 2,3: max]
     - Case 3 [Span 1,3: max / span 2: min]
     - Case 4 [Span 1,3: min / span 2: max]
  8. Draw BMD and SFD diagrams

Example

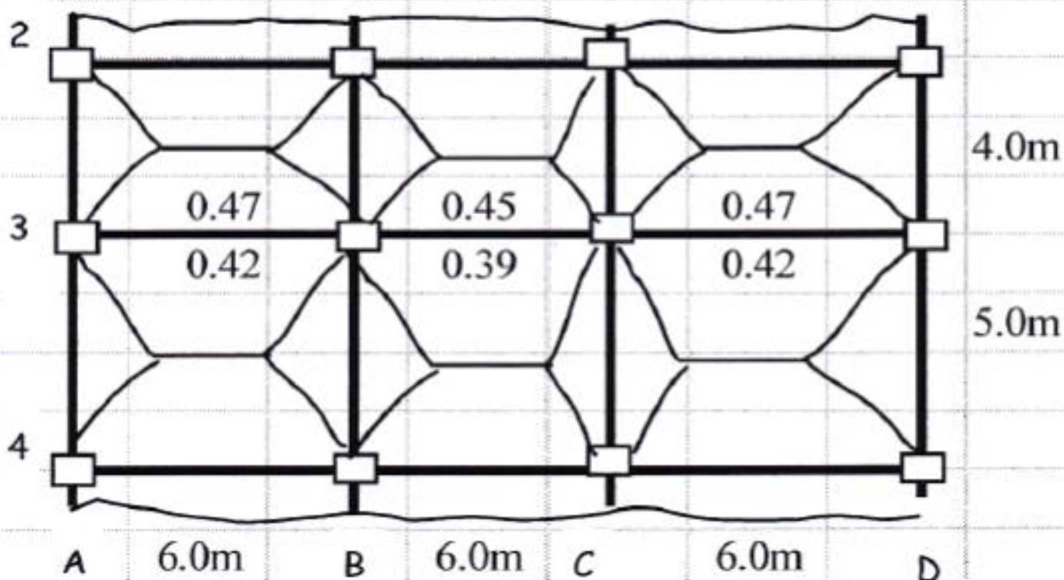
- Action on beam:

Ref.	Calculations				Output
	SPECIFICATION				
	Loading:				
	Finishes, ceiling, services etc.	=	0.75	kN/m^2	
	Density of concrete	=	25	kN/m^3	
	Imposed load	=	4.00	kN/m^2	
	Partition	=	0.50	kN/m^2	
	Dimension:				
	Slab thickness, h	=	150	mm	
	Beam size, b x h	=	250 x 600	mm	
	Column size, b x h	=	300 x 400	mm	

Example

- Action on beam:

Loading distribution:

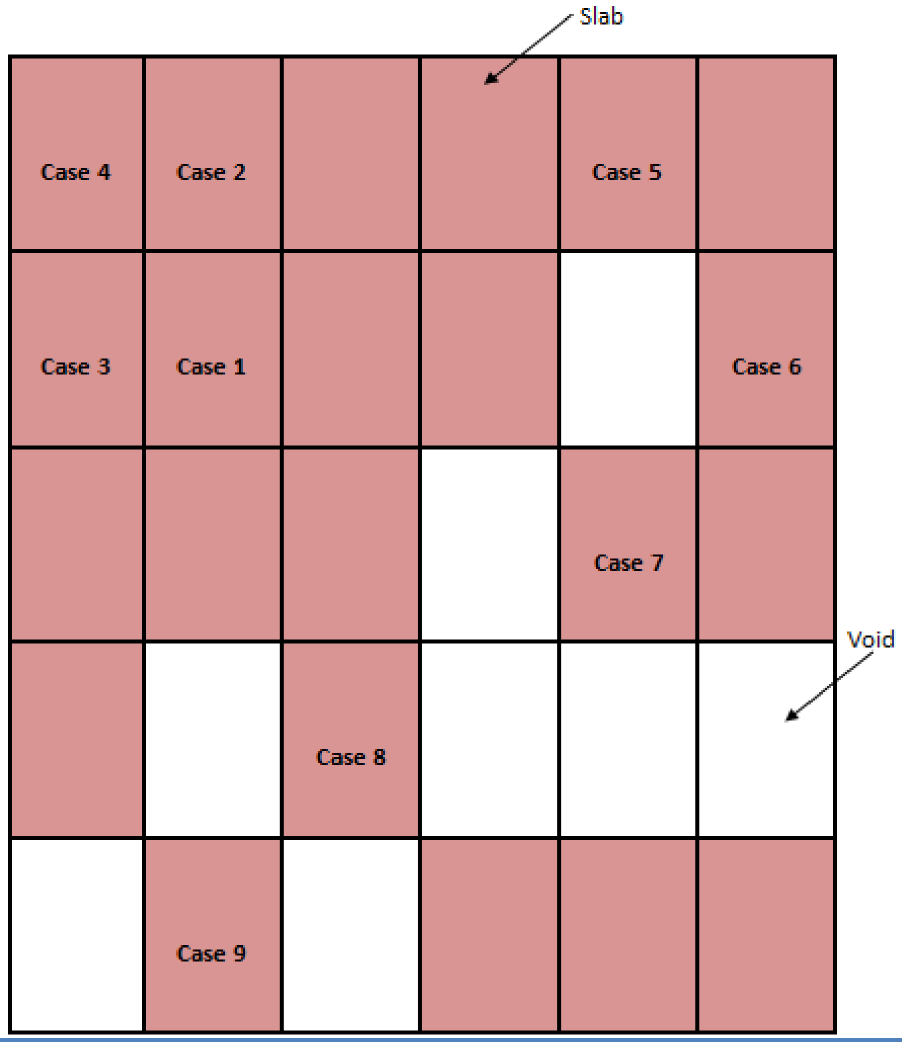


Slab 2-3/A-B :	L_y/L_x	=	6.0 / 4.0 =	1.50	Case 2
Slab 3-4/A-B :	L_y/L_x	=	6.0 / 5.0 =	1.20	Case 2
Slab 2-3/B-C :	L_y/L_x	=	6.0 / 4.0 =	1.50	Case 1
Slab 3-4/B-C :	L_y/L_x	=	6.0 / 5.0 =	1.20	Case 1
Slab 2-3/C-D :	L_y/L_x	=	6.0 / 4.0 =	1.50	Case 2
Slab 3-4/C-D :	L_y/L_x	=	6.0 / 5.0 =	1.20	Case 2

Example

- Action on beam: $v_{sx} = \beta_{vx} n l_x$ $\quad$ $v_{sy} = \beta_{vy} n l_x$

Type of panel location	β_{vx} for values of l_y/l_x								β_{vy}
	1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	
Four edges continuous									
Continuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33
One short edge discontinuous									
Continuous edge	0.36	0.39	0.42	0.44	0.45	0.47	0.50	0.52	0.36
Discontinuous edge	-	-	-	-	-	-	-	-	0.24
One long edge discontinuous									
Continuous edge	0.36	0.40	0.44	0.47	0.49	0.51	0.55	0.59	0.36
Discontinuous edge	0.24	0.27	0.29	0.31	0.32	0.34	0.36	0.38	-
Two adjacent edges discontinuous									
Continuous edge	0.40	0.44	0.47	0.50	0.52	0.54	0.57	0.60	0.40
Discontinuous edge	0.26	0.29	0.31	0.32	0.34	0.35	0.38	0.40	0.26
Two short edges discontinuous									
Continuous edge	0.40	0.43	0.45	0.47	0.48	0.49	0.52	0.54	-
Discontinuous edge	-	-	-	-	-	-	-	-	0.26
Two long edges discontinuous									
Continuous edge	-	-	-	-	-	-	-	-	0.40
Discontinuous edge	0.26	0.30	0.33	0.36	0.38	0.40	0.44	0.47	-
Three edges discontinuous (one long edge continuous)									
Continuous edge	0.45	0.48	0.51	0.53	0.55	0.57	0.60	0.63	-
Discontinuous edge	0.30	0.32	0.34	0.35	0.36	0.37	0.39	0.41	0.29
Three edges discontinuous (one short edge continuous)									
Continuous edge	-	-	-	-	-	-	-	-	0.45
Discontinuous edge	0.29	0.33	0.36	0.38	0.40	0.42	0.45	0.48	0.30
Four edges discontinuous									
Discontinuous edge	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33



Example

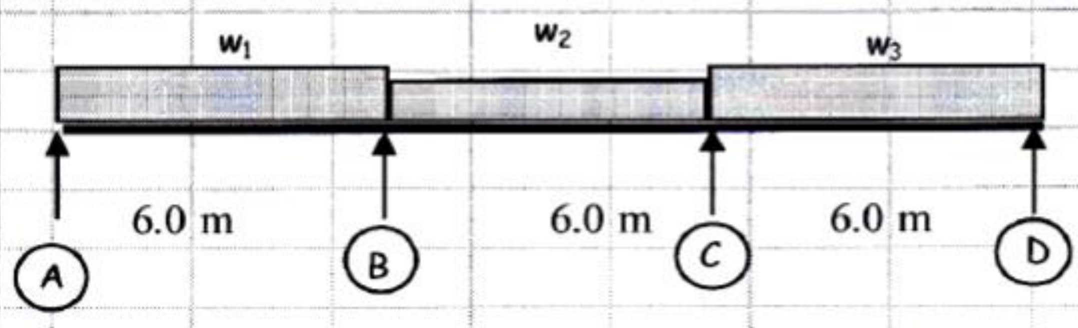
- Action on beam:

ACTIONS

Loads on slab, *n* kN/m² :

Slab selfweight =	0.15	x 25	=	3.75 kN/m²
Finishes, ceiling etc. =				0.75 kN/m²
Characteristic permanent load, g_k =				4.50 kN/m²
Imposed load =				4.00 kN/m²
Partition =				0.50 kN/m²
Characteristic variable load, q_k =				4.50 kN/m²

Loads on beam, *w* kN/m :



Example

- Action on beam:

Span 1 (A-B) : w_1					
Perm. load from slab =	0.47 x	4.50 x	4.0 =	8.46	kN/m
Perm. load from slab =	0.42 x	4.50 x	5.0 =	9.45	kN/m
Beam selfweight =	0.45 x	0.25 x	25 =	2.81	kN/m
Characteristic permanent load, G_k			=	20.72	kN/m
Variable load fr.slab =	0.47 x	4.50 x	4.0 =	8.46	kN/m
Variable load fr. slab =	0.42 x	4.50 x	5.0 =	9.45	kN/m
Chacracteristic variable load, Q_k =				17.91	kN/m
Design load, $1.35G_k + 1.5Q_k$			=	54.84	kN/m
$1.35G_k$			=	27.98	kN/m
Span 2 (B-C) : w_2					
Perm. load from slab =	0.45 x	4.50 x	4.0 =	8.10	kN/m
Perm. load from slab =	0.39 x	4.50 x	5.0 =	8.78	kN/m
Beam selfweight =	0.45 x	0.25 x	25 =	2.81	kN/m
Characteristic permanent load, g_k			=	19.69	kN/m

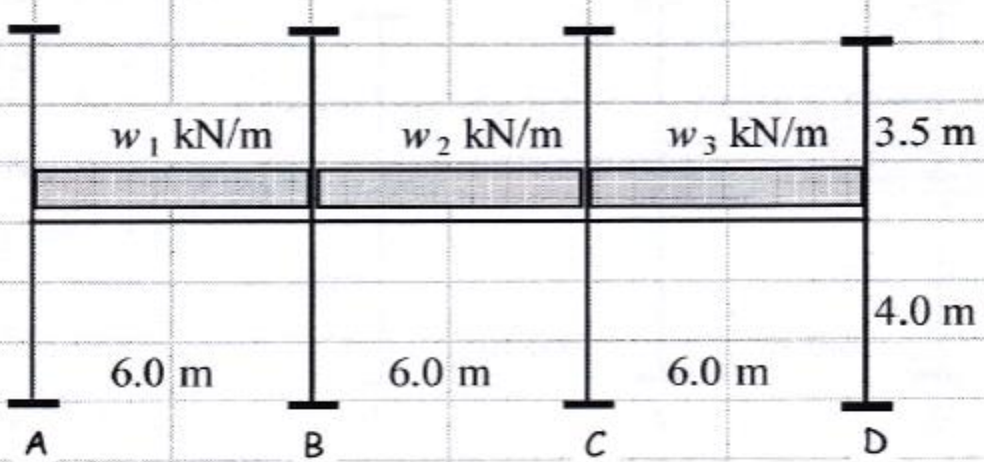
Example

- Action on beam:

Span 2 (B-C) : w_2					
Variable load fr. slab =	0.45 x	4.50 x	4.0 =	8.10	kN/m
Variable load fr. slab =	0.39 x	4.50 x	5.0 =	8.78	kN/m
Chacracteristic variable load, Q_k =				16.88	kN/m
Design load, $1.35G_k + 1.5Q_k$			=	51.89	kN/m
$1.35G_k$			=	26.58	kN/m
Span 3 (C-D) : w_3					
Perm. load from slab =	0.47 x	4.50 x	4.0 =	8.46	kN/m
Perm. load from slab =	0.42 x	4.50 x	5.0 =	9.45	kN/m
Beam selfweight =	0.45 x	0.25 x	25 =	2.81	kN/m
Characteristic permanent load, g_k			=	20.72	kN/m
Variable load from slab	0.47 x	4.50 x	4.0 =	8.46	kN/m
Variable load from slab	0.42 x	4.50 x	5.0 =	9.45	kN/m
Chacracteristic variable load, Q_k =				17.91	kN/m
Design load, $1.35G_k + 1.5Q_k$			=	54.84	kN/m
$1.35G_k$			=	27.98	kN/m

Example

- Analysis of one level sub-fame

Ref.	Calculations	Output	
	Data:		
		Actions:	
		w_1 :	kN/m
		$1.35G_k =$	28.0
		$1.35G_k+1.5Q_k=$	54.8
		w_2 :	
		$1.35G_k =$	26.6
		$1.35G_k+1.5Q_k=$	51.9
		w_3 :	
	Size :	$1.35G_k =$	28.0
	Beam : b x h = 250 x 600 mm	$1.35G_k+1.5Q_k=$	54.8
	Column : b x h = 300 x 400 mm		

Example

- Analysis of one level sub-fame

Momen of Inertia : $I = bh^3/12$					
Beam : I =	250 x	600	^{3}/12 =	4.5 x	10^9 mm^4
Column: I =	300 x	400	^{3}/12 =	1.6 x	10^9 mm^4
Stiffness : $K = I/L$					
Column: K_{cu} =	1.6 x	10^9 /	3500 =	4.6 x	10^5 mm^3
K_{cl} =	1.6 x	10^9 /	4000 =	4.0 x	10^5 mm^3
Beam: K_{AB} =	4.5 x	10^9 /	6000 =	7.5 x	10^5 mm^3
K_{BC} =	4.5 x	10^9 /	6000 =	7.5 x	10^5 mm^3
K_{CD} =	4.5 x	10^9 /	6000 =	7.5 x	10^5 mm^3

Example

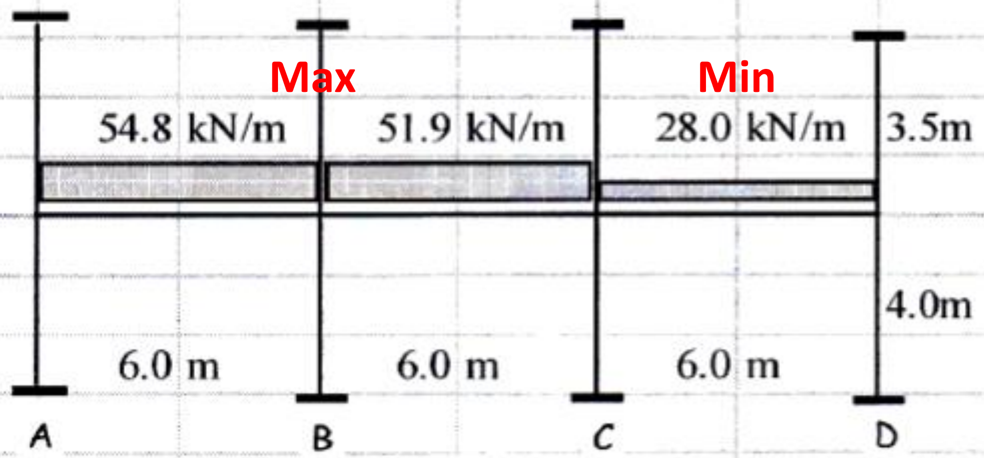
- Analysis of one level sub-fame

Distribution factor : $F = K / \Sigma K$		
Joint A:	$F_{AB} = K_{AB}/(K_{AB} + K_{cu} + K_{cl}) =$	0.47
	$F_{cu} = K_{cu}/(K_{AB} + K_{cu} + K_{cl}) =$	0.28
	$F_{cl} = K_{cl}/(K_{AB} + K_{cu} + K_{cl}) =$	0.25
Joint B:	$F_{BA} = K_{AB}/(K_{AB}+K_{BC}+K_{cu}+K_{cl}) =$	0.32
	$F_{BC} = K_{BC}/(K_{AB}+K_{BC}+K_{cu}+K_{cl}) =$	0.32
	$F_{cu} = K_{cu}/(K_{AB}+K_{BC}+K_{cu}+K_{cl}) =$	0.19
	$F_{cl} = K_{cl}/(K_{AB}+K_{BC}+K_{cu}+K_{cl}) =$	0.17
Joint C:	$F_{CB} = K_{BC}/(K_{BC}+K_{CD}+K_{cu}+K_{cl}) =$	0.32
	$F_{CD} = K_{CD}/(K_{BC}+K_{CD}+K_{cu}+K_{cl}) =$	0.32
	$F_{cu} = K_{cu}/(K_{BC}+K_{CD}+K_{cu}+K_{cl}) =$	0.19
	$F_{cl} = K_{cl}/(K_{BC}+K_{CD}+K_{cu}+K_{cl}) =$	0.17
Joint D:	$F_{DC} = K_{CD}/(K_{CD}+K_{cu}+K_{cl}) =$	0.47
	$F_{cu} = K_{cu}/(K_{CD}+K_{cu+}K_{cl}) =$	0.28
	$F_{cl} = K_{cl}/(K_{CD}+K_{cu}+K_{cl}) =$	0.25

Example: One Level Sub-Frame

- Analysis of one level sub-fame : CASE 1

Ref.	Calculations	Output
	Case 1 : Span 1 & 2 design permanent & variable loads $1.35G_k + 1.5Q_k$	
	Span 3 design permanent loads $1.35G_k$	



Fixed end moment :

$-M_{AB} = M_{BA} = w_1 L_1^2/12$

$= 54.8 \times 6.0^2/12$

$= 164.5$ kNm

$-M_{BC} = M_{CB} = w_2 L_2^2/12$

$= 51.9 \times 6.0^2/12$

$= 155.7$ kNm

$-M_{CD} = M_{DC} = w_1 L_1^2/12$

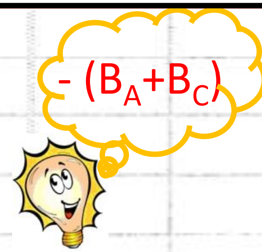
$= 28.0 \times 6.0^2/12$

$= 83.9$ kNm

Example: One Level Sub-Frame

- Analysis of one level sub-fame : CASE 1

Moment distribution												
48.58				-7.90				-9.26				-21.65
0.17				-0.24				0.26				-0.07
1.22				-0.71				0.32				-0.95
0.40				-5.23				4.07				3.25
46.80				-1.72				-13.91				-23.87
0.28	A		B	0.19			C	0.19			D	0.28
0.25	0.47		0.32	0.17	0.32		0.32	0.17	0.32		0.47	0.25
	-164.5		164.5		-155.7		155.7		-83.9		83.9	
40.9	76.8		-2.8	-1.5	-2.8		-22.8	-12.2	-22.8		-39.2	-20.9
	-1.4		38.4		-11.4		-1.4		-19.6		-11.4	
0.4	0.7		-8.6	-4.6	-8.6		6.7	3.6	6.7		5.3	2.8
	-4.3		0.3		3.3		-4.3		2.7		3.3	
1.1	2.0		-1.2	-0.6	-1.2		0.5	0.3	0.5		-1.6	-0.8
	-0.6		1.0		0.3		-0.6		-0.8		0.3	
0.1	0.3		-0.4	-0.2	-0.4		0.4	0.2	0.4		-0.1	-0.1
42.5	-91.1		191.3	-6.9	-176.5		134.2	-8.1	-116.8		40.6	-18.9

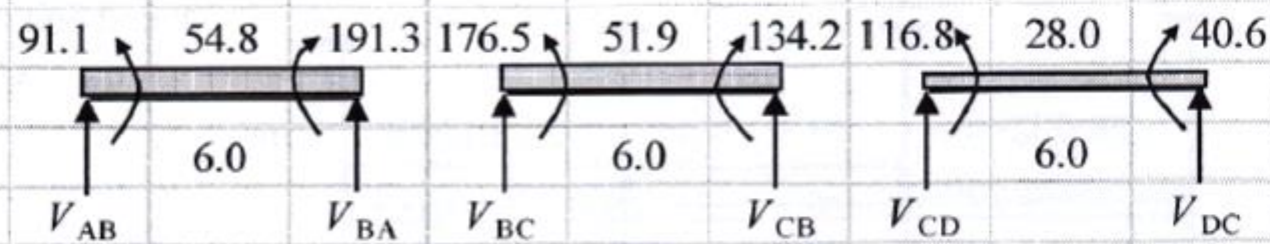


Beam

Column

Example: One Level Sub-Frame

- Analysis of one level sub-fame : CASE 1

Ref.	Calculations	Output
	Shear force :	
		
	ΣM @ B = 0	
	$6.0\, V_{AB}$ - (54.8 x 6.0 x 3.0) + 191.3 - 91.1 = 0	
	V_{AB} = (987.13 - 191.3 +91.1) / 6.0 = 147.8 kN	
	V_{BA} = (54.8 x 6.0) - 147.8 = 181.2 kN	

Example: One Level Sub-Frame

- Analysis of one level sub-fame : CASE 1

$\Sigma M @ C = 0$

$6.0\,V_{BC} - (51.9 \times 6.0 \times 3.0) + 134.2 - 176.5 = 0$

$V_{BC} = (934.03 - 134.2 + 176.5) / 6.0 = 162.7$ kN

$V_{CB} = (51.9 \times 6.0) - 162.7 = 148.6$ kN

$\Sigma M @ D = 0$

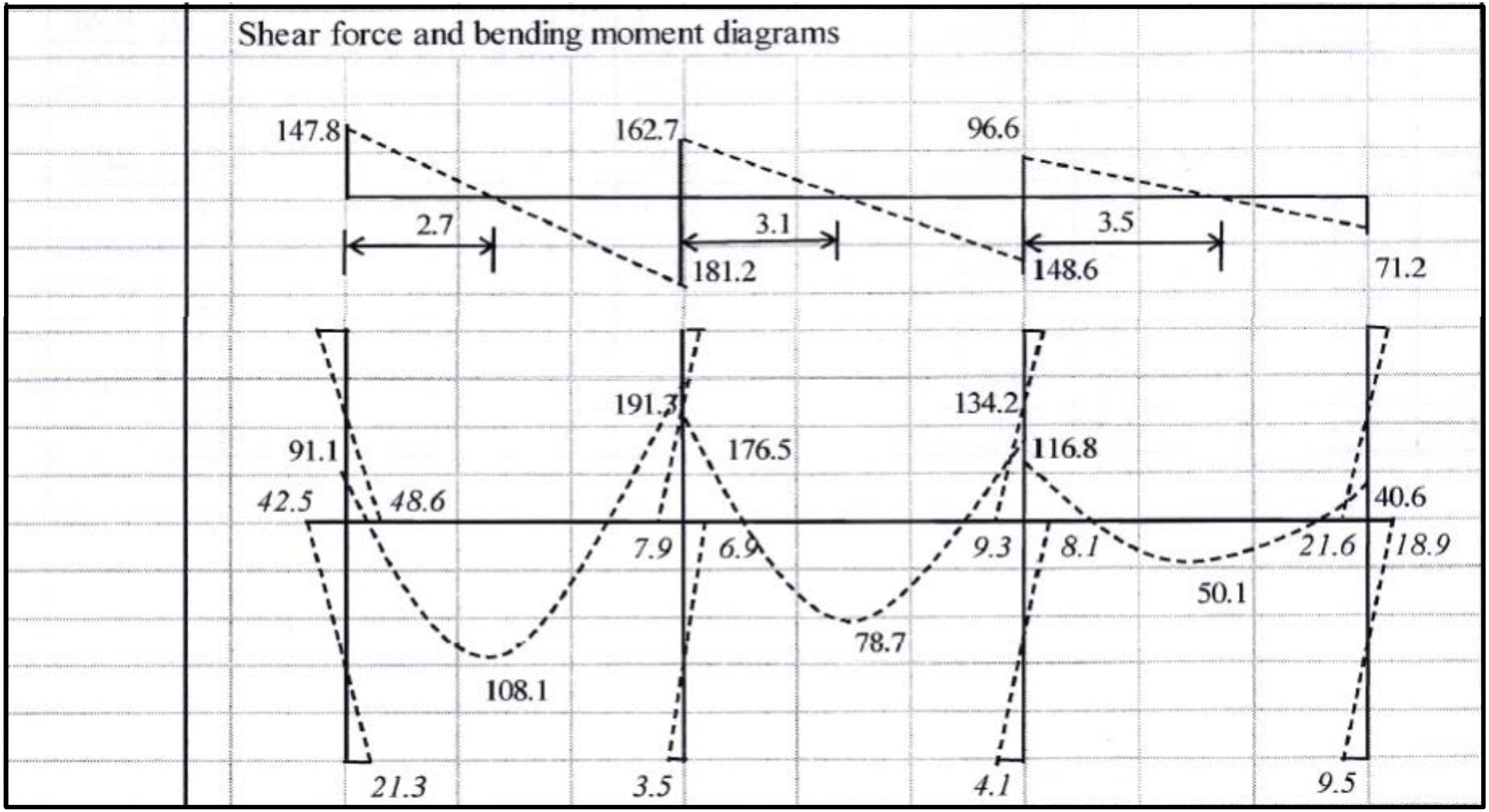
$6.0\,V_{CD} - (28.0 \times 6.0 \times 3.0) - 116.8 + 40.6 = 0$

