Chapter 4

ANALYSIS OF STATICALLY INDETERMINATE STRUCTURE

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4.1 INTRODUCTION

- The problem of indeterminate beam is to determine support reaction. This is because an indeterminate beam can not decide static which have number untraceable (reaction to support) more than 3. A static equilibrium equation is not enough to solve this problem. By such several methods was introduced to determine reaction force, further shear strength and bending moment in beam. Two methods would discuss in this chapter is :

  a) Slope-deflection method

  b) Moment-distribution method

- The analysis of continuous beam and framework with slope deflection would involve following procedure :

  a) Determine end fixed moment, moment result deflection and moment result

  deposit / support shift.

  b) Determine slope at support

  c) Determine moment at support

- Analysis of continuous beam and framework with moment distribution methods would involve following procedure:

  a) Determine strength every member
  b) Determine distribution factor
  c) Determine factor bring side
  d) Determine end fixed moment
  e) Make the moment distribution process and process bring side
  f) Determine moment at support

4.2 SLOPE DEFLECTION

- For indeterminate structure, moment to the end of the member happen from :
  - fixed end moment
  - deflection slope or rotation
  - support shift (support settlement)

To form stated equation, member must be have cross section uniform (homogeneous) among two support.

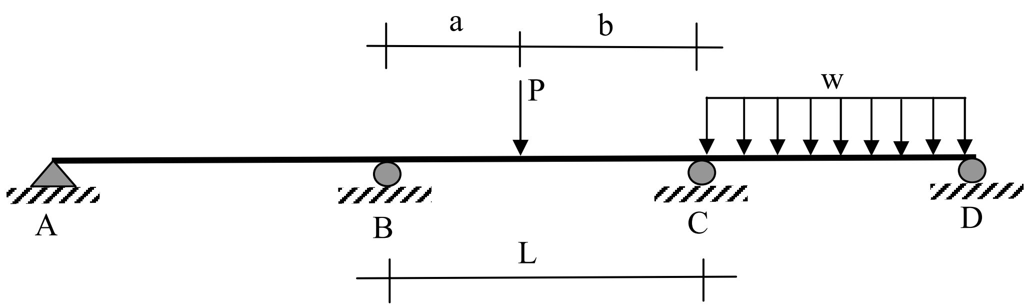
- Redundant (reaction or internal force) make value an unknown or value want to be determined and it is known as method force.

- Deformation can also be used as value an unknown and it is known as method deformation, either it is slope-deflection method.

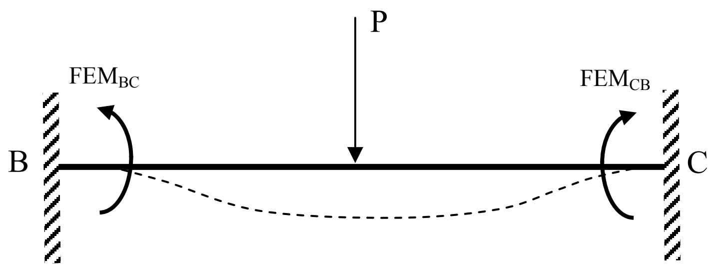
- In this method, slope and deflection to joints would be produced and where further end moment can be decided then.

4.2.1 SLOPE DEFLECTION METHODS

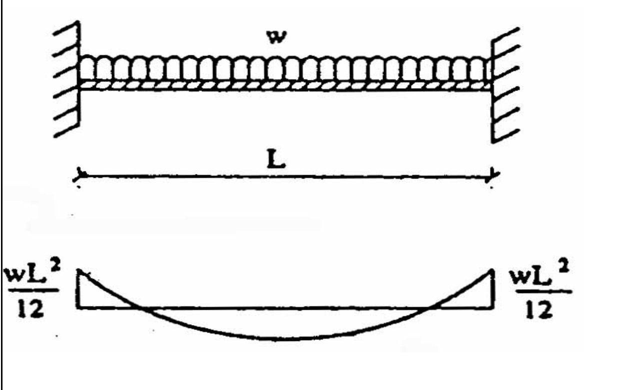
- Consider typical beam BC for continuous beam shown in figure below, moment resultant at the end B and C can published as following :

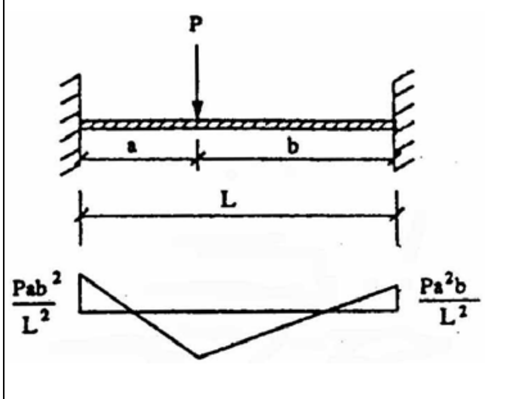


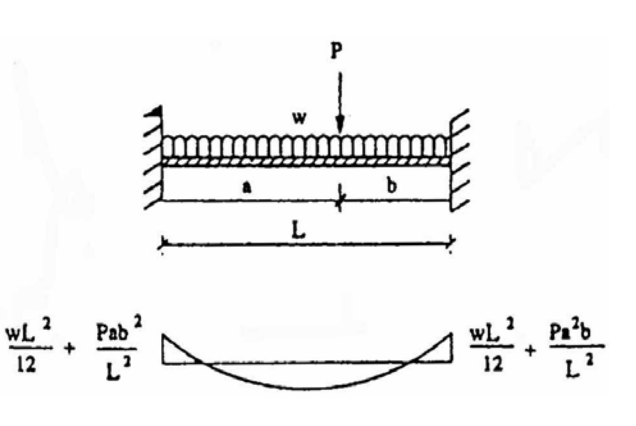
- Fixed end moment (FEM)

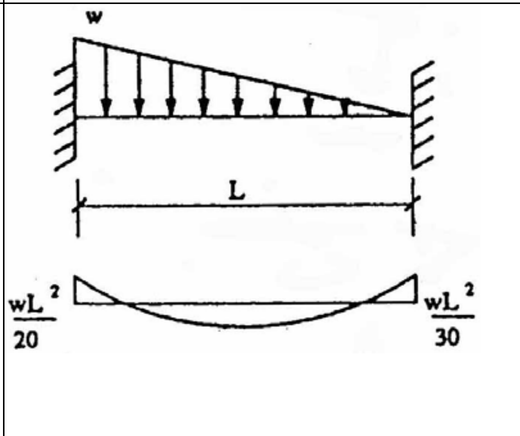


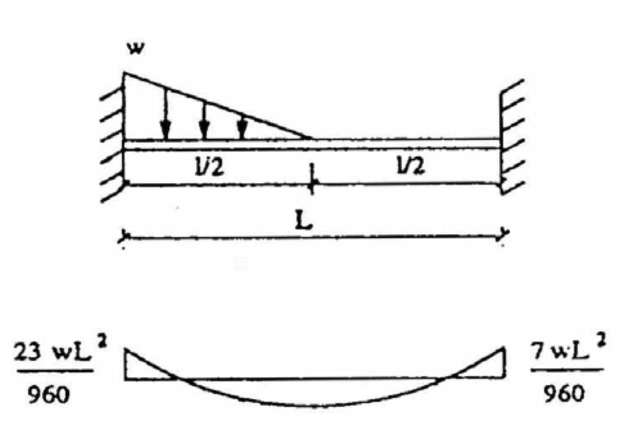
Fixed end moment defined as moment resultant at end to end outside tax incidence member that imposed to stated member when both it member is fixed, there for the rotation to end to end member is zero. Fixed end moment for some circumstances is as below:

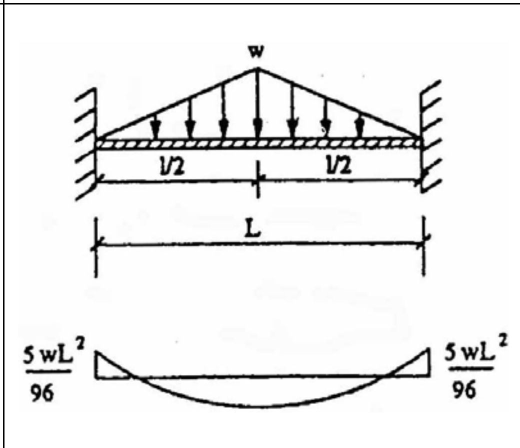


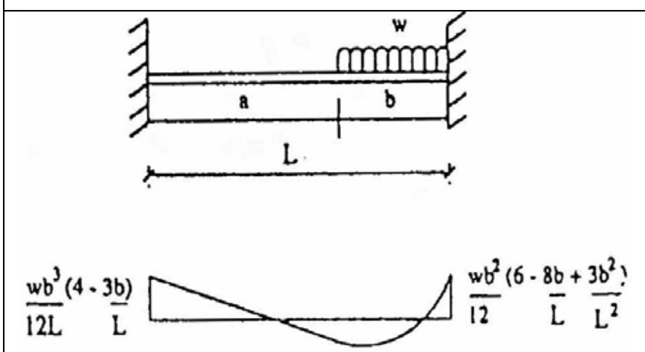


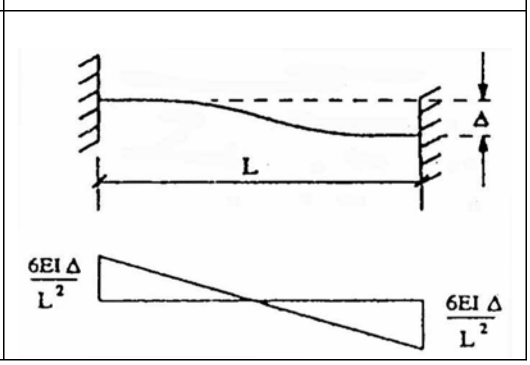




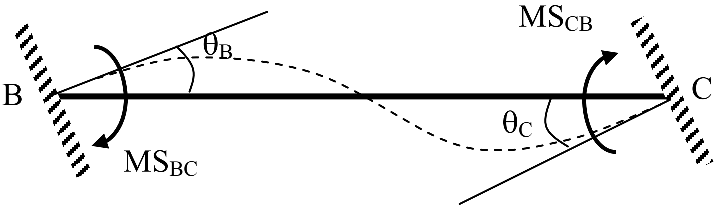








- Moment of Slope (MS)



Slope or rotation that happened at one beam will produce moment. This moment are contrary direction with direction of rotation. End gradient member would be positive if produce rotation clockwise

Moment of slope can phrased as following:

1. End gradient B if C's end control / fixed

$$MS_{BC} = \frac{(4EI\theta_B)}{L} \quad \text{dan} \quad MS_{CB} = \tfrac{1}{2} MS_{BC} = \frac{(2EI\theta_B)}{L}$$

2. End gradient C if B's end control / ikategar

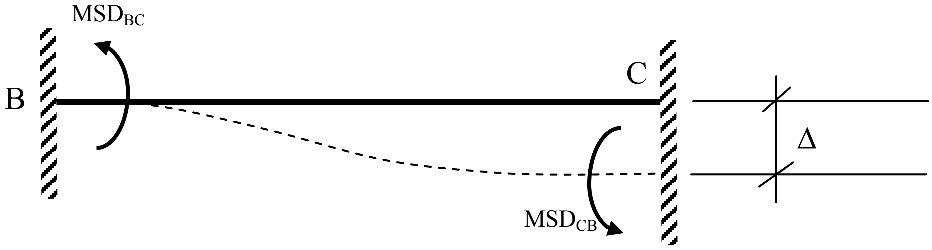
$$MS_{CB} = \frac{(4EI\theta_C)}{L} \quad \text{dan} \quad MS_{BC} = \tfrac{1}{2} MS_{CB} = \frac{(2EI\theta_C)}{L}$$

The results are :

$$MS_{BC}^{cer} = \frac{(4EI\theta_B)}{L} + \frac{(2EI\theta_C)}{L} = \frac{2EI(2\theta_B + \theta_C)}{L}$$

$$MS_{CB}^{cer} = \frac{(4EI\theta_C)}{L} + \frac{(2EI\theta_B)}{L} = \frac{2EI(2\theta_C + \theta_B)}{L}$$

c. Moment of Support Displacement (MSD)



Deformation (displacement) Δ at one end to another end (B's end and C constrained). Result of deformation moment is such following:

$$MSD_{BC} = MSD_{CB} = \frac{-6EI\Delta}{L^2} = \frac{-6EI\delta}{L}$$

$$\text{with } \delta = \frac{\Delta}{L}$$

Being moment resultant to support B for BC's member is moment counting the fixed and moment, slope and displacement, as follows:

$$\begin{aligned}
\therefore M_{BC} &= FEM_{BC} + MS_{BC} + MSD_{BC} \\
&= -\frac{Pab^2}{L^2} + \frac{2EI}{L}\left(2\theta_B + \theta_C\right) - \frac{6EI\delta}{L} \\
&= -\frac{Pab^2}{L^2} + \frac{2EI}{L}\left(2\theta_B + \theta_C - 3\delta\right) \\
&= \pm M^{ht} + \frac{2EI}{L}\left(2\theta_B + \theta_C - 3\delta\right)
\end{aligned}$$

$$\begin{aligned}
\therefore M_{CB} &= FEM_{CB} + MS_{CB} + MSD_{CB} \\
&= \frac{Pab^2}{L^2} + \frac{2EI}{L}\left(2\theta_C + \theta_B\right) - \frac{6EI\delta}{L} \\
&= \frac{Pab^2}{L^2} + \frac{2EI}{L}\left(2\theta_C + \theta_B - 3\delta\right) \\
&= \pm M^{ht} + \frac{2EI}{L}\left(2\theta_C + \theta_B - 3\delta\right)
\end{aligned}$$

Custom sign which are used hereabouts is positive moment if his rotation follow time needle and negative if rotation it anti-clockwise.