$V_{CD} = (503.56 + 116.8 - 40.6) / 6.0 = 96.6$ kN

$V_{DC} = (28.0 \times 6.0) - 96.6 = 71.2$ kN

Example: One Level Sub-Frame

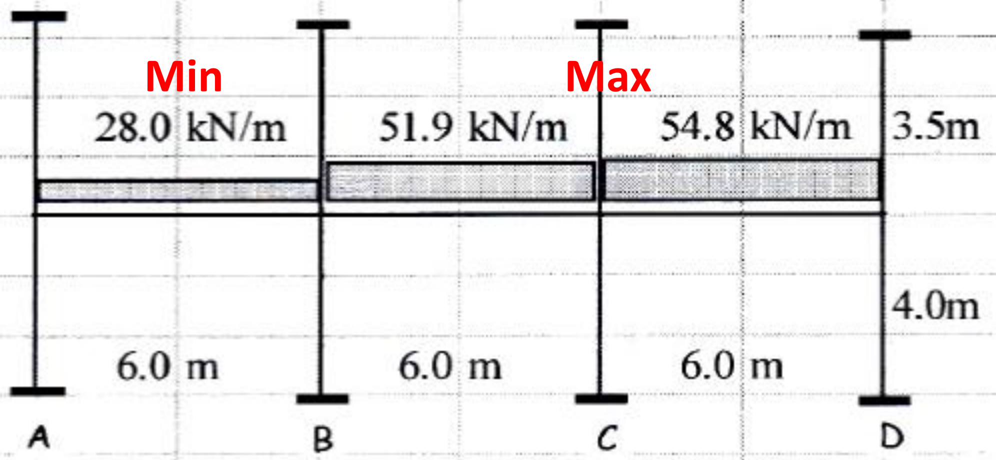
- Analysis of one level sub-fame : CASE 1



Example: One Level Sub-Frame

- Analysis of one level sub-fame : CASE 2

Ref.	Calculations	Output
	Case 2 : Span 2 & 3 design permanent & variable loads $1.35G_k + 1.5Q_k$ Span 1 design permanent loads $1.35G_k$	



Fixed end moment :

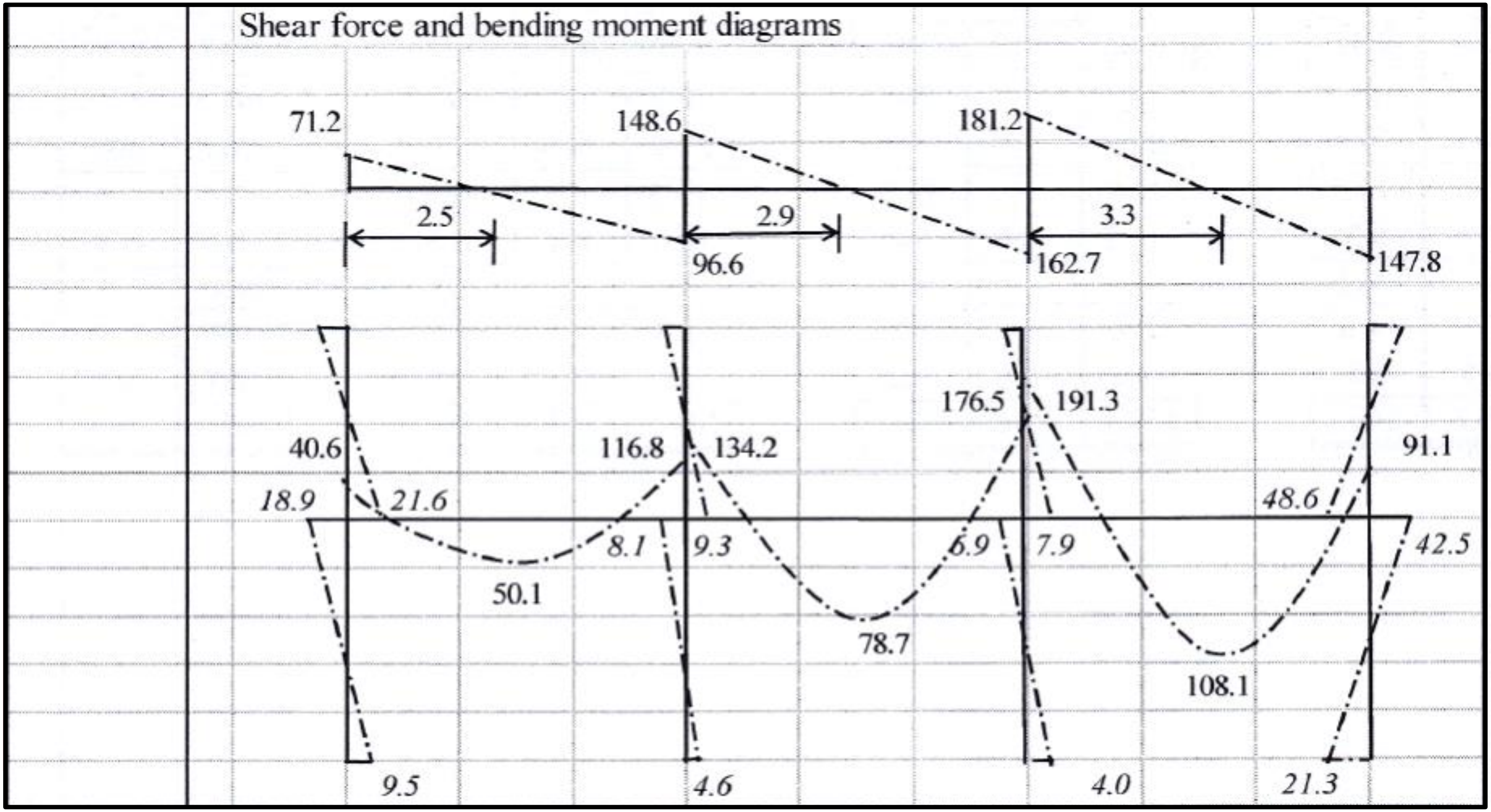
$-M_{AB} = M_{BA} = w_1 L_1^2/12$
$= 28.0 \times 6.0^2/12$
$= 83.9$ kNm

$-M_{BC} = M_{CB} = w_2 L_2^2/12$
$= 51.9 \times 6.0^2/12$
$= 155.7$ kNm

$-M_{CD} = M_{DC} = w_1 L_1^2/12$
$= 54.8 \times 6.0^2/12$
$= 164.5$ kNm

Example: One Level Sub-Frame

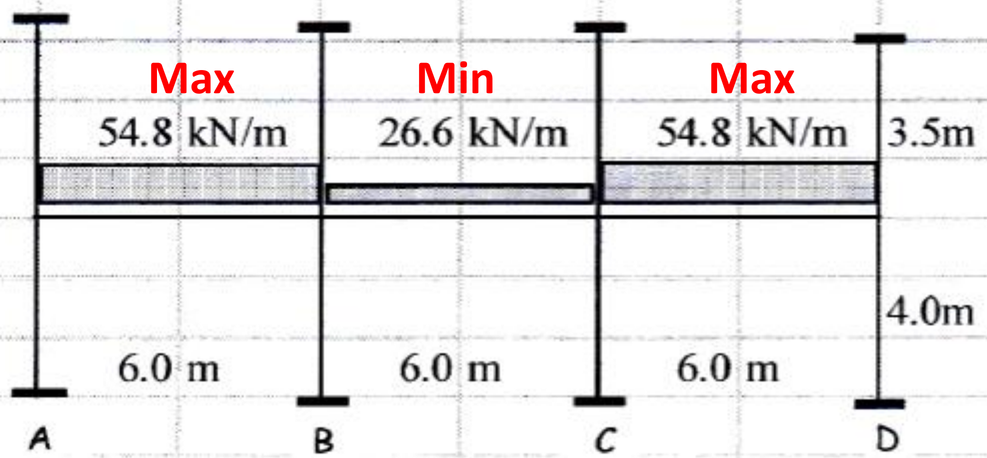
- Analysis of one level sub-fame : CASE 2



Example: One Level Sub-Frame

- Analysis of one level sub-fame : CASE 3

Ref.	Calculations	Output
	Case 3 : Span 1 & 3 design permanent & variable loads $1.35G_k + 1.5Q_k$	
	Span 2 design permanent loads $1.35G_k$	



Fixed end moment :