4.2.2 STEPS OF SOLUTIONS

- Write the slope - deflection equation for each end moment member
- Equilibrium equation obtainable to each joint where the total of joint member is zero (Σ M = 0)

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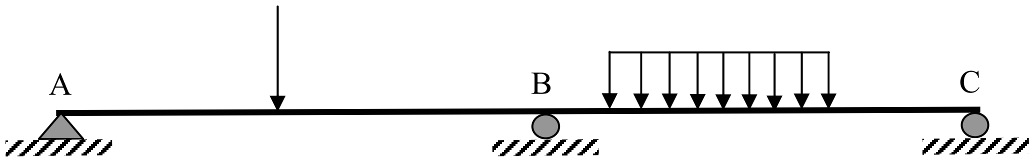

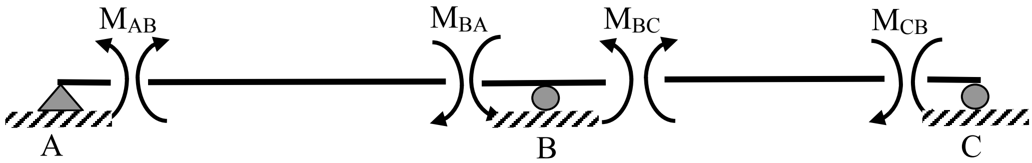

$$\Sigma M_A = M_{AB} = 0$$
$$\Sigma M_B = M_{BA} + M_{BC} = 0$$
$$\Sigma M_C = M_{CB} = 0$$

- Boundary condition placed at fixed end slope, where the displacement (Δ) is equal to zero.
- After slope equation for value determined, all end moment defined with include slope value stated into equation.
- Further member can be sorted for calculation reaction, shear strength and bending moment.
- From here, shear force diagram and bending moment could be sketched.

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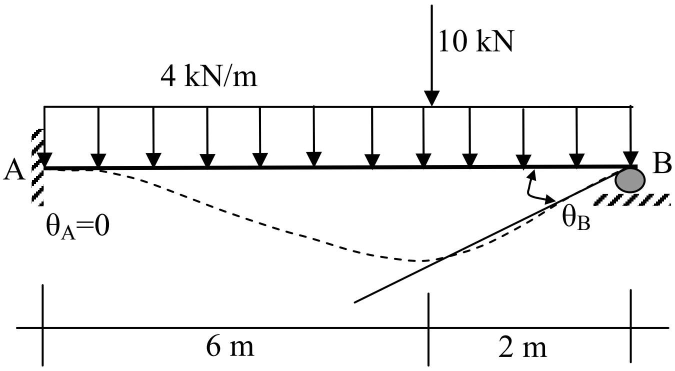
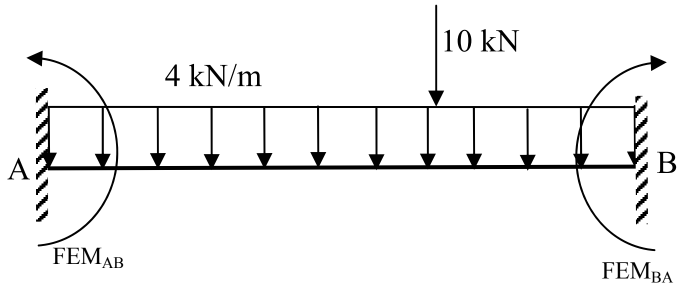
EXAMPLE 4.1

Determine the moment value and shear force to each support and draw shear force diagram (SFD) and bending moment (BMD) for structure beam below. EI's value is constant.



SOLUTION :

- ✓ Finding the fixed end moment (FEM)



$$FEM_{AB} = \frac{-wl^2}{12} - \frac{Pab^2}{l^2} = \frac{-4(8)^2}{12} - \frac{10(6)(2)^2}{8^2} = -25.1\,\text{kNm}$$

$$FEM_{BA} = \frac{+wl^2}{12} + \frac{Pab^2}{l^2} = \frac{4(8)^2}{12} + \frac{10(2)(6)^2}{8^2} = +32.6\,\text{kNm}$$

- ✓ Boundary Condition : $\theta_A = 0$, $\Delta = 0 \rightarrow \delta = 0$, $\theta_B = \ldots?$

- ✓ Slope Deflection Equation :

$$M_{AB} = \pm\, FEM + \frac{2EI}{L}\left(2\theta_A + \theta_B - 3\delta\right)$$

$$= -25.1 + \frac{2EI}{8}\left(2(0) + \theta_B - 3(0)\right) = \frac{EI}{4}(\theta_B) - 25.1$$

$$M_{BA} = \pm\, FEM \; + \frac{2EI}{L}\left(2\theta_B + \theta_A - 3\delta\right)$$
$$= +32.6 + \frac{2EI}{8}\left(2\theta_B + (0) - 3(0)\right)$$
$$= \frac{EI}{2}(\theta_B) + 32.6$$

- ✓ Equilibrium equation

$$\sum M_B = \sum M_{BA} = 0$$

$$\therefore \; \frac{EI}{2}(\theta_B) + 32.6 = 0 \; \text{--------}> \; \theta_B = \frac{-65.2}{EI}$$

- ✓ End of Moment

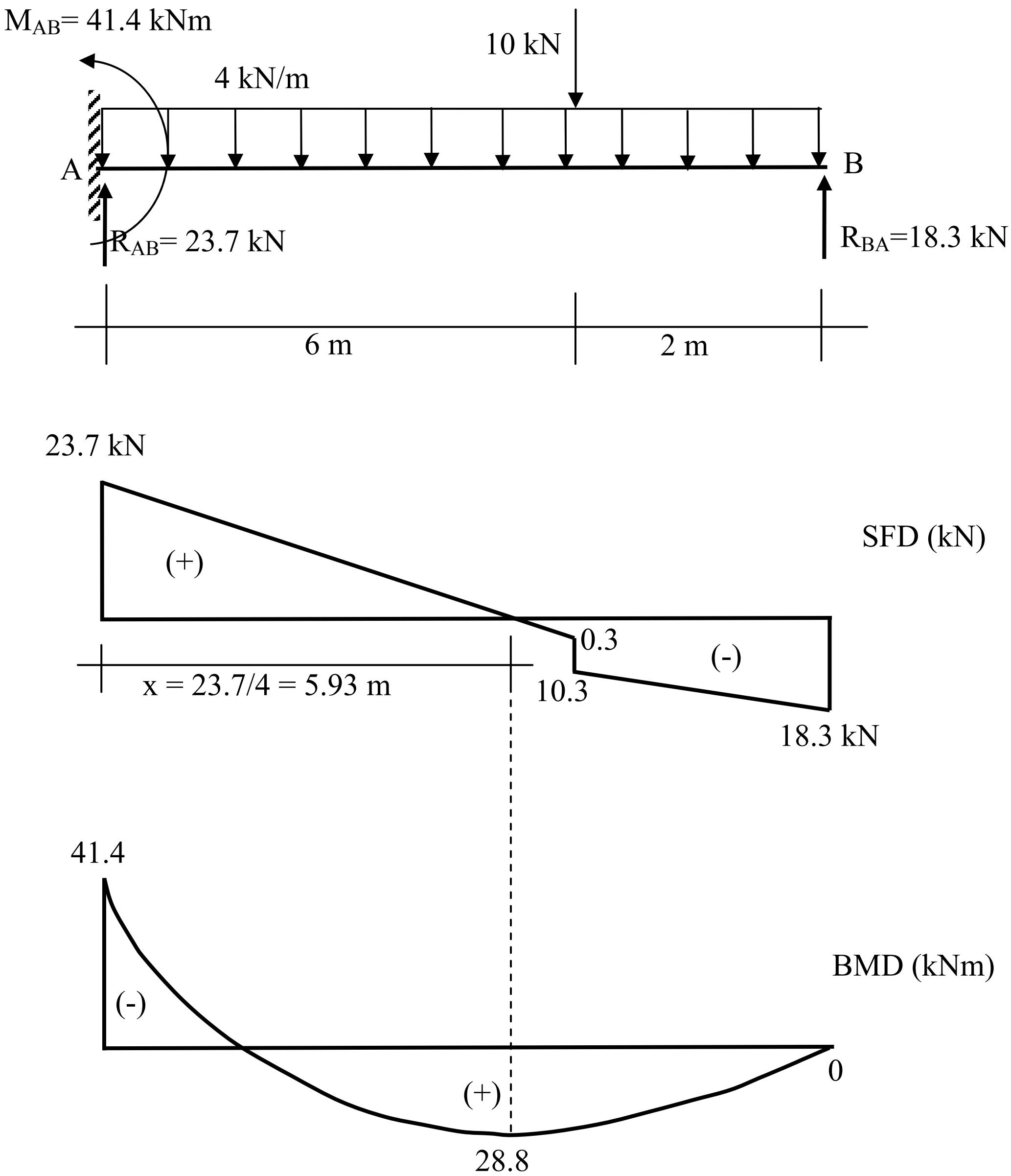
$$M_{AB} = \frac{EI}{4}\left(\frac{-65.2}{EI}\right) - 25.1 = -41.4\,\text{kNm}$$
$$M_{BA} = \frac{EI}{2}\left(\frac{-65.2}{EI}\right) + 32.6 = 0\,\text{kNm}$$

- ✓ Shear Body Diagram and Bending Moment Diagram

Reaction :

$$\sum M_A = 0 \;;$$
$$-41.4 + 10(6) + 4(8)(4) - R_B(8) = 0$$
$$R_B = 18.3\,\text{kN}$$

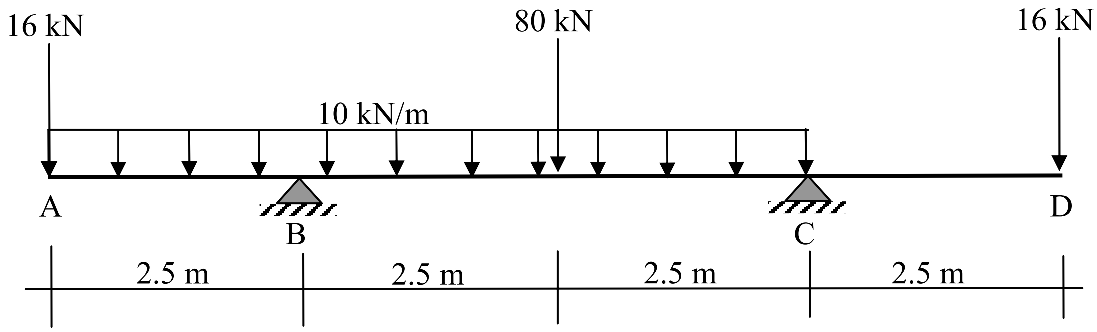
$$\sum M_B = 0 \;;$$
$$-41.4 - 10(2) - 4(8)(4) + R_A(8) = 0$$
$$R_A = 23.7\,\text{kN}$$



M_{AB}= 41.4 kNm
10 kN
4 kN/m
A
B
R_{AB}= 23.7 kN
R_{BA}=18.3 kN
6 m
2 m
23.7 kN
SFD (kN)
(+)
0.3
(-)
x = 23.7/4 = 5.93 m
10.3
18.3 kN
41.4
BMD (kNm)
(-)
0
(+)
28.8

EXAMPLE 4.2

Determine the moment value and shear strength to each support and draw force gambarajah shear (GDR / SFD) and bending moment (GML / BMD) for structure beam below. The EI's value is constant.



Solution :

- ✓ Fixed end moment (FEM)

$$FEM_{AB} = 0$$

$$FEM_{BA} = 16(2.5) + \frac{10(2.5)^2}{2} = +71.25\,kNm$$

$$FEM_{BC} = \frac{-wl^2}{12} - \frac{Pab^2}{l^2} = \frac{-10(5)^2}{12} - \frac{80(2.5)(2.5)^2}{5^2} = -70.83\,kNm$$

$$FEM_{CB} = \frac{+wl^2}{12} + \frac{Pa^2b}{l^2} = \frac{-10(5)^2}{12} + \frac{80(2.5)^2(2.5)}{5^2} = +70.83\,kNm$$

$$FEM_{CD} = -16(2.5) = -40\,kNm$$

$$FEM_{DC} = 0$$

- ✓ At support B and C not happen decline, then $\delta = 0$, angle rotation at θ_B abd θ_C should be determined his value prior.

$$M_{BA} = M_{BA}^{ht} = 71.25\,kNm$$

$$M_{BC} = \pm M_{BC}^{ht} + \frac{2EI}{L}\left(2\theta_B + \theta_C - \frac{3\Delta}{L}\right)$$

$$= -70.83 + \frac{2EI}{5}\left(2\theta_B + \theta_C - 0\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$M_{CB} = \pm FEM_{CB} + \frac{2EI}{L}\left(2\theta_C + \theta_B - \frac{3\Delta}{L}\right)$$

$$= +70.83 + \frac{2EI}{5}\left(2\theta_C + \theta_B - 0\right) \text{......................}(2)$$

$$M_{CD} = FEM_{CD} = -40 \text{ kNm}$$

- ✓ Total Moment at support B is zero / empty, then

$$\sum M_B = 0 \text{ ---> } M_{BA} + M_{BC} = 0$$

$$71.25 - 70.83 + \frac{2EI}{5}\left(2\theta_B + \theta_C\right) = 0$$

$$\frac{2EI}{5}\left(2\theta_B + \theta_C\right) = -0.42$$

$$2\theta_B + \theta_C = -\frac{2.1}{2EI}$$

$$\theta_C = -\frac{2.1}{2EI} - 2\theta_B \text{ ....}(3)$$

- ✓ Total Moment at C is zero, so :

$$\sum M_C = 0 \text{ ---> } M_{CB} + M_{CD} = 0$$

$$70.83 + \frac{2EI}{5}\left(2\theta_C + \theta_B - 0\right) - 40 = 0$$

$$\frac{2EI}{5}\left(2\theta_C + \theta_B\right) = -30.83$$

$$2\theta_C + \theta_B = -\frac{154.15}{2EI} \text{ ....}(4)$$

- ✓ Subtitute (3) into (4) so ,

$$\theta_B = \frac{24.99}{EI} \text{ ....}(5)$$

- ✓ Subtitute (5) into (3) so, $\theta_C = \frac{-2.1}{2EI} - 2(\frac{-24.99}{EI}) = \frac{-51.03}{EI}$

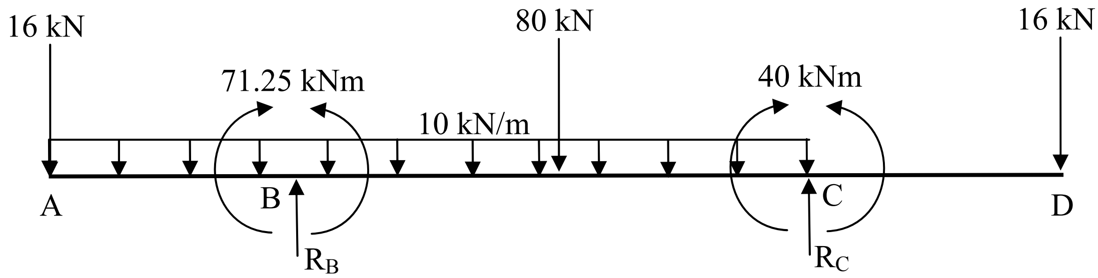
- ✓ End of Moment:

$$M_{BA} = 71.25 \text{ kNm}$$

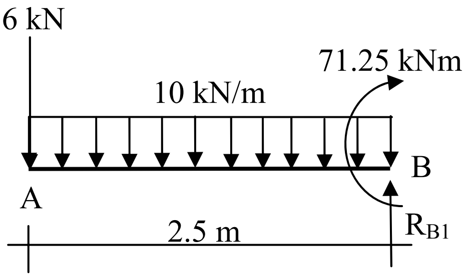
$$M_{BC} = -70.83 + \frac{2EI}{5}\left(2(\frac{24.99}{EI}) + (\frac{-51.03}{EI})\right) = -71.25 \text{ kNm}$$

$$M_{CB} = +70.83 + \frac{2EI}{5}\left(2(\frac{-51.03}{EI}) + \frac{24.99}{EI} - 0\right) = 40\,\text{kNm}$$

$$M_{CD} = -40\,\text{kNm}$$



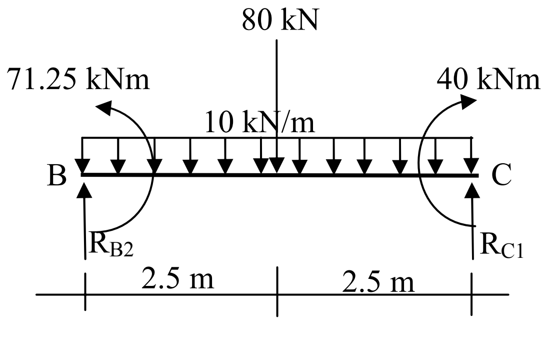
- ✓ Reaction at span AB



$\sum Fy = 0$

$16 + 10(2.5) + R_{B1} = 0 \rightarrow R_{B1} = 41$ kN

- ✓ Reaction at span BC



$\sum M_B = 0$

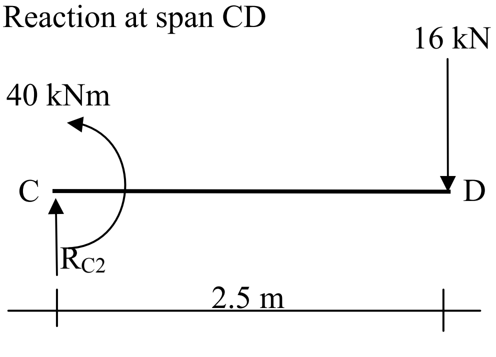
$40-71.25+80(2.5)+10(5)(2.5)-R_{C1}(5) = 0$
$\rightarrow R_{C1} = 58.75$ kN

$\sum M_C = 0$

$40-71.25-80(2.5)-10(5)(2.5)+R_{B2}(5) = 0$
$\rightarrow R_{B2} = 71.25$

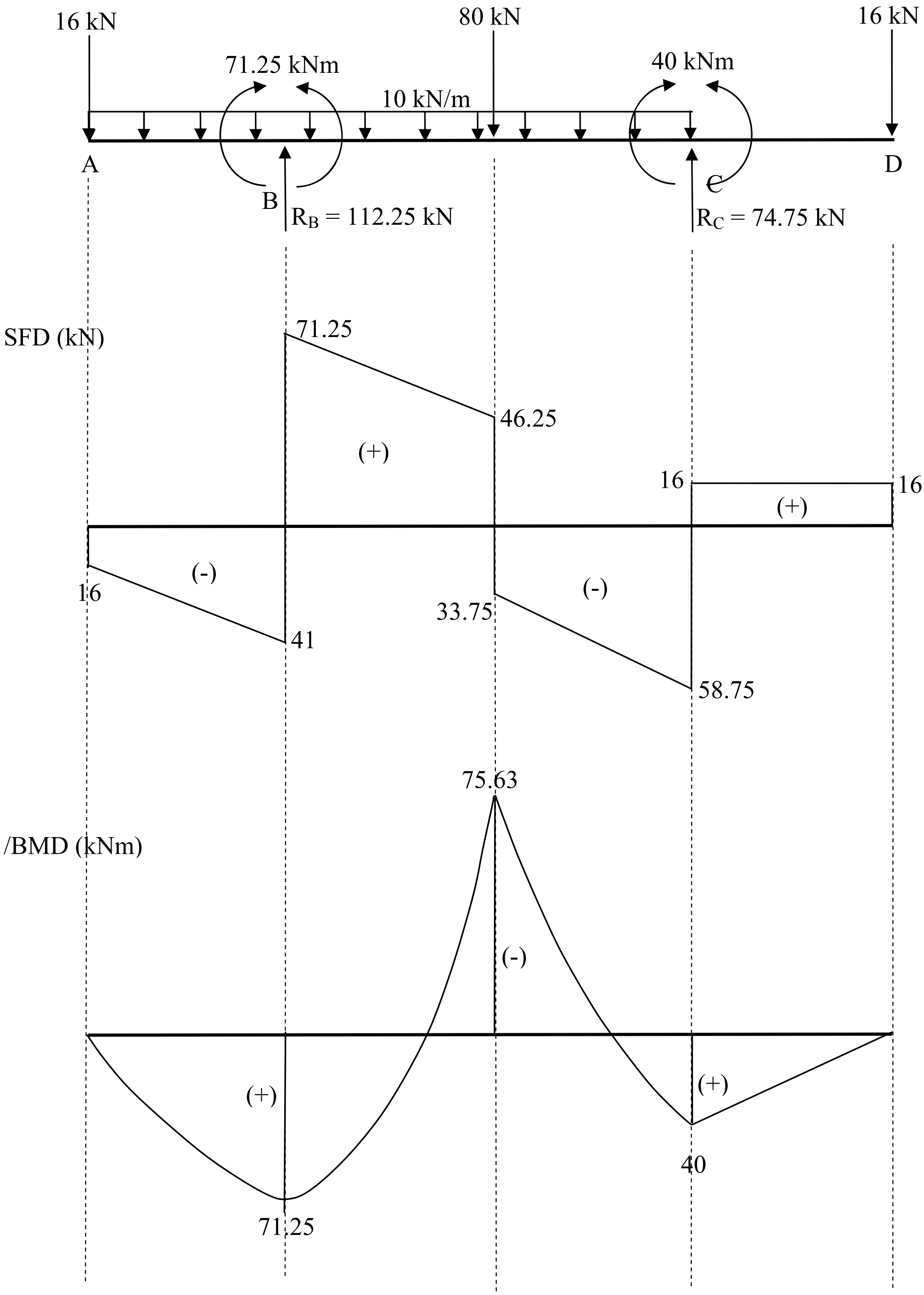
$\sum F_y = 0 \rightarrow 71.25+58.75 = 80 + 10(5)$
130 = 130 ..ok!

- ✓ Reaction at span CD



$\sum F_y = 0 \rightarrow -16 + R_{C2} = 0$
$R_{C2} = 16$ kN

✓ Shear Body Diagram and Bending Moment Diagram



4.3 MOMENT DISTRIBUTION

- Moment distribution method is only involving distribution moments to joint repetitively.

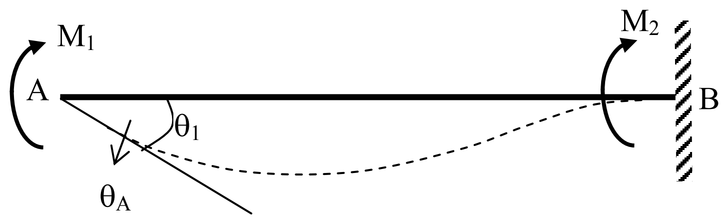
- The accuracy of moment distribution method is dependent to the number repeat which does and usually more than 5 repeat real enough. Right value will be acquired when no more moments that need distributed.

- In general the value is dependent to several factor as :

    - fixed end moment
    - factor bring side
    - strength (factor distribution)

- The Concept of moment distribution method are determining element strength each structure. Strength this depends to:
    - modulus flexibility / elastisitas (E)
    - moment of inertia (I)
    - long element (L)

- Notice a beam



- If M_{AB} imposed at the end A and produce θ_A provable from method slope deflection :

$$M_{AB} = \left(\frac{4EI}{L}\right)\theta_A$$

- To produce 1 unit rotation at A, $\theta_A = 1$

$$M_{AB} = \left(\frac{4EI}{L}\right)$$

Value of $\left(\frac{4EI}{L}\right)$ is Stiffness

$\left(\frac{I}{L}\right)$ is stiffness factor

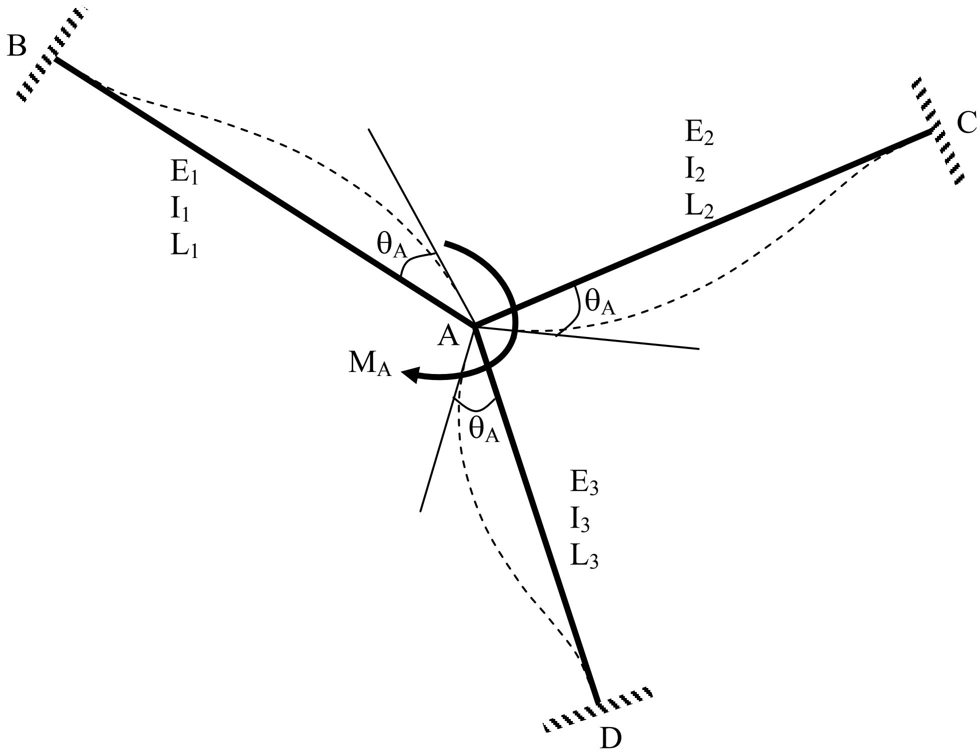
- In other words, stiffness can be defined as moment end required to achieve rotation one unit at one end when other one end is fixed

- Fixed-end moment is moment to end member producing when all joint (support) a structure fixed to prevent rotation anything external load and displacement / shift.

- The M_{AB} will produce of fixed-end moment FEM_{BA} and kwon : $FEM_{BA} = ½ \, FEM_{AB}$

- This value known as Carry Over Factor (CF)

Consider one structure as below



- A moment M_A placed at A until joint A would rotate little and will be distributing M_A to each end member whose joint with A, this rotation also to be effected to end which another to member AB, AC and AD.

  - $M_{AB} = (4EI_1/L_1).\theta_A = K_{AB}.\ \theta_A$
  - $M_{AC} = (4EI_2/L_2).\theta_A = K_{AC}.\ \theta_A$
  - $M_{AD} = (4EI_3/L_3).\theta_A = K_{AD}.\ \theta_A$

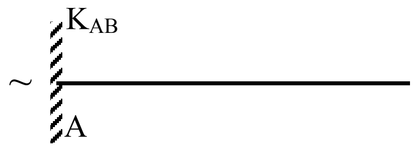
$$\begin{aligned}\Sigma M_A = M_A &= M_{AB} + M_{AC} + M_{AD} \\ &= (K_{AB} + K_{AC} + K_{AD})\ .\theta_A \\ &= (\ \Sigma K\ )\ .\theta_A\end{aligned}$$

$\theta_A = M_A / \Sigma K$ and replaced again
$M_{AB} = (K_{AB} / \Sigma K).M_A$
$M_{AC} = (K_{AC} / \Sigma K).M_A$
$M_{AD} = (K_{AD} / \Sigma K).M_A$

Expression Ki'/ Σ K known as Distribution Factor, where:

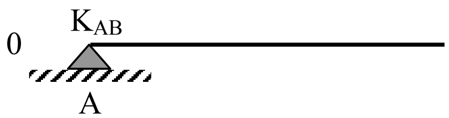
Ki = Stiffness of member
Σ K = Total of Stiffness for all member

- At the end Fixed,



$DF_{AB} = (K_{AB})/(K_{AB} + \sim) = 0$

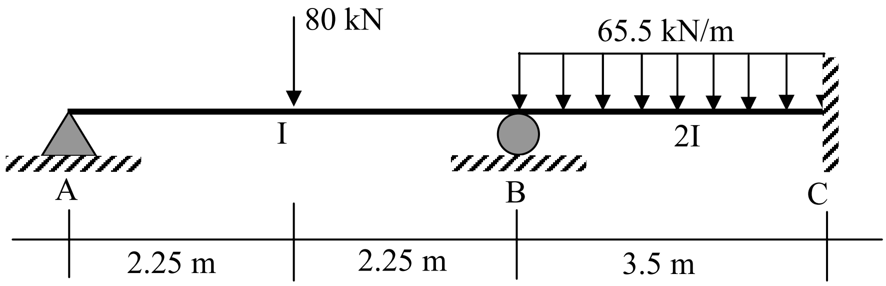
- At the end pinned,



$DF_{AB} = (K_{AB})/(K_{AB} + 0) = 1$

EXAMPLE 4.3

Draw BMD and SFD for the beam below.