$-M_{AB} = M_{BA} = w_1 L_1^2/12$

$= 54.8 \times 6.0^2/12$

$= 164.5$ kNm

$-M_{BC} = M_{CB} = w_2 L_2^2/12$

$= 26.6 \times 6.0^2/12$

$= 79.7$ kNm

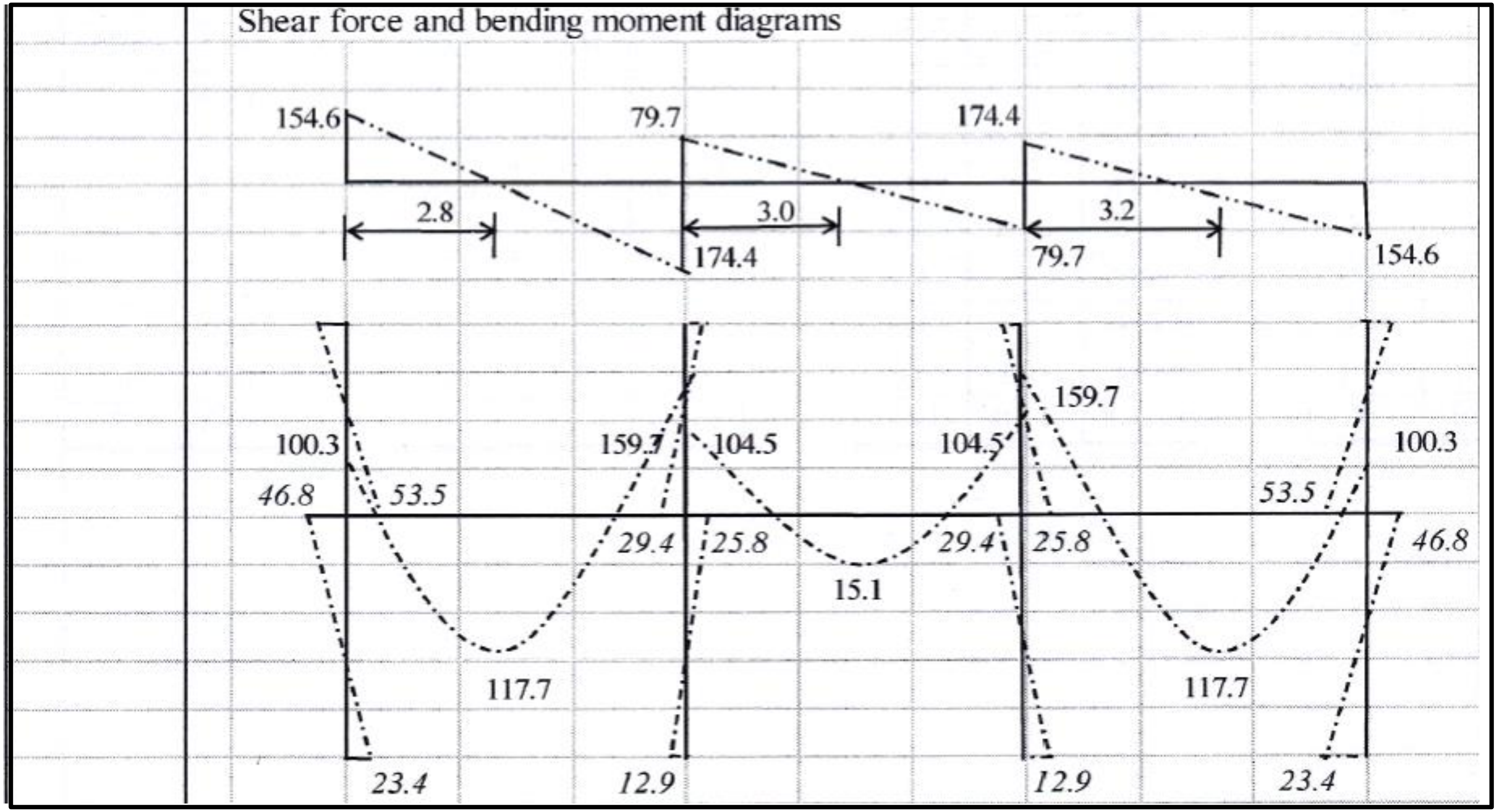
$-M_{CD} = M_{DC} = w_1 L_1^2/12$

$= 54.8 \times 6.0^2/12$

$= 164.5$ kNm

Example: One Level Sub-Frame

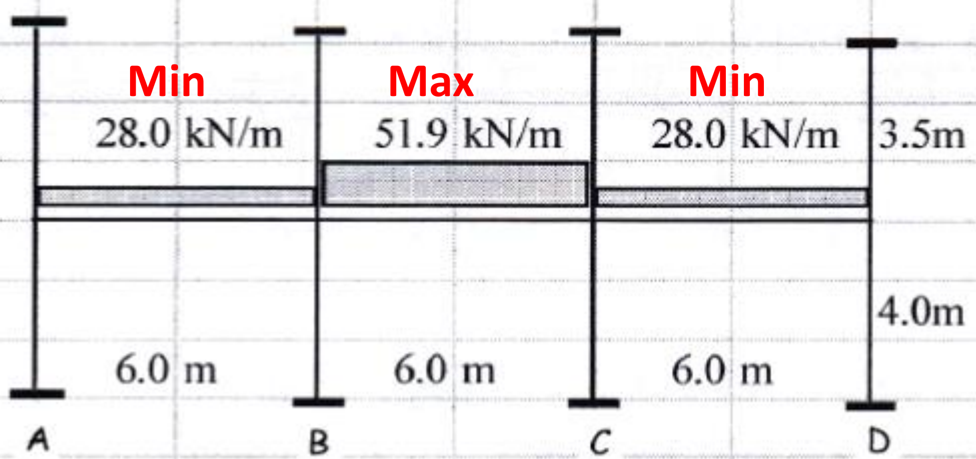
- Analysis of one level sub-fame : CASE 3



Example: One Level Sub-Frame

- Analysis of one level sub-fame : CASE 4

Ref.	Calculations	Output
	Case 4 : Span 2 design permanent & variable loads $1.35G_k + 1.5Q_k$	
	Span 1 & 3 design permanent loads $1.35G_k$	



Fixed end moment :

$-M_{AB} = M_{BA} = w_1 L_1^2/12$

$= 28.0 \times 6.0^2/12$

$= 83.9$ kNm

$-M_{BC} = M_{CB} = w_2 L_2^2/12$

$= 51.9 \times 6.0^2/12$

$= 155.7$ kNm

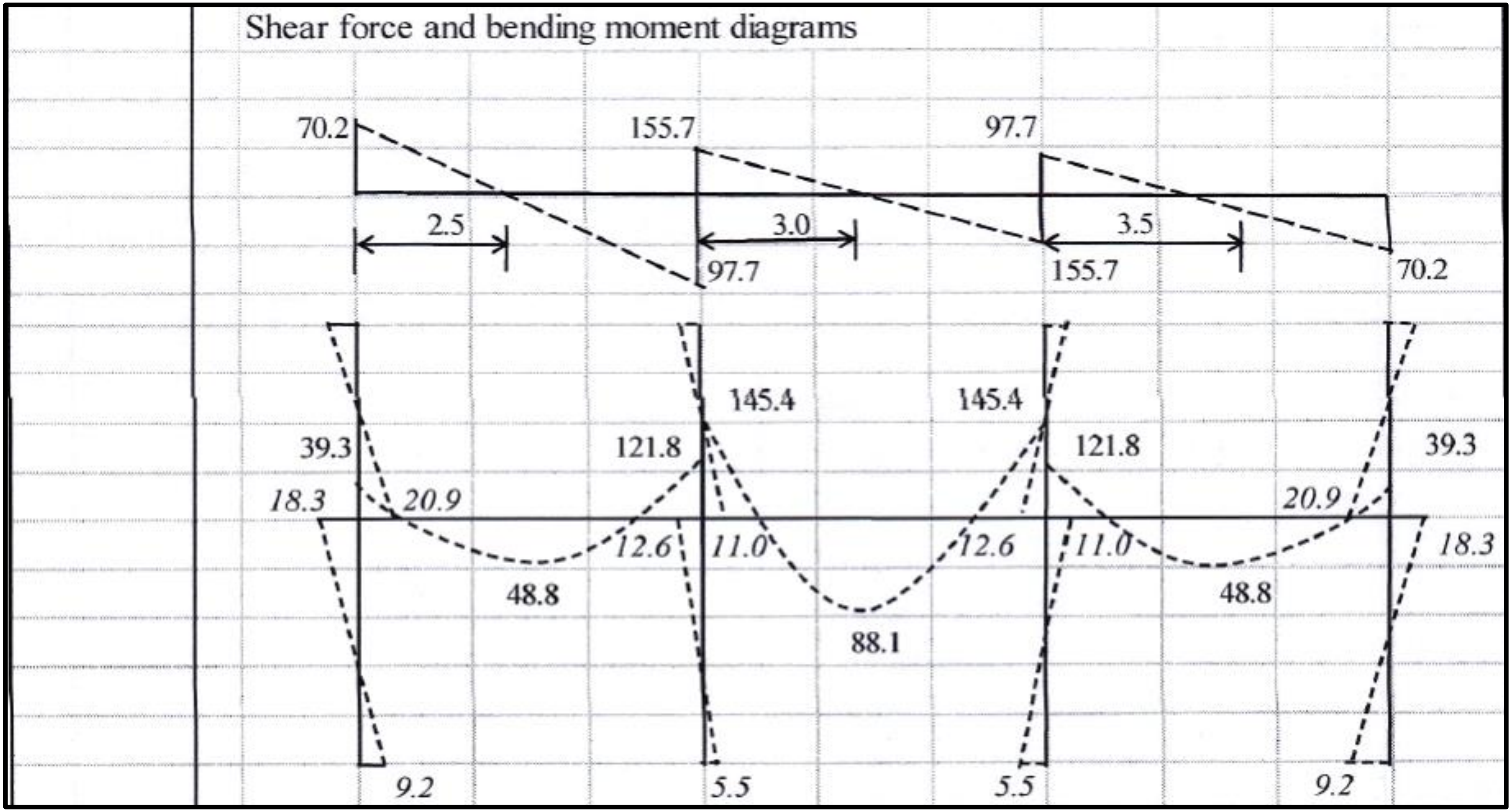
$-M_{CD} = M_{DC} = w_1 L_1^2/12$

$= 28.0 \times 6.0^2/12$

$= 83.9$ kNm

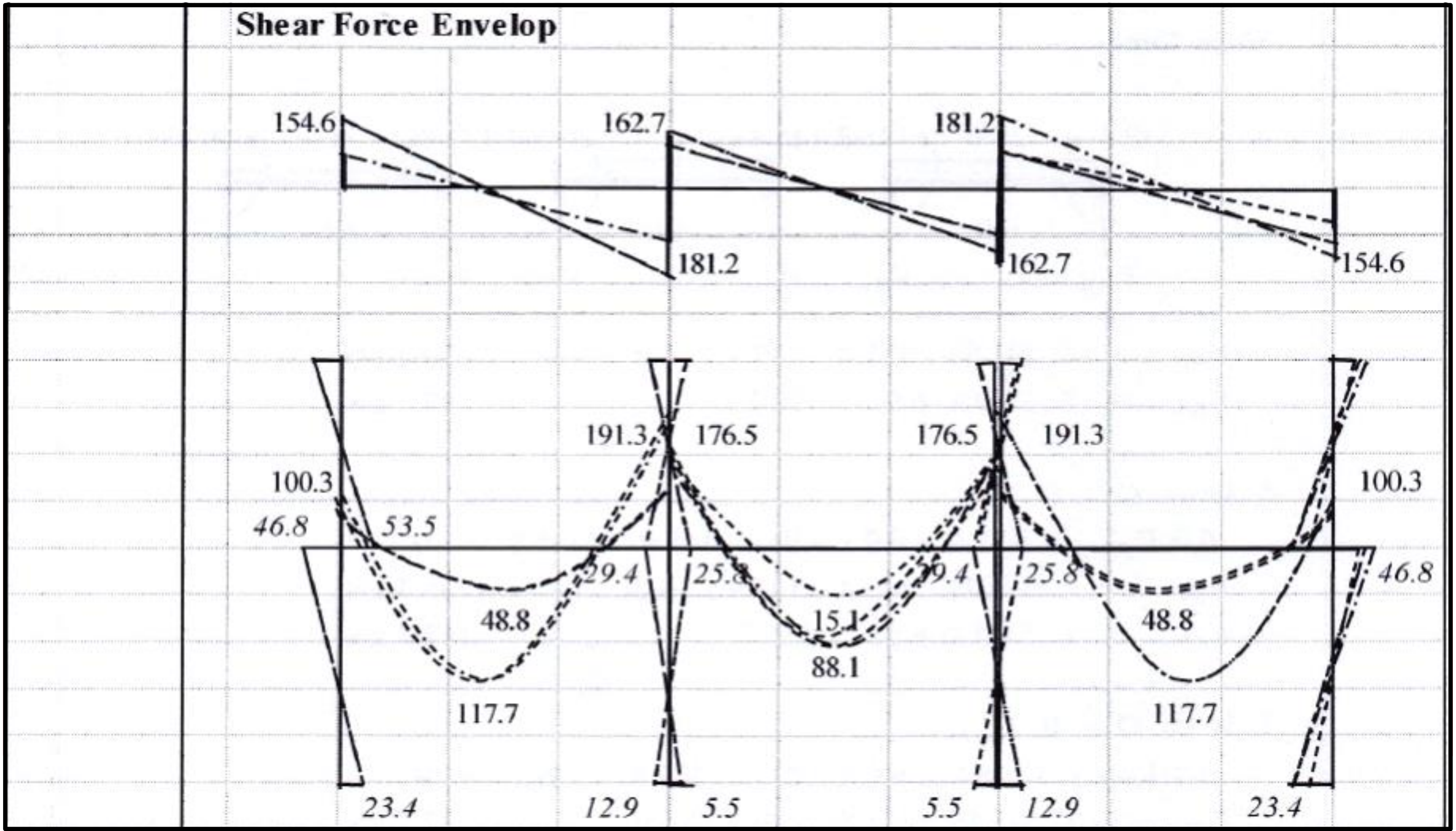
Example: One Level Sub-Frame

- Analysis of one level sub-fame : CASE 4



Example: One Level Sub-Frame

- Analysis of one level sub-fame : ENVELOPE

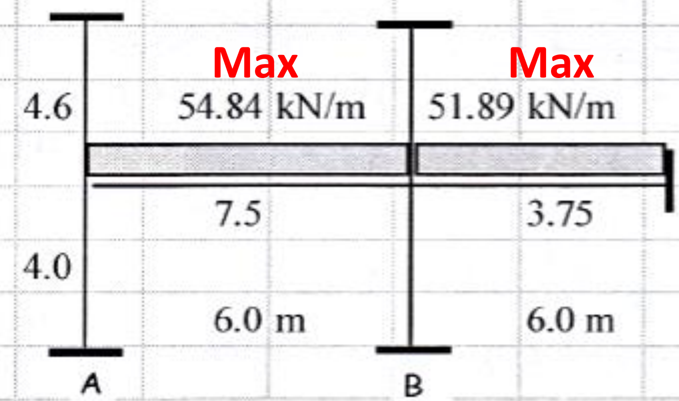


Example: Two Point Sub-Frame

Beam A-B (Beam C-D similar)

Case 1 : Span 1 design permanent & variable loads $1.35G_k + 1.5Q_k$

: Span 2 design permanent & variable loads $1.35G_k + 1.5Q_k$



Fixed end moment :

$-M_{AB} = M_{BA} = w_1 L_1^2/12$

$= 54.8 \times 6.0^2/12$

$= 164.5$ kNm

$-M_{BC} = M_{CB} = w_2 L_2^2/12$

$= 51.9 \times 6.0^2/12$

$= 155.7$ kNm

Distribution factor:

Joint A:	F_{AB} =	7.5 /	(7.5 +	4.6 +	4.0)		=	0.47
	F_{cu} =	4.6 /	(7.5 +	4.6 +	4.0)		=	0.28
	F_{cl} =	4.0 /	(7.5 +	4.6 +	4.0)		=	0.25
Joint B:	F_{BA} =	7.5 /	(7.5 +	3.75 +	4.0 +	4.6)	=	0.38
	F_{BC} =	3.8 /	(7.5 +	3.75 +	4.0 +	4.6)	=	0.19
	F_{cu} =	4.0 /	(7.5 +	3.8 +	4.0 +	4.6)	=	0.20
	F_{cl} =	4.6 /	(7.5 +	3.8 +	4.0 +	4.6)	=	0.23