Solution:

- ✓ Distribution Factor (DF)

A's support regarded as fixed, AB and BC span will have constant equivalent strength is 4 and will disappear during calculation factor this moment distribution. .

$$\text{Stiffness AB}, K_{AB} = K_{BA} = \frac{EI}{L_{AB}} = \frac{EI}{4.5}$$

$$\text{Stiffness BC}, K_{BC} = K_{CB} = \frac{EI}{L_{BC}} = \frac{2EI}{3.5} = \frac{EI}{1.75}$$

At A, assume have AA' beam, but $K_{AA'} = 0$

$DF_{AB} = K_{AB} / (K_{AB} + 0) = 1$

At B,

$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{(EI/4.5)}{(EI/4.5) + (EI/1.75)} = 0.28$$

$$DF_{BC} = \frac{K_{BC}}{K_{BC} + K_{BA}} = \frac{(EI/1.75)}{(EI/4.5) + (EI/1.75)} = 0.72$$

At C, assume have CC' beam, but $K_{CC'} = \approx$

$$DF_{CB} = \frac{K_{CB}}{K_{CB} + \approx} = 0$$

✓ Fixed End Moment(FEM) :

$$FEM_{AB} = -FEM_{BA} = -\frac{Pab^2}{L^2} = \frac{-80(2.25)(2.25)^2}{4.5^2} = -45\,kNm$$

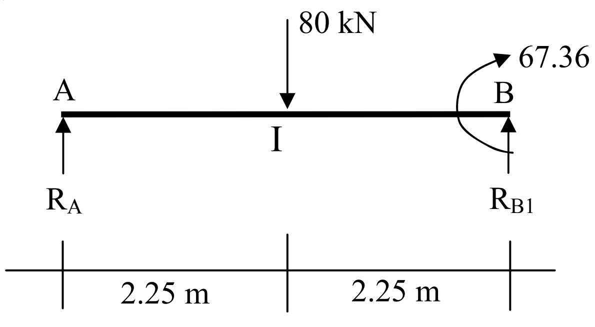
$$FEM_{BC} = -FEM_{CB} = -\frac{wL^2}{12} = \frac{-65.5(3.5)^2}{12} = -66.86\,kNm$$

✓ Moment Distribution

Member	**AB**	**BA**	**BC**	**CB**
CF	**0.5**	**0.5**	**0.5**	**0**
DF	**1**	**0.28**	**0.72**	**0**
FEM	-45	45	-66.86	66.86
Dist.	45	6.12	15.74	0
CO	3.06	22.5	0	7.87
Dist	-3.06	-6.3	-16.2	0
CO	-3.15	-1.53	0	-8.1
Dist	3.15	0.43	1.1	0
CO	0.22	1.58	0	0.55
Dist	-0.22	-0.44	-1.14	0
CO	-0.22	-0.11	0	-0.57
Dist	0.22	0.03	0.08	0
CO	0.02	0.11	0	0.04
Dist	-0.02	-0.03	-0.08	0
End Moment	**0**	**67.36**	**-67.36**	**66.65**

✓ Finding the reaction :

AB Span



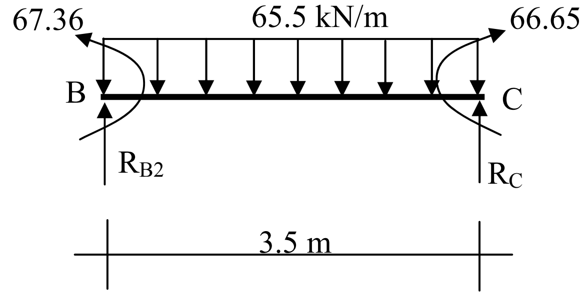
$$\sum M_B = 0 \;\;=====>\;\; 4.5R_A - 80(2.25) + 67.36 = 0\ ,$$

$$\therefore R_A = 25.03\,kN$$

$$\sum F_y = 0 \;\;=====>\;\; R_A + R_{B1} - 80 = 0\,,$$

$$\therefore R_{B1} = 54.97\,kN$$

BC span



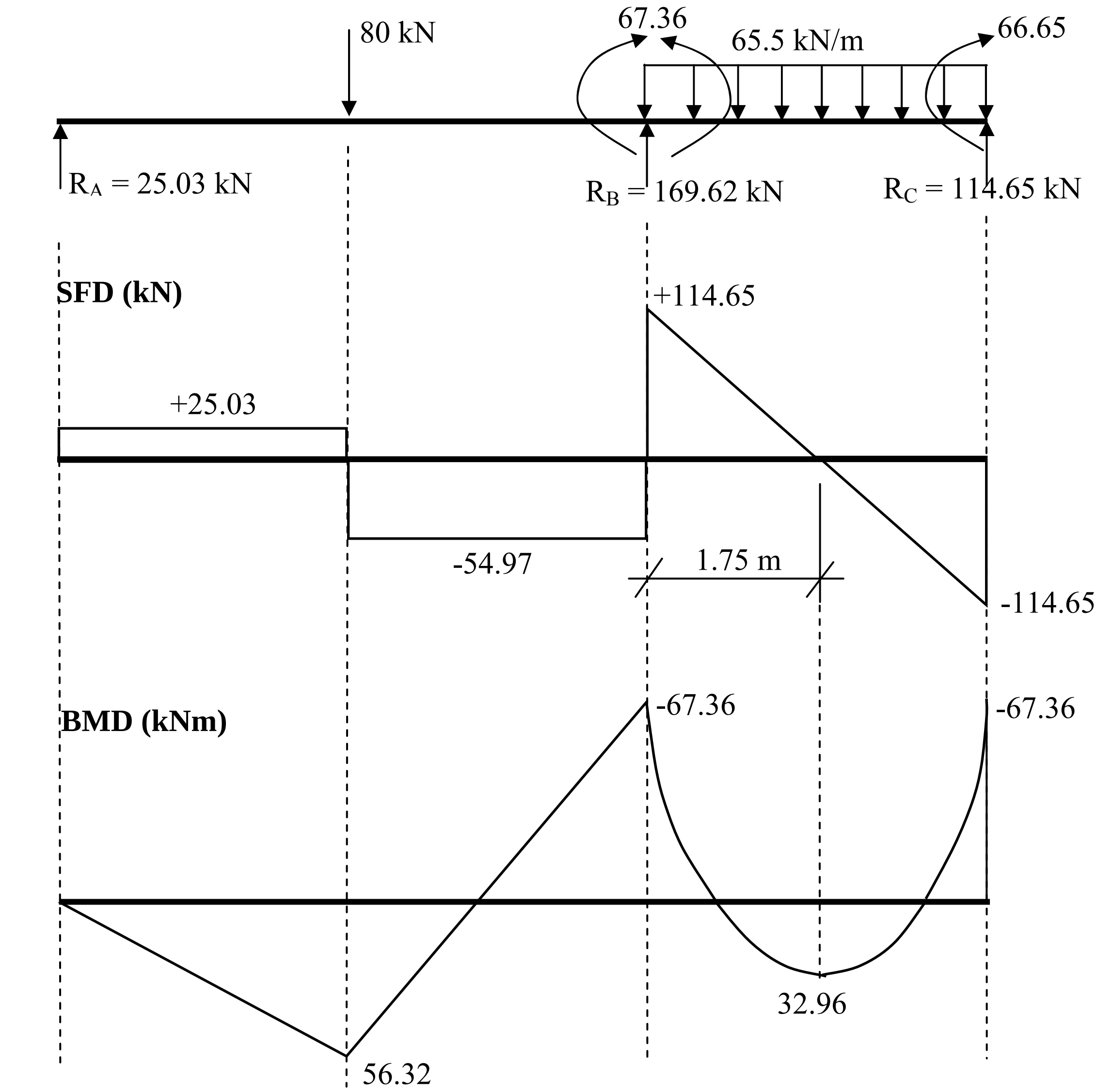
$$\sum M_C = 0 \;=====>\; 3.5R_{B2} - 67.36 + 66.65 - 65.5(3.5)(3.5/2) = 0 \;,$$

$$\therefore R_{B2} = 114.65\,\text{kN}$$

$$\sum F_y = 0 \;=====>\; R_C + R_{B2} - 65.5(3.5) = 0\,,$$

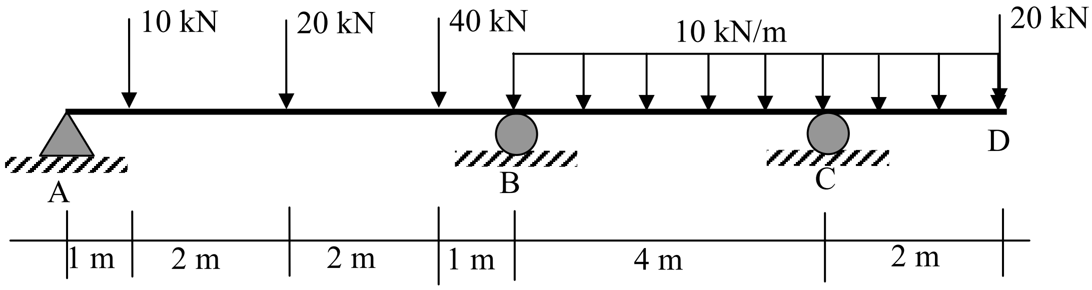
$$\therefore R_C = 114.6\,\text{kN}$$

✓ Shear Force Diagram and Bending Moment Diagram



EXAMPLE 4.4

Draw the BMD and SFD for the beam below.



Solution :

✓ Distribution Factor (DF)

$DF_{AB} = 1$

Joint	Member	K	Σ K	DF
B	BA	4EI/6	5EI/3	0.4
	BC	4EI/4		0.6
C	CB	4EI/4	4EI/4	1.0
	CD	0		0.0

✓ Fixed-end Moment

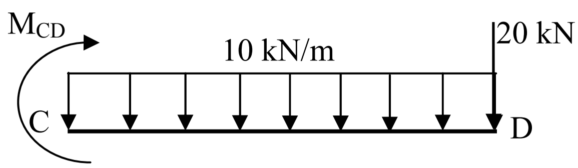
$$FEM_{AB} = \frac{-10(1)(5)^2}{6^2} - \frac{20(3)(3)^2}{6^2} - \frac{40(5)(1)^2}{6^2} = -27.5 \text{ kNm}$$

$$FEM_{BA} = \frac{+10(5)(1)^2}{6^2} + \frac{20(3)(3)^2}{6^2} + \frac{40(1)(5)^2}{6^2} = +44.17 \text{ kNm}$$

$$FEM_{BC} = \frac{-10(4)^2}{12} = -13.33 \text{ kNm}$$

$$FEM_{CB} = +13.33 \text{ kNm}$$

$$FEM_{CD} = -60 \text{ kNm}$$



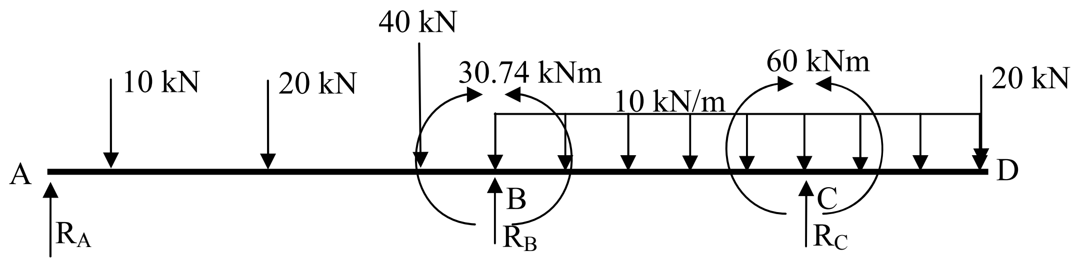
Moment Distribution

$$\sum M_C = 0 \quad ; \quad M_C + 20(2) + 10(2)(1) = 0$$

$$M_C = -60 \text{ kNm}$$

Moment Distribution:

Member	AB	BA	BC	CB	CD
CF	**0,5**	**0,5**	**0,5**	**0,5**	**0**
DF	**1,0**	**0,4**	**0,6**	**1**	**0**
FEM	-27.5	44.17	-13.33	13.33	-60
Dist.	27.5	-12.34	-18.5	46.67	0
CO	-6.17	13.75	23.34	-9.25	0
Dist	6.17	-14.84	-22.25	9.25	0
CO	-7.42	3.09	4.63	-11.13	0
Dist	7.42	-3.09	-4.63	11.13	0
CO	-1.55	3.71	5.57	-2.32	0
Dist	1.55	-3,71	-5.57	2.32	0
CO	-1.86	0.78	1.16	-2.79	0
Dist	1.86	-0.78	-1.16	2.79	0
CO	-0.39	0.93	1.4	-0.58	0
Dist	0.39	-0.93	-1.4	0.58	0
End Moment	**0**	**30.74**	**-30.74**	**60**	**-60**



✓ Reaction

- AB Span

$$\sum M_B = 0 \;\; ; \;\; R_A(6) - 10(5) - 20(3) - 40(1) + 30.74 = 0$$

$$R_A = 19.88 \text{ kN}$$

$$\sum M_A = 0 \;\; ; \;\; -R_{B1}(6) - 10(1) - 20(3) - 40(5) + 30.74 = 0$$

$$R_{B1} = 50.12 \text{ kN}$$

- BC Span

$$\sum M_C = 0 \;\; ; \;\; R_{B2}(4) - 10(4)(2) - 30.74 + 60 = 0$$

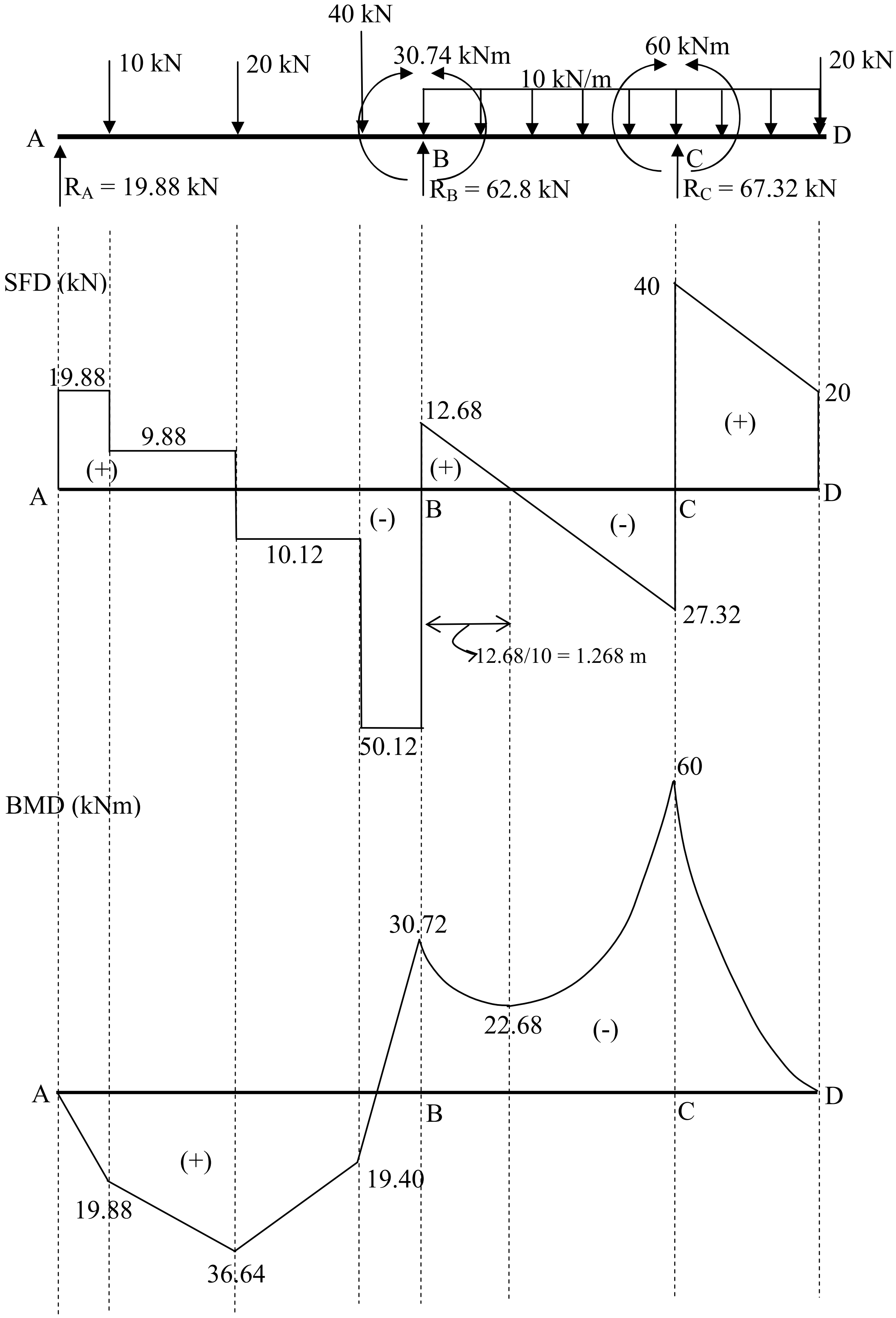
$$R_{B2} = 12.68 \text{ kN}$$

$$\sum M_B = 0 \;\; ; \;\; -R_{C1}(4) - 10(4)(2) - 30.74 + 60 = 0$$

$$R_{C1} = 27.32 \text{ kN}$$

- CD Span

$$\sum F_y = 0 \;\; ; \;\; R_{C2} = 20 + 10(2) = 40 \text{ kN}$$

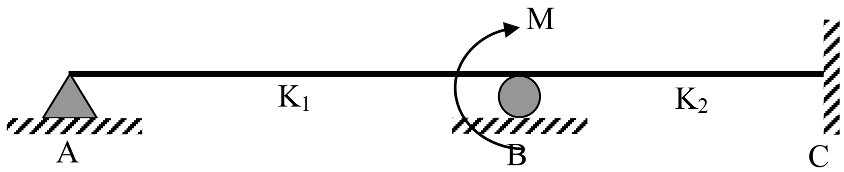

40 kN
10 kN
20 kN
30.74 kNm
60 kNm
20 kN
10 kN/m
A
B
C
D
RA = 19.88 kN
RB = 62.8 kN
RC = 67.32 kN
SFD (kN)
19.88
9.88
(+)
10.12
(-)
50.12
12.68
(+)
(-)
27.32
40
20
(+)
12.68/10 = 1.268 m
BMD (kNm)
19.88
36.64
19.40
(+)
30.72
22.68
(-)
60

4.4 MODIFY STIFFNESS FOR PIN OR ROLA SUPPORT AT THE END CASE

- To facilitate moment distribution process so that focus more precisely, supply beam continuous with either or both of them support pin or roller, distribution method this moment may be modified.

- Moment to pin or roller which is located at the end beam must to cost null / empty. Because of that, not need do process bring side to support no could bear / arrest moment.

- A continuous beam, with M is out of balance moment introduced at point B; this moment must distribute to member BA and BC.



Member AB have extension pin at A, where $M_{AB} = 0$. Modify should be done upper factor distribution is take into totally moment at the end span, $M_{AB} = 0$. E is constant

✓ As Known,

$$M_{AB} = 2K_1(2\theta_A + \theta_B) = 0 \quad ==> \theta_A = -\theta_B / 2$$

$$M_{BA} = 2K_1(2\theta_B + \theta_A) = 2K_1(2\theta_B - \theta_B / 2) = 3K_1\theta_B$$

$$M_{BC} = 2K_2(2\theta_B + \theta_C) = 4K_2\theta_B$$

✓ At B support,

$$M = M_{AB} + M_{BC} = (3EK_1 + 4EK_2)\theta_B$$

$$===> \theta_B = \frac{M}{(3EK_1 + 4EK_2)}$$

$$M_{BA} = \left(\frac{3EK_1}{3EK_1 + 4EK_2}\right)M$$

with $\left(\frac{3EK_1}{3EK_1 + 4EK_2}\right) = \left(\frac{(3/4)K_1}{(3/4)K_1 + K_2}\right) = DF_{BA}$ (distribution factor for BA) and

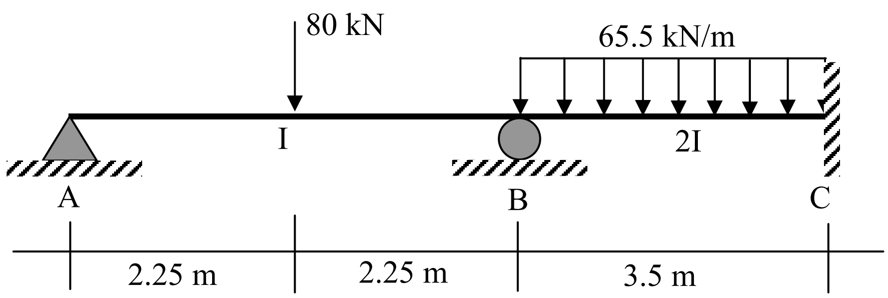
$$M_{BC} = \left(\frac{4EK_1}{3EK_1 + 4EK_2} \right) M$$

with $$\left(\frac{4EK_1}{3EK_1 + 4EK_2} \right) = \left(\frac{(1)K_1}{(3/4)K_1 + 4K_2} \right) = DF_{BC}$$ (distribution factor for BC)

For span beam have end fixed and one more end are joint with pin, stiffness for span have extension pin stated is ¾ from original stiffness while calculation factor distribution.

EXAMPLE 4.5

By comparing to Example 4.3, determine end moment after moment distribution using the stiffness modifies method.



Solution :

- ✓ Distribution Factor (DF)

$$\text{Stiffness AB}, K_{AB} = K_{BA} = \left(\frac{3}{4}\right)\frac{EI}{4.5} = 0.17EI$$

$$\text{Stiffness BC}, K_{BC} = K_{CB} = \frac{EI}{L_{BC}} = \frac{E(2I)}{3.5} = 0.57EI$$

At A, assume have AA' beam, but $K_{AA'} = 0$

$DF_{AB} = K_{AB} / (K_{AB} + 0) = 1$

At B,

$$DF_{BA} = \frac{K_{BA}}{K_{BA} + K_{BC}} = \frac{(0.17EI)}{(0.17EI) + (0.57EI)} = 0.23$$

$$DF_{BC} = \frac{K_{BC}}{K_{BC} + K_{BA}} = \frac{(0.57EI)}{(0.17EI) + (0.57EI)} = 0.77$$

At C, assume have CC' beam, but $K_{CC'} = \approx$

$$DF_{CB} = \frac{K_{CB}}{K_{CB} + \approx} = 0$$

✓ Fixed End Moment :

$$FEM_{AB} = -FEM^{t}_{BA} = -\frac{Pab^2}{L^2} = \frac{-80(2.25)(2.25)^2}{4.5^2} = -45\,kNm$$

$$FEM^{t}_{BC} = -FEM_{CB} = -\frac{wL^2}{12} = \frac{-65.5(3.5)^2}{12} = -66.86\,kNm$$

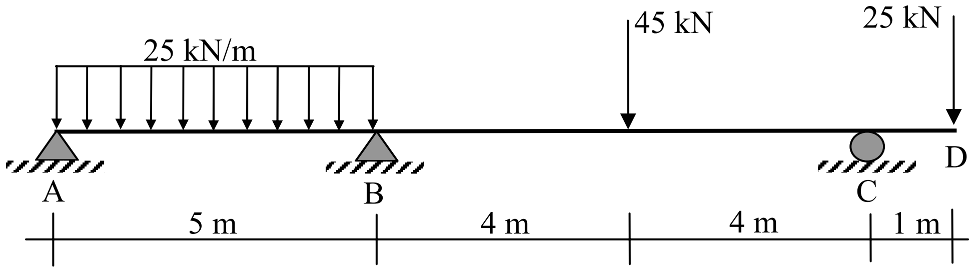
✓ Moment Distribution

Member	AB	BA	BC	CB
CF	**0.5**	**0**	**0.5**	**0**
DF	**1**	**0.23**	**0.77**	**0**
FEM Dist.	-45 45	45 5.03	-66.86 16.83	66.86 0
CO Dist		22.5 -5.18	0 -17.32	8.42 0
CO Dist		0 0	0 0	-8.6 0
End Moment	**0**	**67.35**	**-67.35**	**66.68**

✓ Make a comparison between Example 4.3 and Example 4.5, distribute moment with using the modified method are more disposed and quick compared to with use the means ordinary stiffness.

EXAMPLE 4.6

Determine reaction to each support and draw SFD and BMD if in the structure below, the displacement at support B descends 4 mm. Constant EI by beam, with the value EI = 3000kNm^2.



Solution :

- ✓ Displacement at support B will impact to value fixed-end moment. The Formula for moment of displacement is $6.EI\Delta/L^2$, where Δ is the displacement.

- ✓ Distribution Factor

$$\text{Modified Stiffness AB}, K_{AB} = K_{BA} = (3/4)(I/5) = 3I/20$$

$$\text{Modified Stiffness BC}, K_{BC} = K_{CB} = (3/4)(I/8) = 3I/32$$

$$\text{Modified Stiffness CD}, K_{CD} = 0$$

At A, $DF_{AB} = \dfrac{K_{AB}}{K_{AB}+0} = 1$

$$DF_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{3/20}{(3/20)+(3/32)} = 8/13$$

At B

$$DF_{BC} = \frac{K_{BC}}{K_{BC}+K_{BA}} = \frac{3/32}{(3/32)+(3/20)} = 5/13$$

At C

$$DF_{CB} = \frac{K_{CB}}{K_{CB}+K_{CD}} = \frac{3/32}{(3/32)+0} = 1$$

✓ Fixed End Moment

$$FEM_{AB} = -\frac{wL^2}{12} - \frac{6EI\Delta}{L^2} = -\frac{25(5)^2}{12} - \frac{6(3000)(4x10^{-3})}{5^2} = -54.96\,\text{kNm}$$

$$FEM_{BA} = 52.08 - 2.88 = 49.2\,\text{kNm}$$

$$FEM_{BC} = -\frac{Pab^2}{L^2} + \frac{6EI\Delta}{L^2} = -\frac{45(4)(4)^2}{8^2} + \frac{6(3000)(4x10^{-3})}{8^2} = -43.87\,\text{kNm}$$

$$FEM_{CB} = 45 + 1.13 = 46.13\,\text{kNm}$$

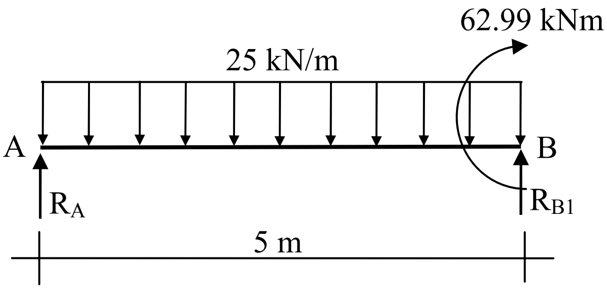
$$FEM_{CD} = -25(1) = -25\,\text{kNm}$$

✓ Moment Distribution

Anggota	**AB**	**BA**	**BC**	**CB**	**CD**
CF	**0,5**	**0**	**0**	**0,5**	**0**
DF	**1,0**	**8/13**	**5/13**	**1**	**0**
FEM	-54.96	49.2	-43.87	46.13	-25
Dist.	54.96	-3.28	-2.05	-21.3	0
CO	-6.17	27.48	-10.57	-9.25	0
Dist		-10.41	-6.5		
End of Moment	**0**	**62.99**	**-62.99**	**25**	**-25**

✓ Reaction:

- Span AB

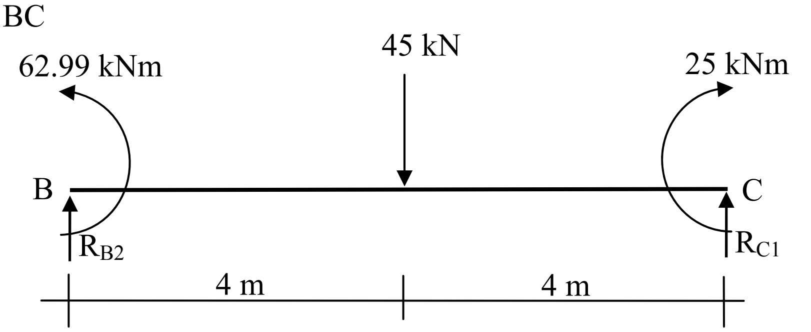


$$\sum M_B = 0 \quad =====> \quad R_A(5) - 25(5)(2.5) + 62.99 = 0$$

$$R_A = 49.9\,\text{kN}$$

$$\sum F_y = 0 \quad =====> \quad R_A + R_{B1} - 25(5) = 0\ ; \quad R_{B1} = 75.1\,\text{kN}$$

- Span BC

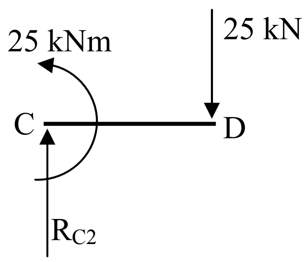


$$\sum M_B = 0 \quad =====> \; -R_{C1}(8) + 45(4) - 62.99 + 25 = 0$$

$$R_{C1} = 17.75\,\text{kN}$$

$$\sum F_y = 0 \quad =====> \; R_{B2} + R_{C1} - 45 = 0 \;; \qquad R_{B2} = 27.25\,\text{kN}$$