Example: Two Point Sub-Frame

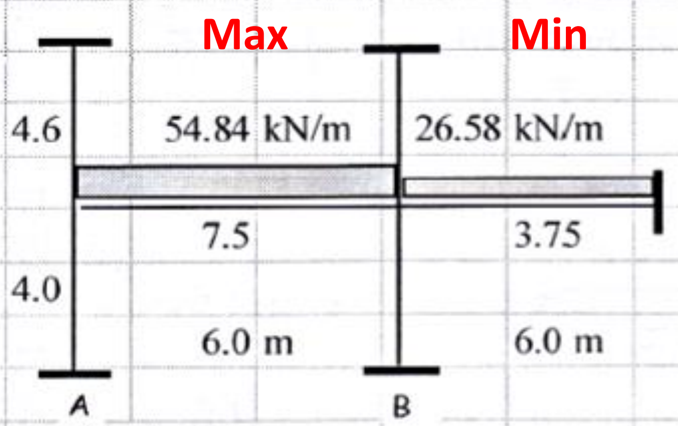
Moment distribution

49.36				-9.95	
0.02				-0.34	
2.07				-0.08	
0.48				-7.75	
46.80				-1.79	
0.28	A		B	0.20	
0.25	0.47		0.38	0.23	0.19
	-164.5		164.5		-155.7
40.9	76.8		-3.3	-2.0	-1.7
	-1.7		38.4		0.0
0.4	0.8		-14.5	-8.9	-7.3
	-7.3		0.4		0.0
1.8	3.4		-0.1	-0.1	-0.1
	-0.1		1.7		0.0
0.0	0.0		-0.6	-0.4	-0.3
43.2	-92.6		186.3	-11.4	-165.0

Example: Two Point Sub-Frame

Case 2: Span 1 design permanent & variable loads $1.35G_k + 1.5Q_k$

: Span 2 design permanent loads $1.35G_k$



Fixed end moment :

$-M_{AB} = M_{BA} = w_1 L_1^2/12$

$= 54.8 \times 6.0^2/12$

$= 164.5$ kNm

$-M_{BC} = M_{CB} = w_2 L_2^2/12$

$= 26.6 \times 6.0^2/12$

$= 79.7$ kNm

Distribution factor:

Joint A:	F_{AB} =	7.5 /	(7.5 +	4.6 +	4.0)		=	0.47
	F_{cu} =	4.6 /	(7.5 +	4.6 +	4.0)		=	0.28
	F_{cl} =	4.0 /	(7.5 +	4.6 +	4.0)		=	0.25
Joint B:	F_{BA} =	7.5 /	(7.5 +	3.75 +	4.0 +	4.6)	=	0.38
	F_{BC} =	3.8 /	(7.5 +	3.75 +	4.0 +	4.6)	=	0.19
	F_{cu} =	4.0 /	(7.5 +	3.8 +	4.0 +	4.6)	=	0.20
	F_{cl} =	4.6 /	(7.5 +	3.8 +	4.0 +	4.6)	=	0.23

Example: Two Point Sub-Frame

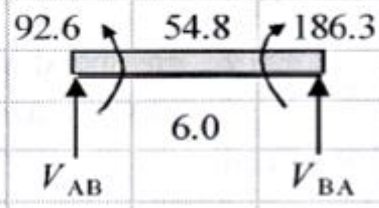
Moment distribution

53.63			-25.95	
0.20			-0.34	
2.07			-0.76	
4.56			-7.75	
46.80			-17.11	
0.28	A	B	0.20	
0.25	0.47	0.38	0.23	0.19
	-164.5	164.5		-79.7
40.9	76.8	-32.1	-19.6	-16.0
	-16.0	38.4		0.0
4.0	7.5	-14.5	-8.9	-7.3
	-7.3	3.7		0.0
1.8	3.4	-1.4	-0.9	-0.7
	-0.7	1.7		0.0
0.2	0.3	-0.6	-0.4	-0.3
46.9	-100.6	159.7	-29.7	-104.1

Example: Two Point Sub-Frame

Shear force :

Case 1



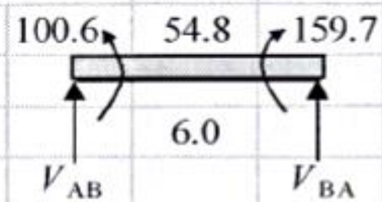
$\Sigma M @ B = 0$

$6.0\, V_{AB} - (54.8 \times 6.0 \times 3.0) + 186.3 - 92.6 = 0$

$V_{AB} = (987.13 - 186.3 + 92.6) / 6.0 = 148.9$ kN

$V_{BA} = (54.8 \times 6.0) - 148.9 = 180.2$ kN

Case 2



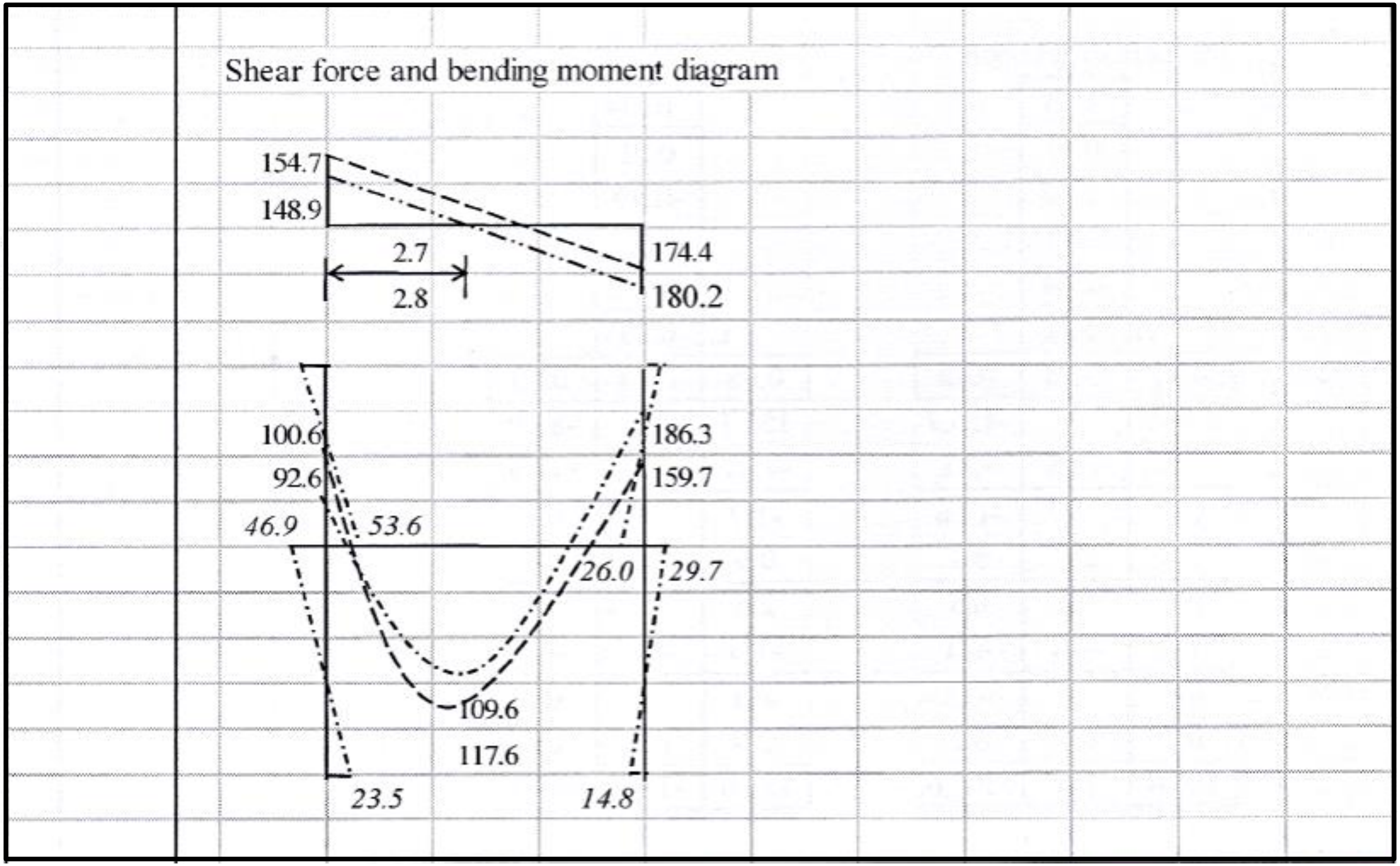
$\Sigma M @ B = 0$

$6.0\, V_{AB} - (54.8 \times 6.0 \times 3.0) + 159.7 - 100.6 = 0$

$V_{AB} = (987.13 - 159.7 + 100.6) / 6.0 = 154.7$ kN

$V_{BA} = (54.8 \times 6.0) - 154.7 = 174.4$ kN

Example: Two Point Sub-Frame

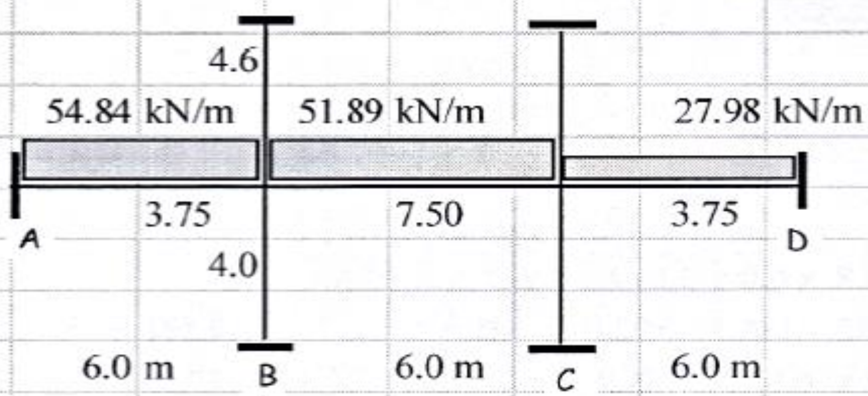


Example: Two Point Sub-Frame

Beam B-C

Case 1 : Span 1 and 2 design permanent & variable loads $1.35G_k + 1.5Q_k$

: Span 3 design permanent load $1.35G_k$



Fixed end moment :

$-M_{AB} = M_{BA} = w_1 L_1^2/12$

$= 54.8 \times 6.0^2/12$

$= 164.5$ kNm

$-M_{BC} = M_{CB} = w_2 L_2^2/12$

$= 51.9 \times 6.0^2/12$

$= 155.7$ kNm

$-M_{CD} = M_{DC} = w_3 L_3^2/12$

$= 28.0 \times 6.0^2/12$

$= 83.9$ kNm

Distribution factor:

Joint B:	F_{BA} =	3.75 / (3.75 + 4.57 + 4.0 + 7.5) =	0.19
	F_{BC} =	7.50 / (3.75 + 4.57 + 4.0 + 7.5) =	0.38
	F_{cu} =	4.57 / (3.75 + 4.57 + 4.0 + 7.5) =	0.23
	F_{cl} =	4.00 / (3.75 + 4.57 + 4.0 + 7.5) =	0.20
Joint C:	F_{CB} =	7.50 / (3.75 + 7.50 + 4.0 + 4.6) =	0.38
	F_{CD} =	3.75 / (3.75 + 7.50 + 4.0 + 4.6) =	0.19
	F_{cu} =	4.57 / (3.75 + 7.50 + 4.0 + 4.6) =	0.23
	F_{cl} =	4.00 / (3.75 + 7.50 + 4.0 + 4.6) =	0.20

Example: Two Point Sub-Frame

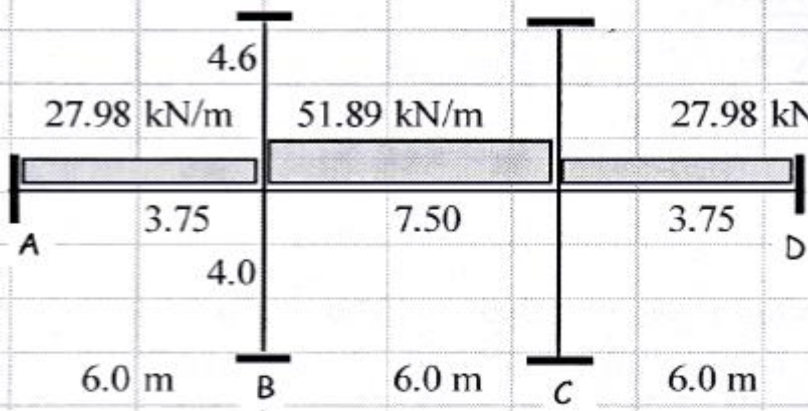
Moment distribution

	1.13			-16.74	
	0.11			0.01	
	-0.07			-0.59	
	3.13			0.39	
	-2.04			-16.55	
B	0.23		C	0.23	
0.19	0.20	0.38	0.38	0.20	0.19
164.5		-155.7	155.7		-83.9
-1.7	-1.8	-3.3	-27.1	-14.5	-13.6
0.0		-13.6	-1.7		0.0
2.6	2.7	5.1	0.6	0.3	0.3
0.0		0.3	2.6		0.0
-0.1	-0.1	-0.1	-1.0	-0.5	-0.5
0.0		-0.5	-0.1		0.0
0.1	0.1	0.2	0.0	0.0	0.0
165.4	1.0	-167.6	129.0	-14.6	-97.7