- Span CD

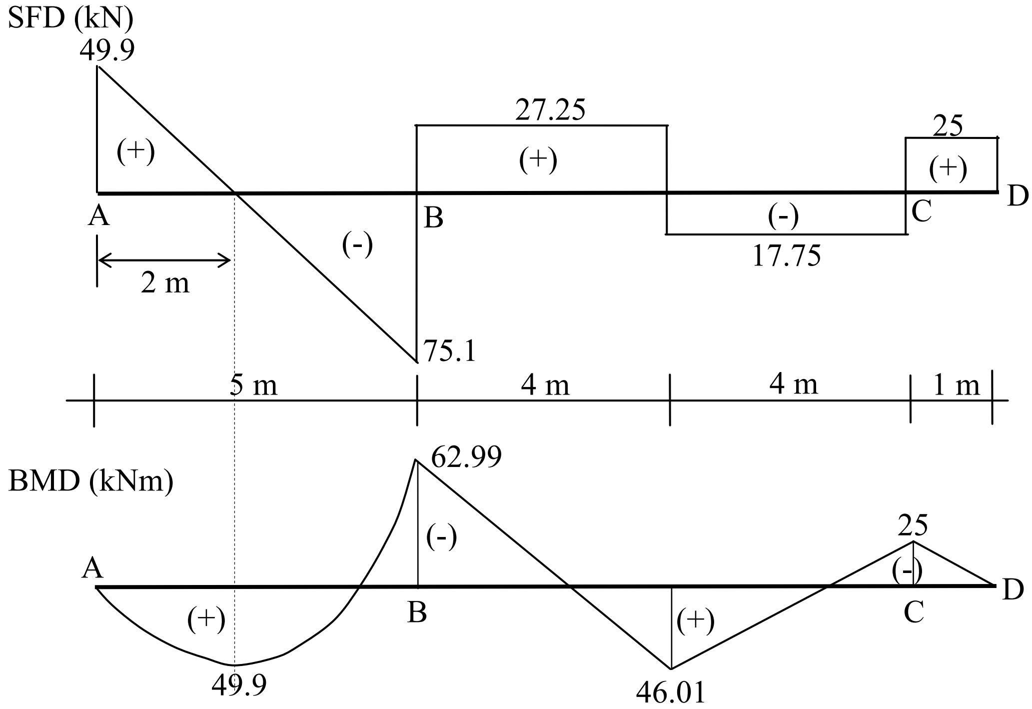


$\sum F_y = 0$

$R_{C2} - 25 = 0$

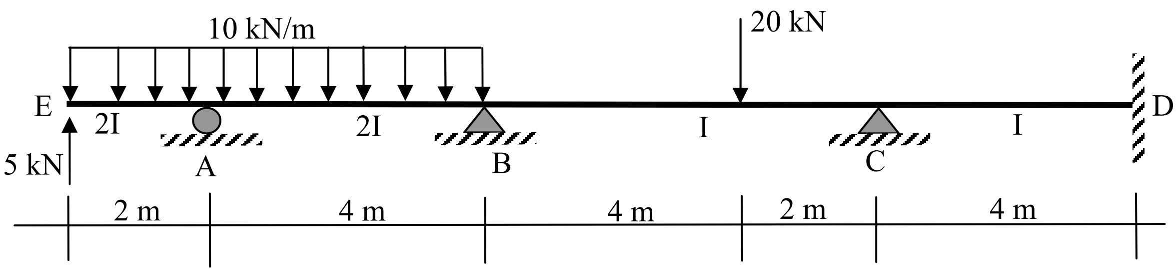
$R_{C2} = 25$ kN

✓ Shear Force Diagram and Bending Moment Diagram



EXAMPLE 4.7

By using the moment distribution method, draw the shear force diagram and bending moment diagram for continuous beam below. The E value is constant.



Solution :

✓ Distribution Factor (DF)

$DF_{DC} = 0$; $DF_{AB} = 1$; $DF_{AE} = 0$

Join	Member	K	Σ K	DF
B	BA	$\left(\frac{3}{4}\right)\frac{E(2I)}{4} = 0.375EI$	0.545EI	0.69
	BC	$\frac{EI}{6} = 0.17EI$		0.31
C	CB	$\frac{EI}{6} = 0.17EI$	0.42EI	0.40
	CD	$\frac{EI}{4} = 0.25EI$		0.60

✓ Fixed-End Moment

$$FEM_{AE} = FEM_{AE} = 10(2)(1) - 5(2) = 10\ \text{kNm}$$

$$FEM_{AB} = \frac{-10(4)^2}{12} = -13.33\ \text{kNm}$$

$$FEM_{BA} = \frac{10(4)^2}{12} = 13.33\ \text{kNm}$$

$$FEM_{BC} = -\frac{20(4)(2)^2}{6^2} = -8.89\ \text{kNm}$$

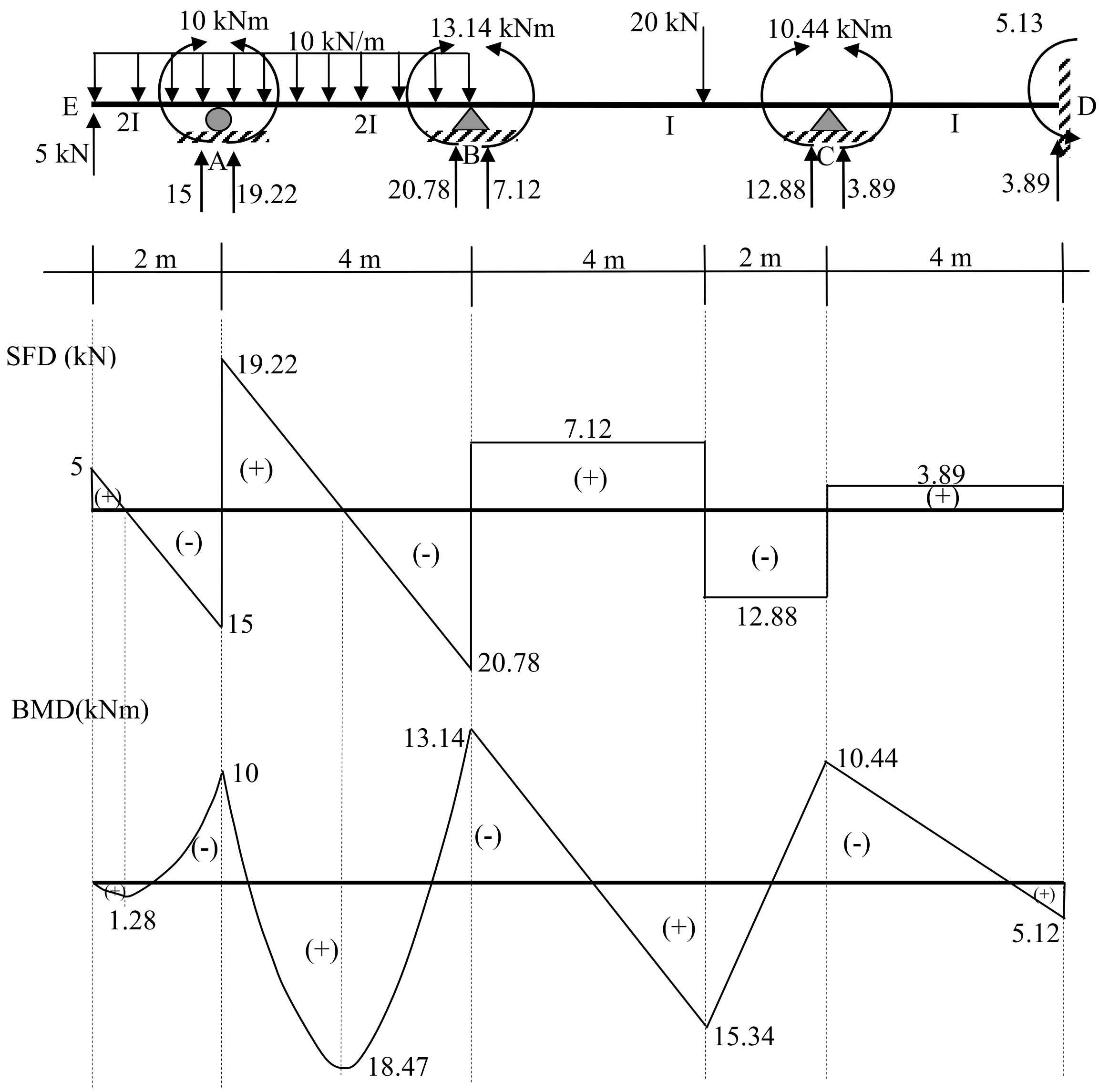
$$FEM_{CB} = \frac{20(4)^2(2)}{6^2} = 17.78\ \text{kNm}$$

$$FEM_{CD} = FEM_{DC} = 0$$

✓ Moment Distribution

Joint	E	A		B		C		D
Member	**EA**	**AE**	**AB**	**BA**	**BC**	**CB**	**CD**	**DC**
CF	**0**	**0**	**0.5**	**0**	**0.5**	**0.5**	**0.5**	**0**
DF	**0**	**0**	**1**	**0.69**	**0.31**	**0.4**	**0.6**	**0**
FEM	0	10	-13.33	13.33	-8.89	17.78	0	0
Dist.	0	0	3.33	-3.06	-1.38	-7.11	-10.67	0
CO		0	0	1.67	-3.56	-0.69	0	-5.34
Dist		0	0	1.3	0.59	0.28	0.41	0
CO		0	0	0	0.14	0.30	0	0.21
Dist		0	0	-0.10	-0.04	-0.12	-0.18	0
End of Moment	**0**	**10**	**-10**	**13.14**	**-13.14**	**10.44**	**-10.44**	**-5.13**

✓ SFD and BMD

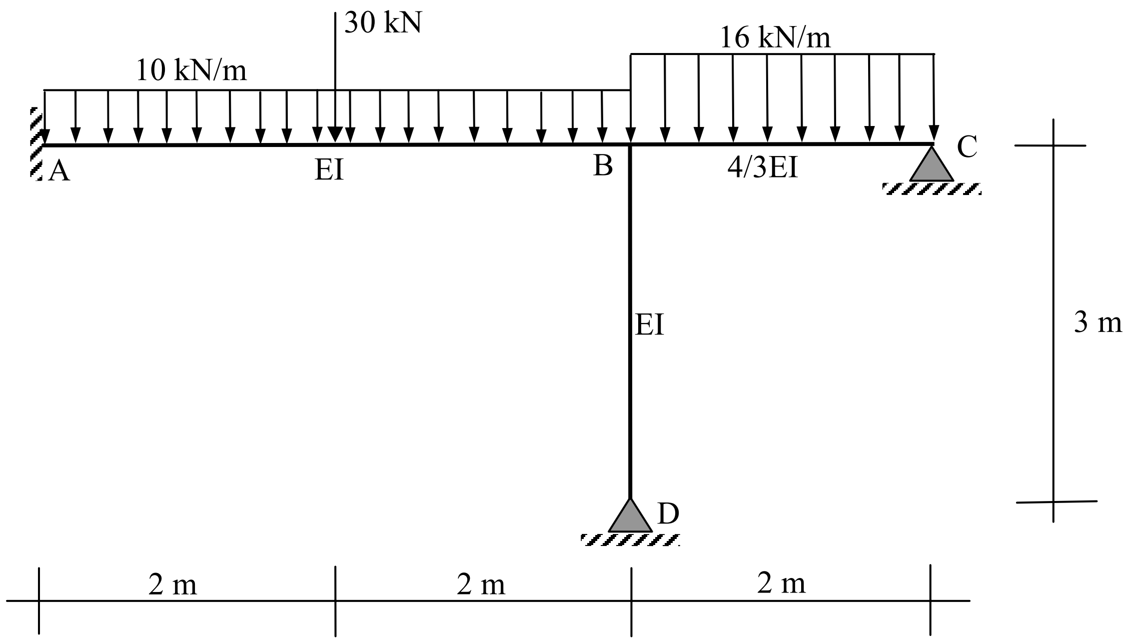


4.5 MOMENT DISTRIBUTION METHOD FOR RIGID NON-SWAY FRAME

- In general, approach steps involved is the same as to beam. More practical by using modified stiffness.
- Rigid non-sway frame is the state where deformation / rigid framework movement will not cause joint or extension or join framework shift ($\Delta = 0$).
- Case-study to rigid frames this make equivalent as in the case beam.

<u>EXAMPLE 4.8</u>

Draw shear force and bending moment diagram to rigid frame structure as below.



SOLUTION :

✓ Distribution Factor (DF)

$DF_{AB} = 0$; $DF_{CB} = DF_{DB} = 1$

Joint	Member	K	Σ K	DF
B	BA	4EI/4 = 1EI	4EI	1/4 = 0.25
	BC	3/4.(4(4/3.EI)/2) = 2EI		2/4 = 0.50
	BD	3/4.(4EI/3) = 1EI		1/4 = 0.25

- ✓ Fixed End Moment

$$FEM_{AB} = \frac{-10(4)^2}{12} - \frac{30(2)(2)^2}{4^2} = -28.33\,\text{kNm}$$

$$FEM_{BA} = \frac{10(4)^2}{12} + \frac{30(2)(2)^2}{4^2} = 28.33\,\text{kNm}$$

$$FEM_{BC} = \frac{-16(2)^2}{12} = -5.33\,\text{kNm}$$

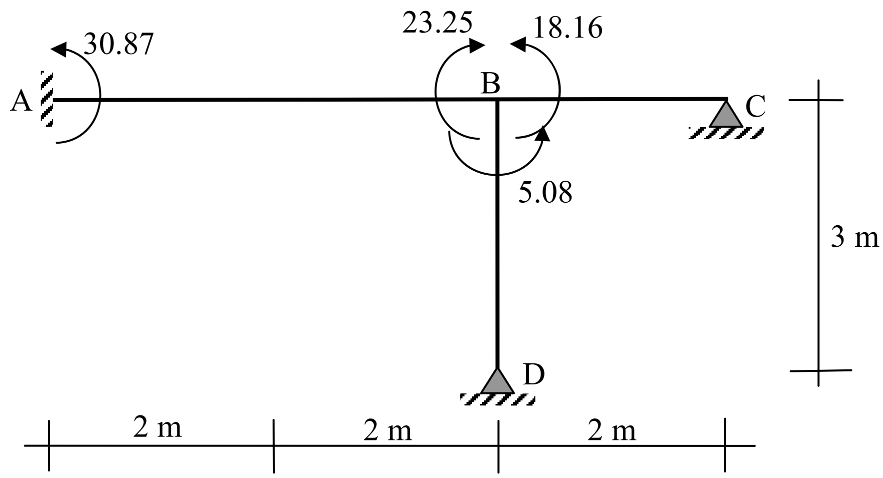
$$FEM_{BC} = \frac{16(2)^2}{12} = 5.33\,\text{kNm}$$

$$FEM_{BD} = FEM_{DB} = 0$$

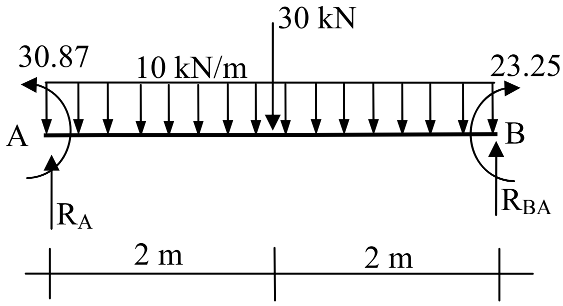
- ✓ Moment Distribution

Join	**A**	**B**			**C**	**D**
Member	**AB**	**BA**	**BC**	**BD**	**CB**	**DB**
CF	**0**	**0.5**	**0**	**0**	**0.5**	**0.5**
DF	**0**	**0,25**	**0,5**	**0,25**	**1**	**1**
FEM	-28,33	28,33	-5,33	0	5,33	0
Dist.	0	-5,75	-11,5	-5,75	-5,33	0
CO	-2,88	0	-2,67	0	0	0
Dist	0	0,67	1,34	0,67	0	0
CO	0,34	0	0	0	0	0
Dist	0	0	0	0	0	0
End Moment	**-30,87**	**23,25**	**-18,16**	**-5,08**	**0**	0