Example: Two Point Sub-Frame

Case 2 : Span 2 design permanent & variable loads $1.35G_k + 1.5Q_k$

: Span 1 and 3 design permanent load $1.35G_k$



Fixed end moment :

$-M_{AB} = M_{BA} = w_1L_1^2/12$

$= 28.0 \times 6.0^2/12$

$= 83.9$ kNm

$-M_{BC} = M_{CB} = w_2L_2^2/12$

$= 51.9 \times 6.0^2/12$

$= 155.7$ kNm

$-M_{CD} = M_{DC} = w_3L_3^2/12$

$= 28.0 \times 6.0^2/12$

$= 83.9$ kNm

Distribution factor:

Joint B:	F_{BA} =	3.75 / (3.75 + 4.57 + 4.0 + 7.5) =	0.19
	F_{BC} =	7.50 / (3.75 + 4.57 + 4.0 + 7.5) =	0.38
	F_{cu} =	4.57 / (3.75 + 4.57 + 4.0 + 7.5) =	0.23
	F_{cl} =	4.00 / (3.75 + 4.57 + 4.0 + 7.5) =	0.20
Joint C:	F_{CB} =	7.50 / (3.75 + 7.50 + 4.0 + 4.6) =	0.38
	F_{CD} =	3.75 / (3.75 + 7.50 + 4.0 + 4.6) =	0.19
	F_{cu} =	4.57 / (3.75 + 7.50 + 4.0 + 4.6) =	0.23
	F_{cl} =	4.00 / (3.75 + 7.50 + 4.0 + 4.6) =	0.20

Example: Two Point Sub-Frame

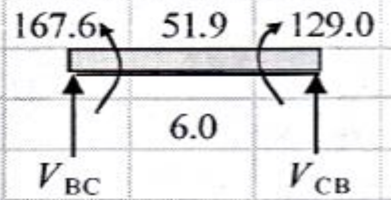
Moment distribution

	20.38				-20.38	
	0.11				-0.11	
	0.59				-0.59	
	3.13				-3.13	
	16.55				-16.55	
B	0.23			C	0.23	
0.19	0.20	0.38		0.38	0.20	0.19
83.9		-155.7		155.7		-83.9
13.6	14.5	27.1		-27.1	-14.5	-13.6
0.0		-13.6		13.6		0.0
2.6	2.7	5.1		-5.1	-2.7	-2.6
0.0		-2.6		2.6		0.0
0.5	0.5	1.0		-1.0	-0.5	-0.5
0.0		-0.5		0.5		0.0
0.1	0.1	0.2		-0.2	-0.1	-0.1
100.6	17.8	-138.9		138.9	-17.8	-100.6

Example: Two Point Sub-Frame

Shear force :

Case 1



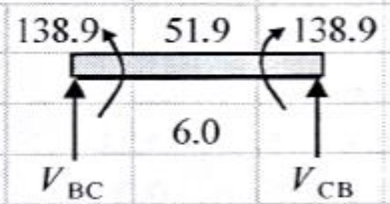
ΣM @ C = 0

$6.0\ V_{BC}$ - (51.9 x 6.0 x 3.0) + 129.0 - 167.6 = 0

V_{BC} = (934.03 - 129.0 +167.6) / 6.0 = 162.1 kN

V_{CB} = (51.9 x 6.0) - 162.1 = 149.3 kN

Case 2



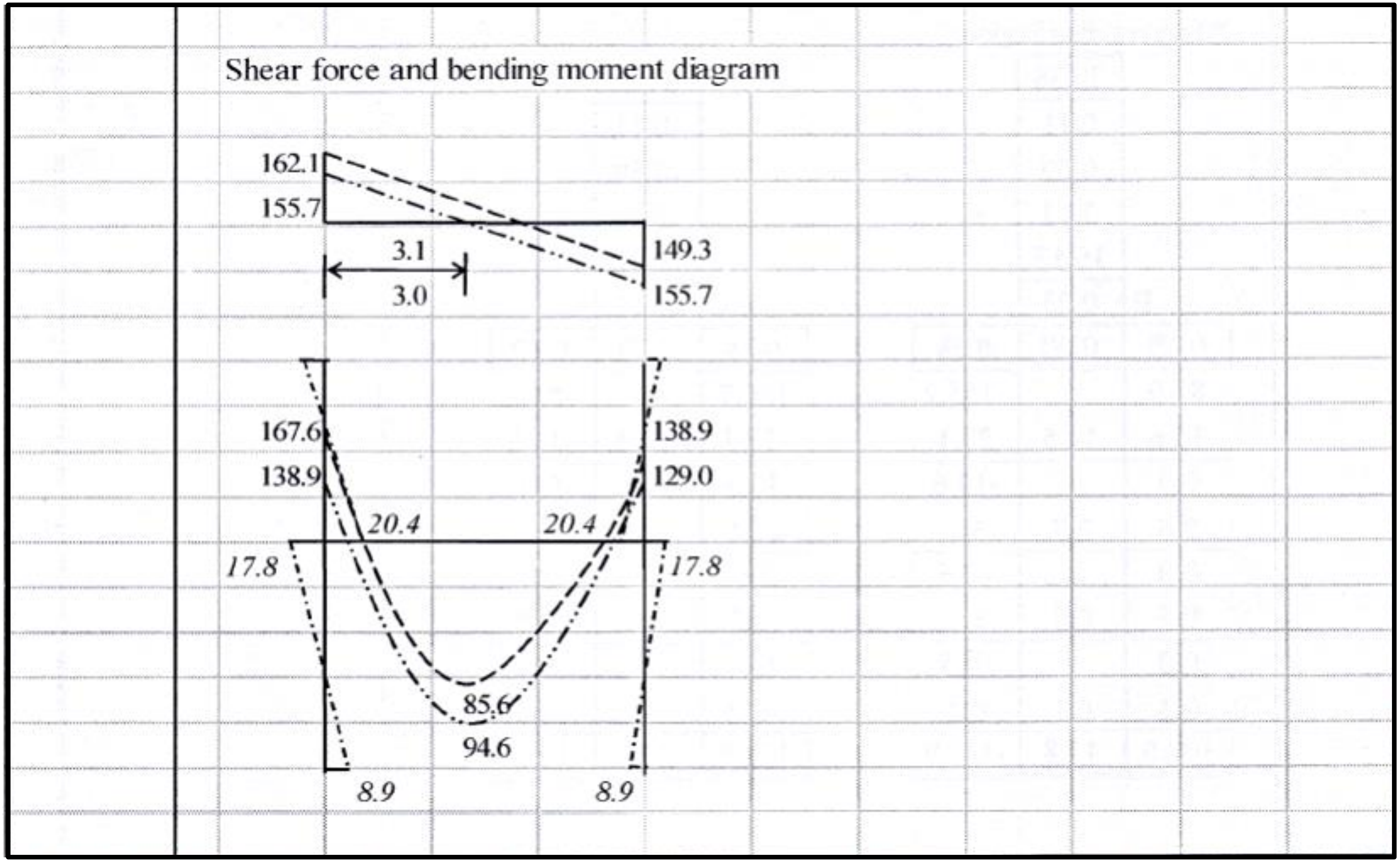
ΣM @ C = 0

$6.0\ V_{BC}$ - (51.9 x 6.0 x 3.0) + 138.9 - 138.9 = 0

V_{AB} = (934.03 - 138.9 +138.9) / 6.0 = 155.7 kN

V_{BA} = (51.9 x 6.0) - 155.7 = 155.7 kN

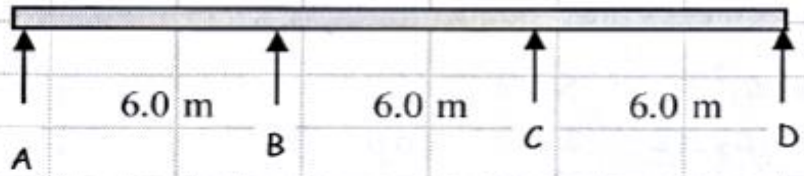
Example: Two Point Sub-Frame



Example: One Point Sub-Frame

Continuous Beam :

Bending moment and shear force in beam.



Moment of Inertia :

$I = bh^3/12 = 250 \times 600^3/12 = 4.50 \times 10^9 \text{ mm}^4$

Stiffness :

AB : $k_{AB} = 0.75\, I/L = 3.38 \times 10^9 / 6000 = 5.63 \times 10^5 \text{ mm}^3$

BC : $k_{BC} = I/L = 4.50 \times 10^9 / 6000 = 7.5 \times 10^5 \text{ mm}^3$

CD: $k_{CD} = 0.75\, I/L = 3.38 \times 10^9 / 6000 = 5.63 \times 10^5 \text{ mm}^3$

Example: One Point Sub-Frame

Distribution factor:

Joint A: $F_{AB} = k_{BA}/(k_{AB} + 0)$

$= 5.63 / (5.63 + 0.00) = 1.00$

Joint B: $F_{BA} = k_{AB}/(k_{AB} + k_{BC})$

$= 5.63 / (5.63 + 7.50) = 0.43$

$F_{BC} = k_{BC}/(k_{AB} + k_{BC})$

$= 7.50 / (5.63 + 7.50) = 0.57$

Joint C: $F_{BC} = k_{BC}/(k_{BC} + k_{CD})$

$= 7.50 / (7.5 + 5.63) = 0.57$

$F_{CD} = k_{CD}/(k_{BC} + k_{CD})$

$= 5.63 / (7.5 + 5.63) = 0.43$

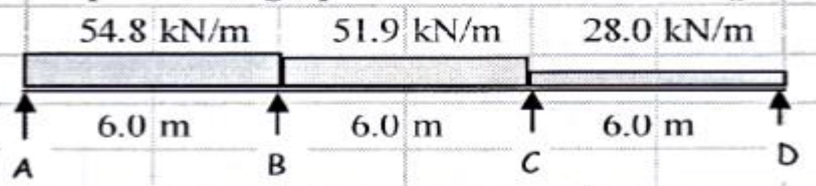
Joint D: $F_{DC} = k_{CD}/(k_{CD} + 0)$

$= 5.63 / (5.63 + 0.00) = 1.00$

Example: One Point Sub-Frame

Case 1 : Span 1 & 2 design permanent & variable loads $1.35G_k + 1.5Q_k$

Span 3 design permanent loads $1.35G_k$



Fixed End Moment :

$-M_{AB} = M_{BA} = w_1 L_1^2/12 =$ 54.84 x 6.0 ^{2} / 12 = 164.5 kNm

$-M_{BC} = M_{CB} = w_2 L_2^2/12 =$ 51.89 x 6.0 ^{2} / 12 = 155.7 kNm

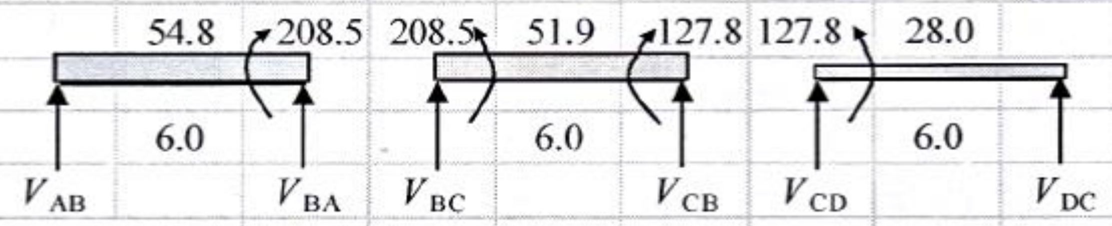
$-M_{CD} = M_{DC} = w_3 L_3^2/12 =$ 27.98 x 6.0 ^{2} / 12 = 83.9 kNm

Moment Distribution :

0.00	1.00	0.43	0.57	0.57	0.43	1.00	0.00
	-164.5	164.5	-155.7	155.7	-83.9	83.9	
	164.5	-3.8	-5.1	-41.0	-30.7	-83.9	
		82.3	-20.5	-2.5	-42.0		
		-26.5	-35.3	25.4	19.1		
			12.7	-17.6			
		-5.4	-7.3	10.1	7.6		
			5.0	-3.6			
		-2.2	-2.9	2.1	1.6		
			1.0	-1.4			
		-0.4	-0.6	0.8	0.6		
	0.0	208.5	-208.5	127.8	-127.8	0.0	

Example: One Point Sub-Frame

Shear force :



$\Sigma M @ B = 0$

$6.0\,V_{AB} - (54.8 \times 6.0 \times 3.0) + 208.5 = 0$

$V_{AB} = (987.13 - 208.5) / 6.0 = 129.8$ kN

$V_{BA} = (54.8 \times 6.0) - 129.8 = 199.3$ kN

$\Sigma M @ C = 0$

$6.0\,V_{BC} - (51.9 \times 6.0 \times 3.0) + 127.8 - 208.5 = 0$

$V_{BC} = (934.03 - 127.8 + 208.5)/6.0 = 169.1$ kN

$V_{CB} = (51.9 \times 6.0) - 169.1 = 142.2$ kN

$\Sigma M @ D = 0$

$6.0\,V_{CD} - (28.0 \times 6.0 \times 3.0) - 127.8 = 0$

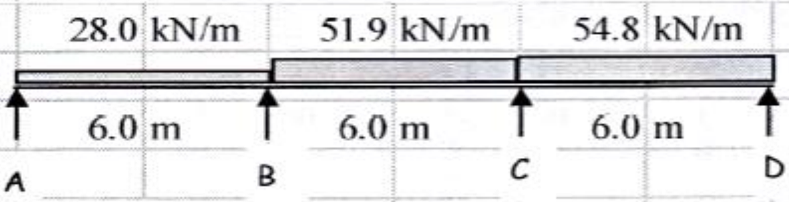
$V_{CD} = (503.56 + 127.8) / 6.0 = 105.2$ kN

$V_{DC} = (28.0 \times 6.0) - 105.2 = 62.6$ kN

Example: One Point Sub-Frame

Case 2: Span 2 & 3 design permanent & variable loads $1.35G_k + 1.5Q_k$

Span 1 design permanent loads $1.35G_k$



Fixed End Moment :

$-M_{AB} = M_{BA} = w_1 L_1^2/12 =$ 27.98 x 6.0 ^{2} / 12 = 83.9 kNm

$-M_{BC} = M_{CB} = w_2 L_2^2/12 =$ 51.89 x 6.0 ^{2} / 12 = 155.7 kNm

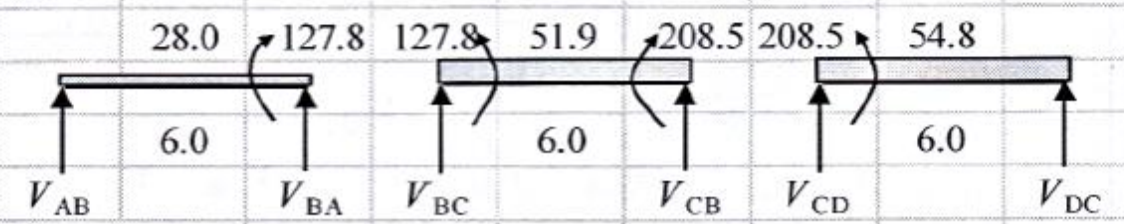
$-M_{CD} = M_{DC} = w_3 L_3^2/12 =$ 54.84 x 6.0 ^{2} / 12 = 164.5 kNm

Moment Distribution :

0.00	1.00	0.43	0.57	0.57	0.43	1.00	0.00
	-83.9	83.9	-155.7	155.7	-164.5	164.5	
	83.9	30.7	41.0	5.1	3.8	-164.5	
		42.0	2.5	20.5	-82.3		
		-19.1	-25.4	35.3	26.5		
			17.6	-12.7			
		-7.6	-10.1	7.3	5.4		
			3.6	-5.0			
		-1.6	-2.1	2.9	2.2		
			1.4	-1.0			
		-0.6	-0.8	0.6	0.4		
	0.0	127.8	-127.8	208.5	-208.5	0.0	

Example: One Point Sub-Frame

Shear force :



$\Sigma M @ B = 0$

$6.0\,V_{AB} - (28.0 \times 6.0 \times 3.0) + 127.8 = 0$

$V_{AB} = (503.56 - 127.8) / 6.0 = 62.6$ kN

$V_{BA} = (28.0 \times 6.0) - 62.6 = 105.2$ kN

$\Sigma M @ C = 0$

$6.0\,V_{BC} - (51.9 \times 6.0 \times 3.0) + 208.5 - 127.8 = 0$

$V_{BC} = (934.03 - 208.5 + 127.8)/6.0 = 142.2$ kN

$V_{CB} = (51.9 \times 6.0) - 142.2 = 169.1$ kN

$\Sigma M @ D = 0$

$6.0\,V_{CD} - (54.8 \times 6.0 \times 3.0) - 208.5 = 0$

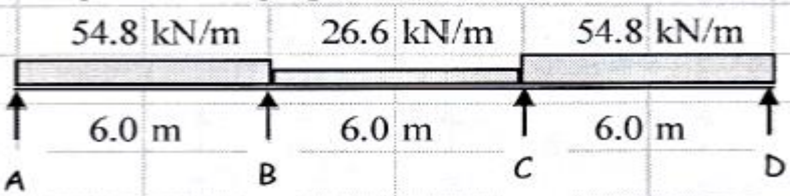
$V_{CD} = (987.13 + 208.5) / 6.0 = 199.3$ kN

$V_{DC} = (54.8 \times 6.0) - 199.3 = 129.8$ kN

Example: One Point Sub-Frame

Case 3: Span 1 & 3 design permanent & variable loads $1.35G_k + 1.5Q_k$

Span 2 design permanent loads $1.35G_k$



Fixed End Moment :

$-M_{AB} = M_{BA} = w_1 L_1^2/12 =$ 54.84 x 6.0^2 / 12 = 164.5 kNm

$-M_{BC} = M_{CB} = w_2 L_2^2/12 =$ 26.58 x 6.0^2 / 12 = 79.7 kNm

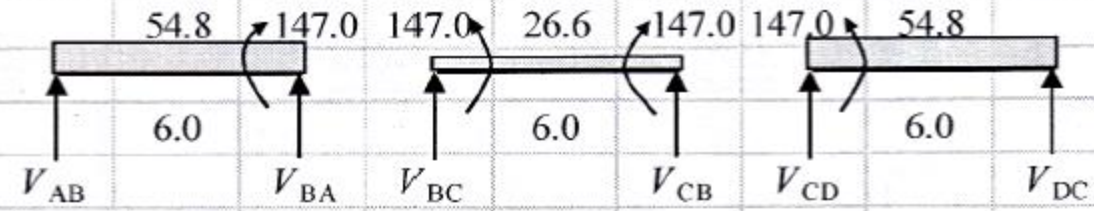
$-M_{CD} = M_{DC} = w_3 L_3^2/12 =$ 54.84 x 6.0^2 / 12 = 164.5 kNm

Moment Distribution :

0.00	1.00	0.43	0.57	0.57	0.43	1.00	0.00
	-164.5	164.5	-79.7	79.7	-164.5	164.5	
	164.5	-36.3	-48.4	48.4	36.3	-164.5	
		82.3	24.2	-24.2	-82.3		
		-45.6	-60.8	60.8	45.6		
			30.4	-30.4			
		-13.0	-17.4	17.4	13.0		
			8.7	-8.7			
		-3.7	-5.0	5.0	3.7		
			2.5	-2.5			
		-1.1	-1.4	1.4	1.1		
	0.0	147.0	-147.0	147.0	-147.0	0.0	

Example: One Point Sub-Frame

Shear force :

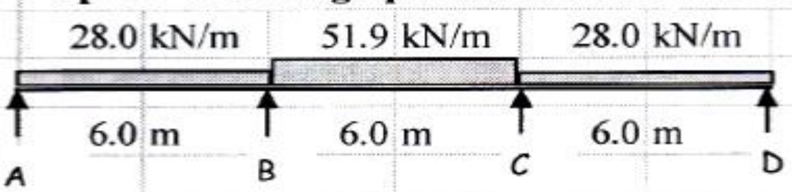


ΣM @ B = 0							
$6.0\, V_{AB}$	-	(54.8	x 6.0 x	3.0) +	147.0	= 0	
V_{AB}	=	(987.13	- 147.0	) / 6.0	=	140.0	kN
V_{BA}	=	(54.8 x	6.0) -	140.0	=	189.0	kN
ΣM @ C = 0							
$6.0\, V_{BC}$	-	(26.6	x 6.0 x	3.0) +	147.0	- 147.0	= 0
V_{BC}	=	(478.41	- 147.0	+147.0	)/6.0 =	79.7	kN
V_{CB}	=	(26.6 x	6.0) -	79.7	=	79.7	kN
ΣM @ D = 0							
$6.0\, V_{CD}$	-	(54.8	x 6.0 x	3.0) -	147.0	= 0	
V_{CD}	=	(987.13	+147.0	) / 6.0	=	189.0	kN
V_{DC}	=	(54.8 x	6.0) -	189.0	=	140.0	kN

Example: One Point Sub-Frame

Case 4: Span 2 design permanent & variable loads $1.35G_k + 1.5Q_k$

Span 1 & 3 design permanent loads $1.35G_k$



Fixed End Moment :

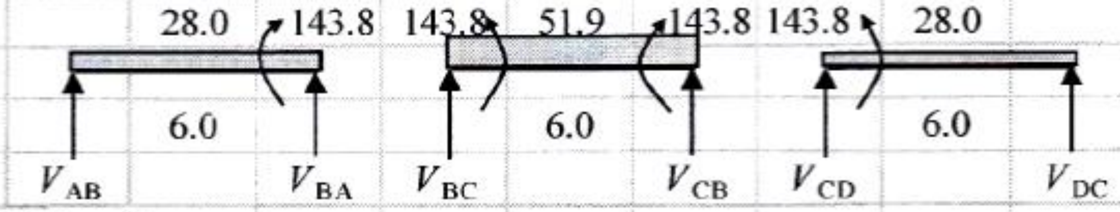
$-M_{AB} = M_{BA} = w_1 L_1^2/12 =$	27.98	x 6.0^2 / 12	=	83.9 kNm
$-M_{BC} = M_{CB} = w_2 L_2^2/12 =$	51.89	x 6.0^2 / 12	=	155.7 kNm
$-M_{CD} = M_{DC} = w_3 L_3^2/12 =$	27.98	x 6.0^2 / 12	=	83.9 kNm

Moment Distribution :

0.00	1.00	0.43	0.57	0.57	0.43	1.00	0.00
	-83.9	83.9	-155.7	155.7	-83.9	83.9	
	83.9	30.7	41.0	-41.0	-30.7	-83.9	
		42.0	-20.5	20.5	-42.0		
		-9.2	-12.3	12.3	9.2		
			6.1	-6.1			
		-2.6	-3.5	3.5	2.6		
			1.8	-1.8			
		-0.8	-1.0	1.0	0.8		
			0.5	-0.5			
		-0.2	-0.3	0.3	0.2		
	0.0	143.8	-143.8	143.8	-143.8	0.0	

Example: One Point Sub-Frame

Shear force :



ΣM @ B = 0

$6.0\,V_{AB} - (28.0 \times 6.0 \times 3.0) + 143.8 = 0$

$V_{AB} = (503.56 - 143.8) / 6.0 = 60.0$ kN

$V_{BA} = (28.0 \times 6.0) - 60.0 = 107.9$ kN

ΣM @ C = 0

$6.0\,V_{BC} - (51.9 \times 6.0 \times 3.0) + 143.8 - 143.8 = 0$

$V_{BC} = (934.03 - 143.8 + 143.8)/6.0 = 155.7$ kN

$V_{CB} = (51.9 \times 6.0) - 155.7 = 155.7$ kN

ΣM @ D = 0

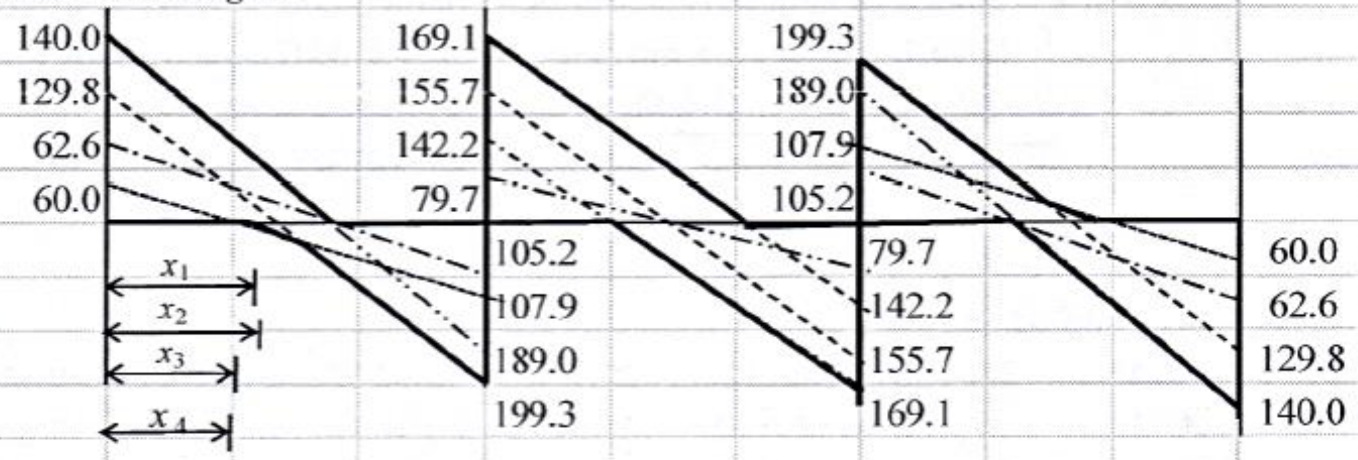
$6.0\,V_{CD} - (28.0 \times 6.0 \times 3.0) - 143.8 = 0$

$V_{CD} = (503.56 + 143.8) / 6.0 = 107.9$ kN

$V_{DC} = (28.0 \times 6.0) - 107.9 = 60.0$ kN

Example: One Point Sub-Frame

Shear Force Diagram

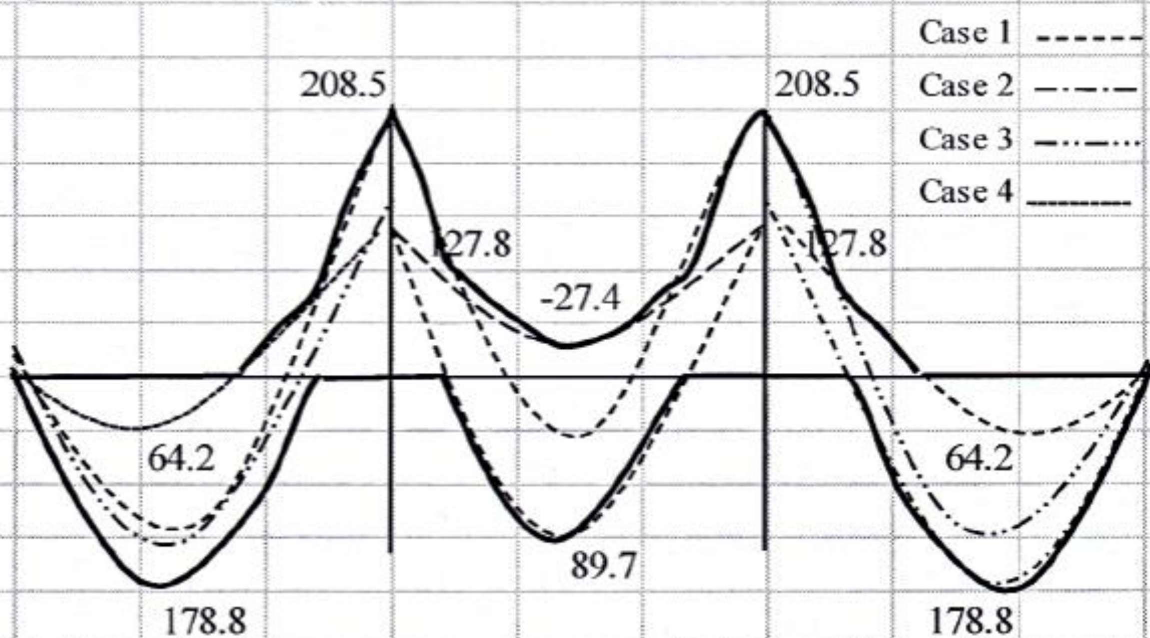


Distance				Mid-span moment				
Span 1				Span 1				
x_1 =	129.8 /	54.8 =	2.4 m	M_1 =	129.8x	2.4 / 2	=	153.6 kNm
x_2 =	62.6 /	28.0 =	2.2 m	M_2 =	62.6x	2.2 / 2	=	70.1 kNm
x_3 =	140.0 /	54.8 =	2.6 m	M_3 =	140.0x	2.6 / 2	=	178.8 kNm
x_4 =	60.0 /	28.0 =	2.1 m	M_4 =	60.0x	2.1 / 2	=	64.2 kNm
Span 2				Span 2				
x_1 =	169.1 /	51.9 =	3.3 m	M_1 =	(169.1	x3.3/2)	-208.5	= 67.1 kNm
x_2 =	142.2 /	51.9 =	2.7 m	M_2 =	(142.2	x2.7/2)	-127.8	= 67.1 kNm
x_3 =	79.7 /	26.6 =	3.0 m	M_3 =	(79.7	x3.0/2)	-147.0	-= 27.4 kNm
x_4 =	155.7 /	51.9 =	3.0 m	M_4 =	(155.7	x3.0/2)	-143.8	= 89.7 kNm

Example: One Point Sub-Frame

Span 3				Span 3
$x_1 =$	105.2 / 28.0 = 3.8 m		$M_1 =$	(105.2 x3.8/2) - 127.8 =70.1 kNm
$x_2 =$	199.3 / 54.8 = 3.6 m		$M_2 =$	(199.3 x3.6/2) - 208.5 =153.6 kNm
$x_3 =$	189.0 / 54.8 = 3.4 m		$M_3 =$	(189.0 x3.4/2) - 147.0 =178.8 kNm
$x_4 =$	107.9 / 28.0 = 3.9 m		$M_4 =$	(107.9 x3.9/2) - 143.8 =64.2 kNm

Bending Moment Diagram



Example: One Point Sub-Frame

Sub-frame: Bending Moment in Columns

Momen of Inertia : $I = bh^3/12$

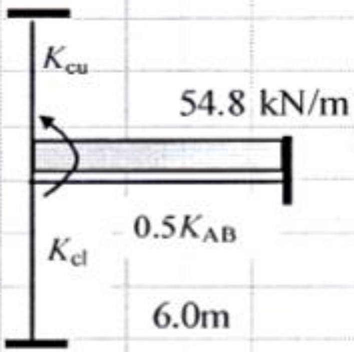
Column: I =	300	x 400	^{3} /12 =	1.6 x 10^9 mm^4
Beam : I =	250	x 600	^{3} /12 =	4.5 x 10^9 mm^4

Stiffness : $K = I/L$

Column$_{lower}$: K_{cl} =	1.6 x	10^9 /	3500 =	4.6 x 10^5 mm^3
Column$_{upper}$: K_{cu} =	1.6 x	10^9 /	4000 =	4.0 x 10^5 mm^3
Beam: K_{AB} =	4.5 x	10^9 /	6000 =	7.5 x 10^5 mm^3
K_{BC} =	4.5 x	10^9 /	6000 =	7.5 x 10^5 mm^3
K_{CD} =	4.5 x	10^9 /	6000 =	7.5 x 10^5 mm^3

Example: One Point Sub-Frame

Column A



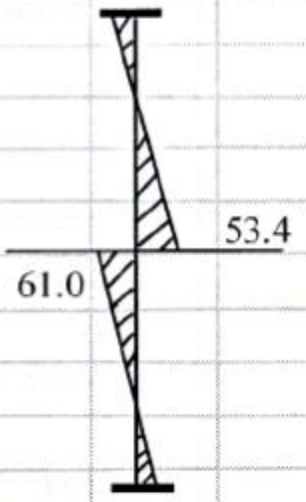
$$FEM = wL^2/12$$
$$= 54.8 \times 6.0^2/12$$
$$= 164.52 \text{ kNm}$$

Moment in upper column,

$$M = FEM \times K_{cu}/(K_{cu} + K_{cl} + K_b/2)$$
$$= 164.5 \times 4.0/(4.0 + 4.6 + 3.8)$$
$$= 53.4 \text{ kNm}$$

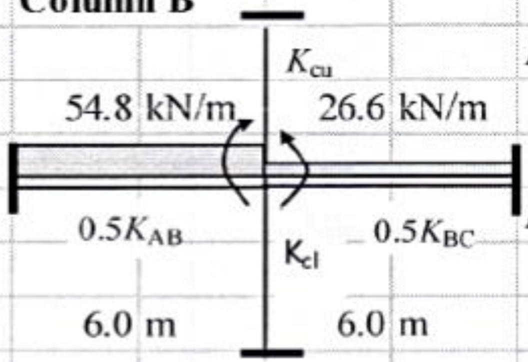
Moment in lowercColumn,

$$M = FEM \times K_{cl}/(K_{cu} + K_{cl} + K_b/2)$$
$$= 164.5 \times 4.6/(4.0 + 4.6 + 3.8)$$
$$= 61.0 \text{ kNm}$$



Example: One Point Sub-Frame

Column B



$FEM_1 = wL^2/12$

$= 54.8 \times 6.0^2/12$

$= 164.5$ kNm

$FEM_2 = wL^2/12$

$= 26.6 \times 6.0^2/12$

$= 79.7$ kNm

$\Delta M = FEM_1 - FEM_2 = 164.5 - 79.7 = 84.8$ kNm

Moment in upper column,

$M = \Delta M \times K_{cu} / (K_{cu} + K_{cl} + 0.5K_{AB} + 0.5K_{BC})$

$= 84.8 \times 4.0 / (4.0 + 4.6 + 3.75 + 3.75)$

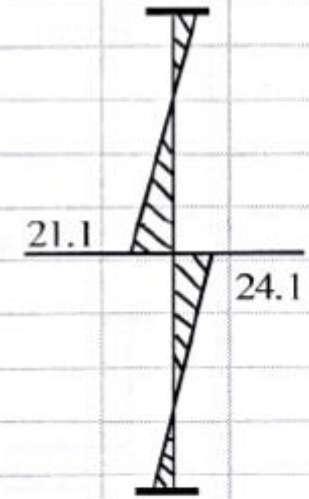
$= 21.1$ kNm

Moment in lower column,

$M = \Delta M \times K_{cl} / (K_{cu} + K_{cl} + 0.5K_{AB} + 0.5K_{BC})$

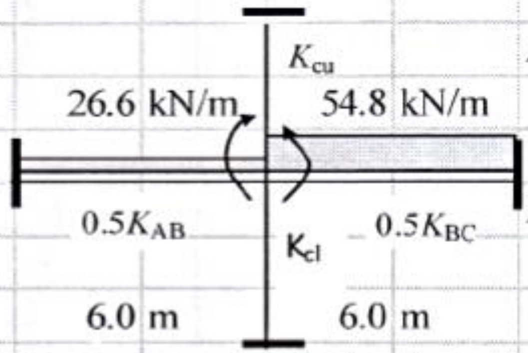
$= 84.8 \times 4.6 / (4.0 + 4.6 + 3.75 + 3.75)$

$= 24.1$ kNm



Example: One Point Sub-Frame

Column C



$FEM_1 = wL^2/12$

$= 26.6 \times 6.0^2/12$

$= 79.7$ kNm

$FEM_2 = wL^2/12$

$= 54.8 \times 6.0^2/12$

$= 164.5$ kNm

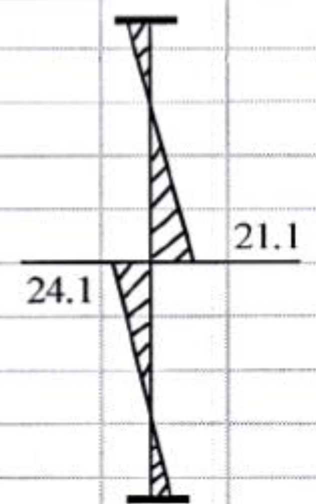
$\Delta M = FEM_1 - FEM_2 = 164.5 - 79.7 = 84.8$ kNm

Moment in upper column,

$M = \Delta M \times K_{cu} / (K_{cu} + K_{cl} + 0.5K_{BC} + 0.5K_{CD})$

$= 84.8 \times 4.0 / (4.0 + 4.6 + 3.75 + 3.75)$

$= 21.1$ kNm



Moment in lower column,

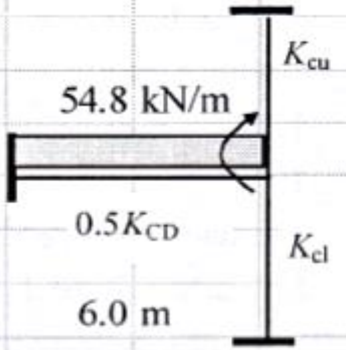
$M = \Delta M \times K_{cl} / (K_{cu} + K_{cl} + 0.5K_{BC} + 0.5K_{CD})$

$= 84.8 \times 4.6 / (4.0 + 4.6 + 3.75 + 3.75)$

$= 24.1$ kNm

Example: One Point Sub-Frame

Column D



$FEM = wL^2/12$

$= 54.8 \times 6.0^2/12$

$= 164.52$ kNm

Moment in upper column,

$M = FEM \times K_{cu}/(K_{cu} + K_{cl} + 0.5K_{CD})$

$= 164.5 \times 4.0 / (4.0 + 4.6 + 3.8)$

$= 53.4$ kNm

Moment in lowercColumn,

$M = FEM \times K_{cl}/(K_{cu} + K_{cl} + 0.5K_{CD})$

$= 164.5 \times 4.6 / (4.0 + 4.6 + 3.8)$

$= 61.0$ kNm