- ✓ Reaction:



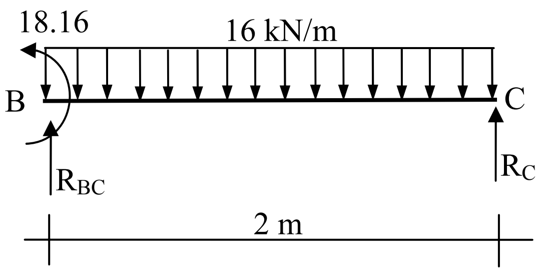
- Span AB



$R_A = 36.19$ kN

$R_{BA} = 33.09$ kN

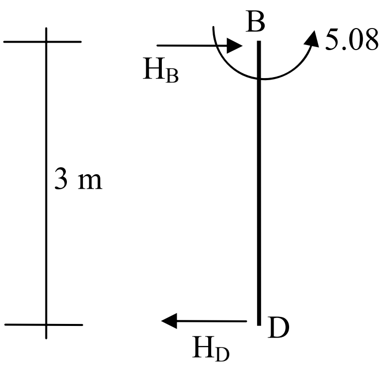
- Span BC



$R_{BC} = 25.08$ kN

$R_C = 6.92$ kN

- Span BD

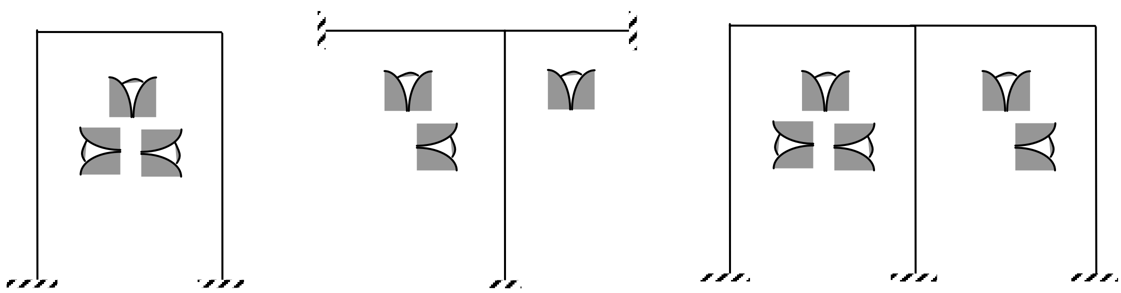


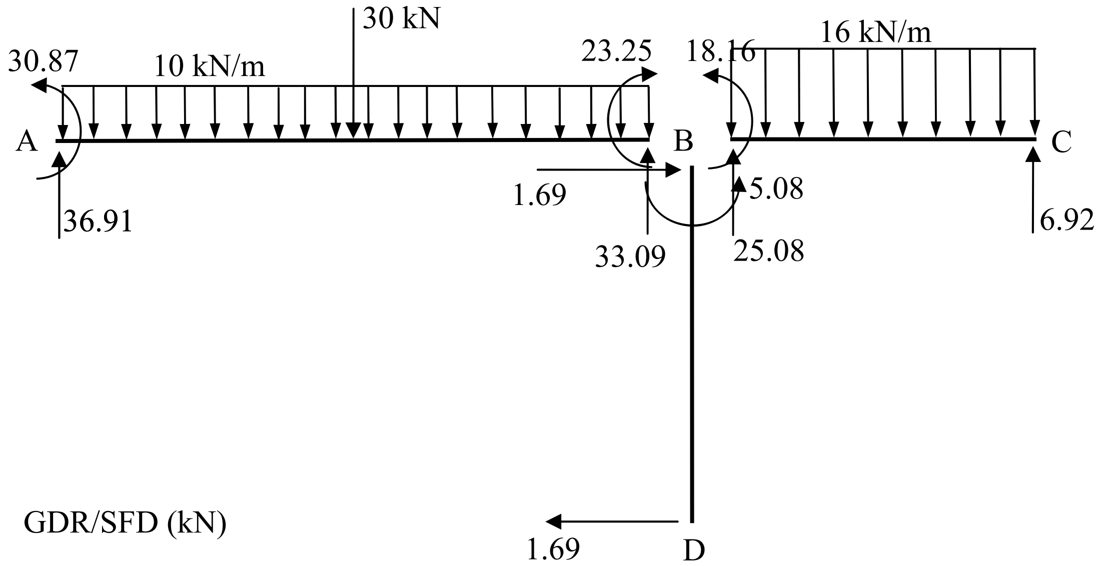
$\sum M_B = 0 \quad ; \quad H_D(3) - 5.08 = 0 \text{-------} > H_D = 1.69$ k

$\sum F_X = 0 \quad ; \quad H_B = 1.69$ kN

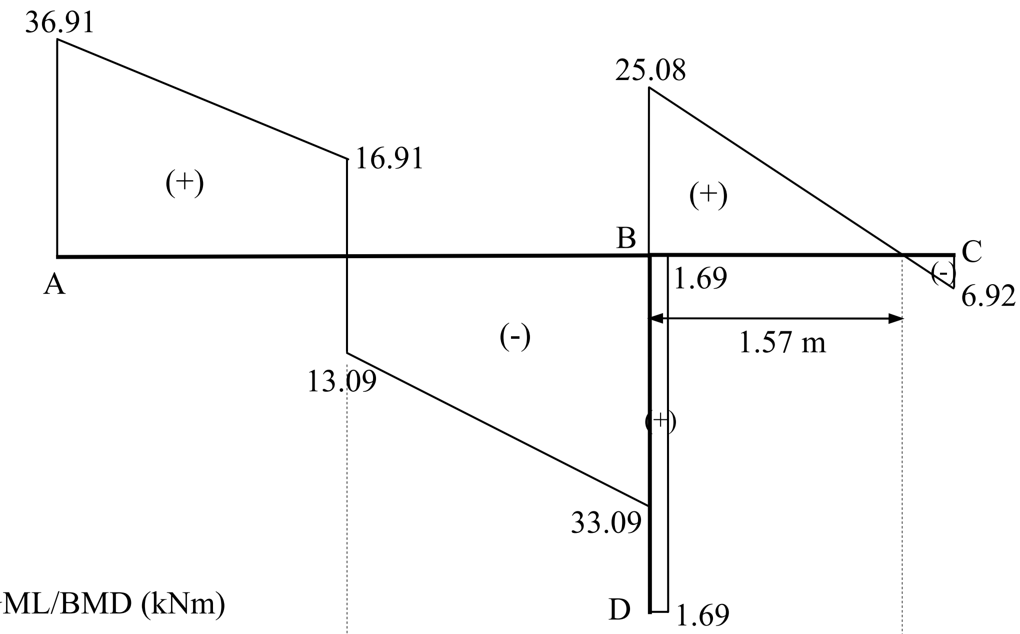
✓ Shear Force Diagram and Bending Moment Diagram

Viewed for SFD and BMD for frame :

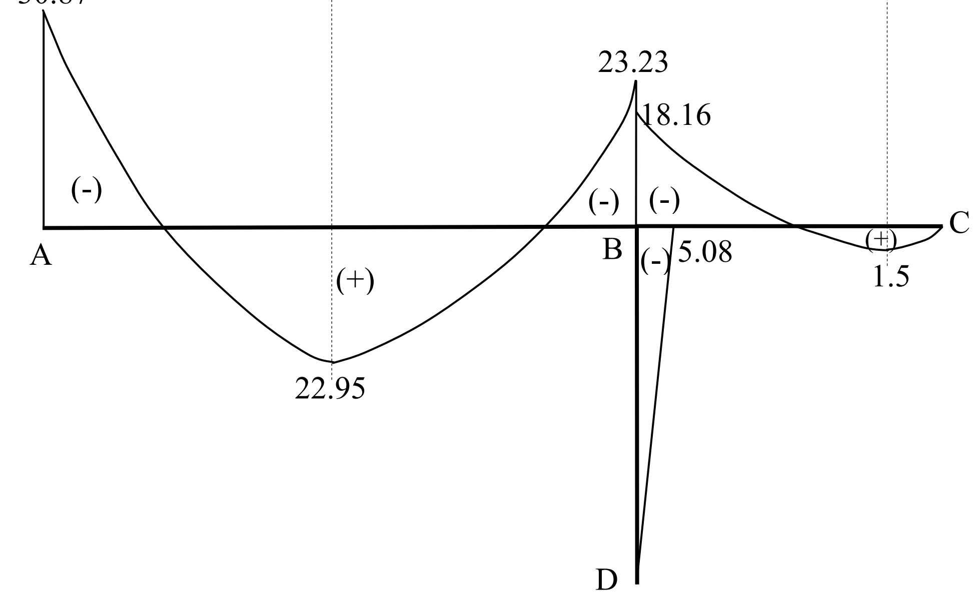




GDR/SFD (kN)

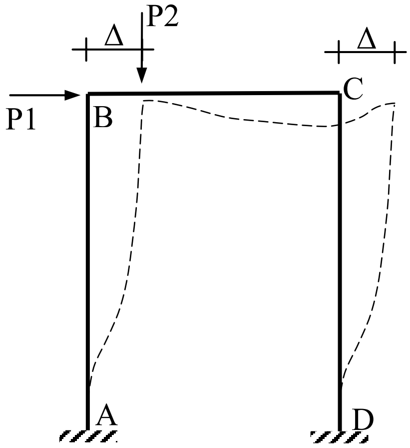


GML/BMD (kNm)

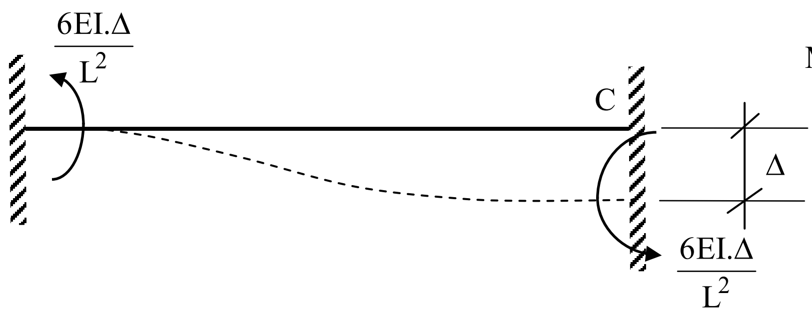


4.6 MOMENT DISTRIBUTION METHOD FOR RIGID FRAME WITH SWAY

- Many case, where we find out framework will experience some movement those mentioned as sway
- Sway would cause happen it moment lurch to which shift happen (Δ).

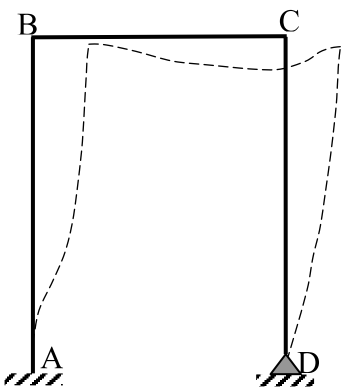


- Moment due to sway (MS) with shift (Δ) could be phrased as:



$$M^S = \frac{-6EI.\Delta}{L^2}$$

- If support / end is pin or roller, then MS becomes

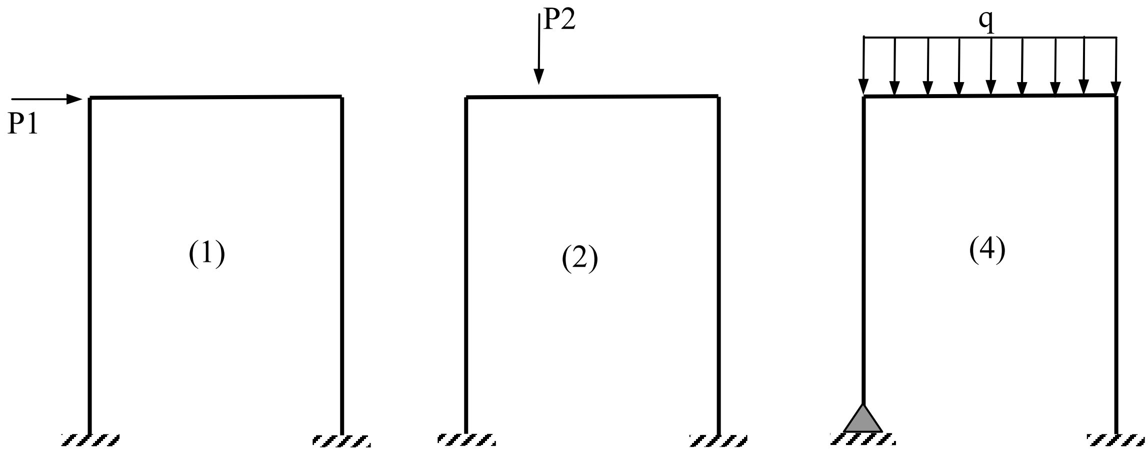


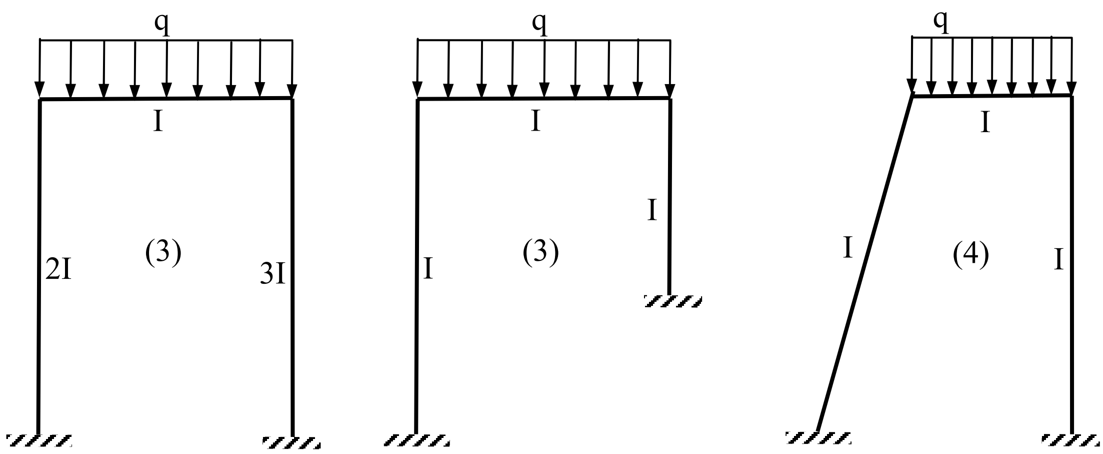
$$M^S_{CD} = \frac{-3EI\Delta}{L^2}$$

$$M^S_{DC} = 0$$

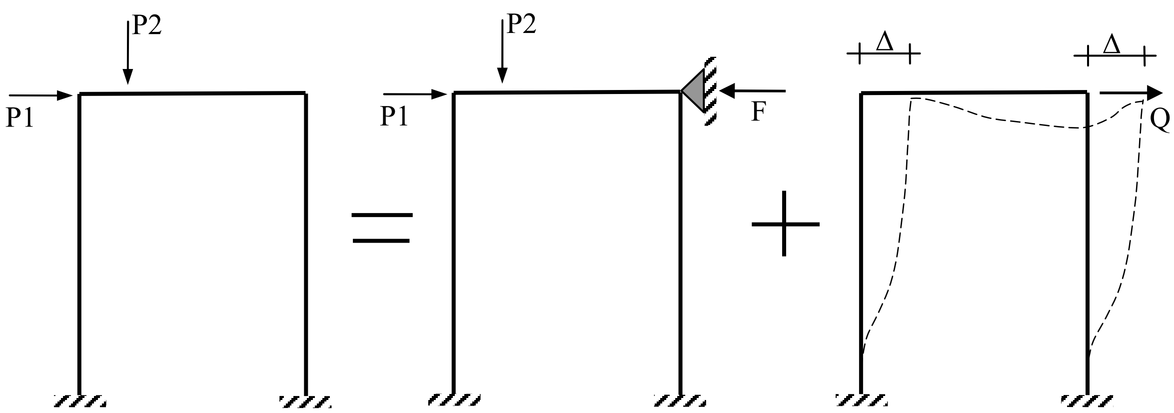
- Sway to the rigid frame will occur because of :

1. Horizontal Load
2. Vertical Load / upright that does not symmetry
3. Unsymmetrical frame system from the aspect form (geometry) or material (inertia)
4. Unequal support
5. Shift to extension or support





- In general, solution should do deep 2 level and this stages then combined (overlap principal) to get solution another situation in fact.



- First stage, framework with no sway caused by constraint that imposed to top end column. In this regard moment distribution can do as usual. Consider, final moment that was found was M1 (moment no sway).

- Second stages sway framework due to Q counter direction to constraint discharged. In this regard problem have burden horizon to extension only. Speak omen final that was found was M2 (moment with sway).

- Then actual moment is = M = M1 + X .M2

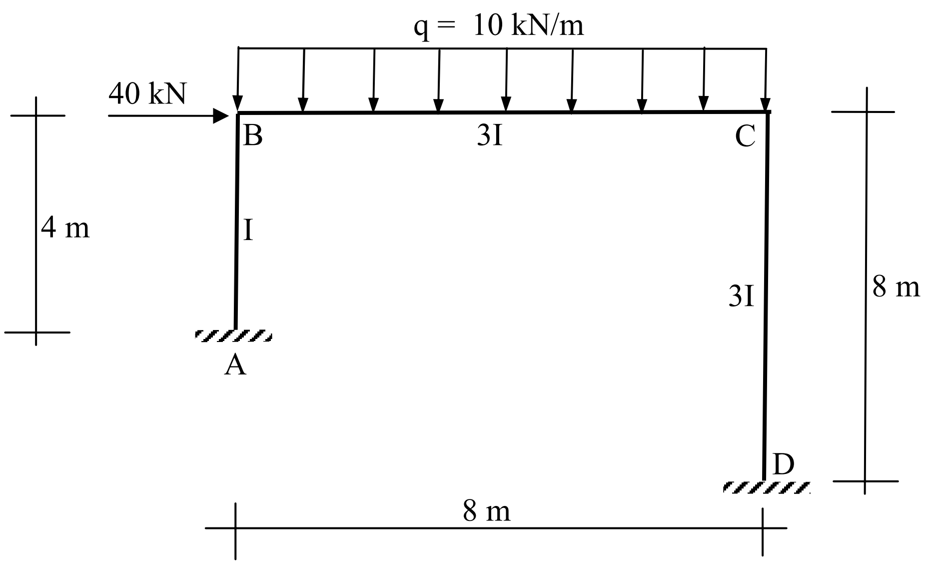
Where X is correction value if Δ are considered have something other value than the real value. Hence, if F is the ability those found from the case no sway and Q is from the sway case, then:

$$F - X.Q = 0$$

$$X = F/Q$$

EXAMPLE 4.9

Draw shear force and bending moment diagram to rigid frame structure as below. The value of E is constant and assume EIΔ = 160

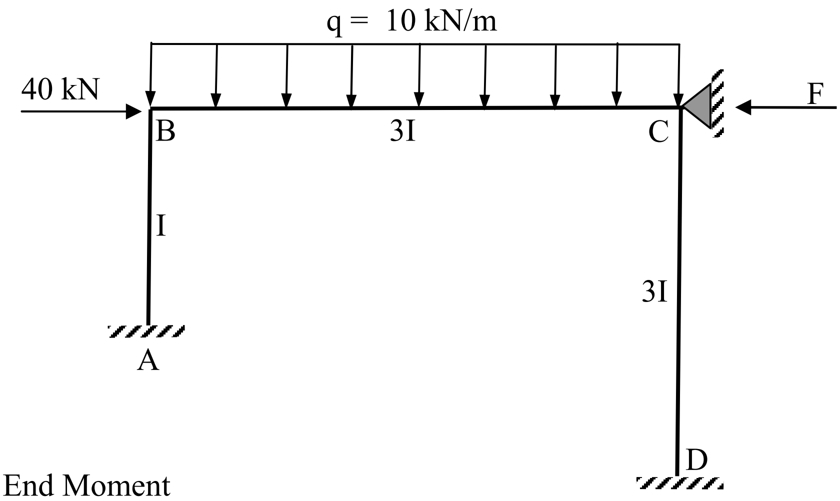


Solution :

✓ Distribution Factor (DF) $DF_{AB} = DF_{DC} = 0$

Joint	Member	K	Σ K	DF
B	BA	4EI/4	20EI/8	0.4
	BC	4E(3I)/8		0.6
C	CB	4E(3I)/8	24EI/8	0.5
	CD	4E(3I)/8		0.5

FRAME WITHOUT SWAY



- ✓ Fixed End Moment

$$FEM_{AB} = FEM_{BA} = FEM_{CD} = FEM_{DC} = 0$$

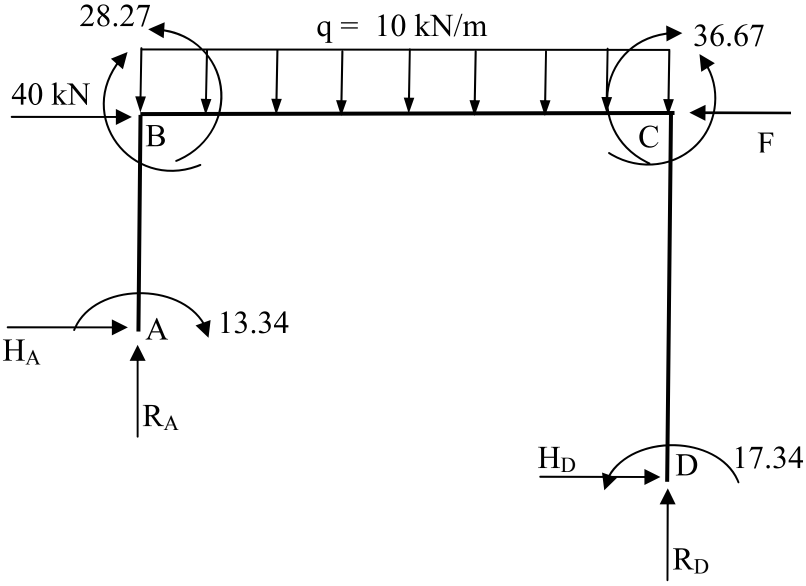
$$FEM_{BC} = \frac{-10(8)^2}{12} = -53.33\,\text{kNm}$$

$$FEM_{CB} = 53.33\,\text{kNm}$$

- ✓ Moment Distribution

Member	**AB**	**BA**	**BC**	**CB**	**CD**	**DC**
CF	**0**	**0.5**	**0.5**	**0.5**	**0.5**	**0**
DF	**0**	**0.4**	**0.6**	**0.5**	**0.5**	**0**
FEM	0	0	-53.33	53.33	0	0
Dist.	0	21.33	32	-26.67	-26.67	0
CO	10.67	0	-13.34	16	0	-13.34
Dist	0	5.34	8	-8	-8	0
CO	2.67	0	-4	4	0	-4
Dist	0	1.6	2.4	-2	-2	0
End Moment	**13.34**	**28.27**	**-28.27**	**36.66**	**-36.67**	**-17.34**

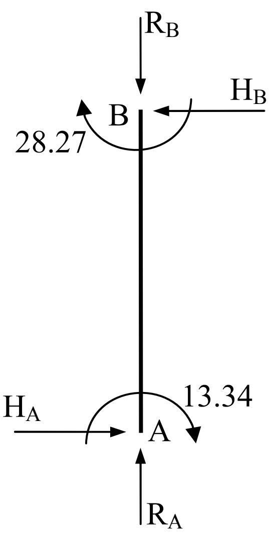
✓ Calculation for F value



$$\sum Fx = 0 \quad ; \quad 40 + H_A + H_D - F = 0$$

$$F = 40 + H_A + H_D$$

- Column AB

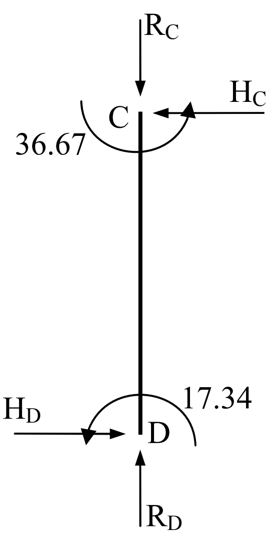


$\sum M_B = 0$

$28.27 + 13.34 - H_A(4) = 0$

$H_A = 10.40$ kN (→)

Column CD



$\sum M_C = 0$

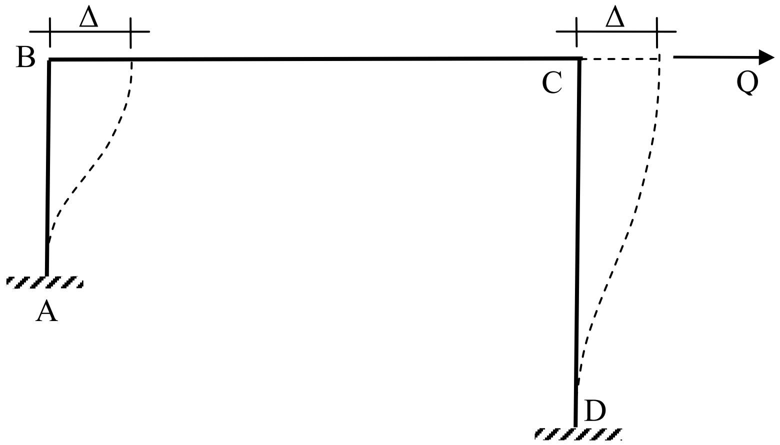
$- 36.67 - 17.34 - H_D(8) = 0$

$H_D = -6.75$ kN

$= 6.75$ kN (←)

* $F = 40 + 10.40 - 6.75 = 43.65$ kN

- ✓ **FRAME WITH SWAY**



- ✓ Moment due to sway

$$M_{AB}^{S} = M_{BA}^{S} = \frac{-6EI\Delta}{L^2} = \frac{-6(160)}{4^2} = -60$$

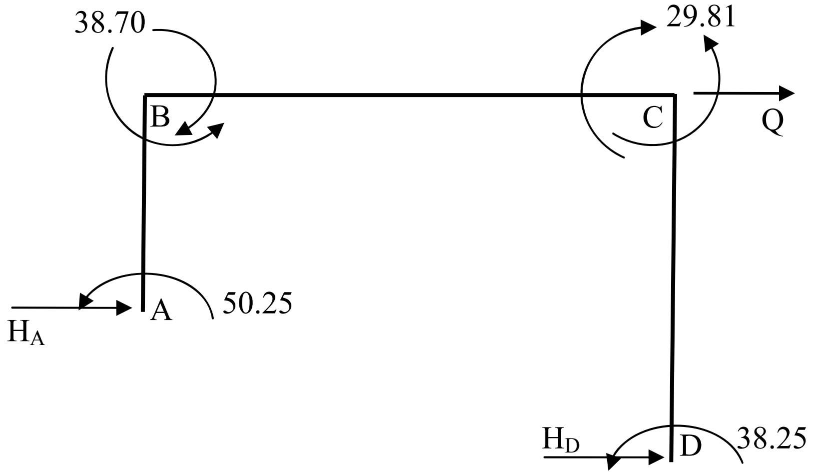
$$M_{CD}^{S} = M_{DC}^{S} = \frac{-6E(3I)\Delta}{L^2} = \frac{-18(160)}{8^2} = -45$$

$$M_{BC}^{S} = M_{CB}^{S} = 0$$

- ✓ Moment Distribution

Member	AB	BA	BC	CB	CD	DC
CF	**0**	**0.5**	**0.5**	**0.5**	**0.5**	**0**
DF	**0**	**0.4**	**0.6**	**0.5**	**0.5**	**0**
FEM	-60	-60	0	0	-45	-45
Dist.	0	24	36	22.5	22.5	0
CO	12	0	11.25	18	0	11.25
Dist	0	-4.5	-6.75	-9	-9	0
CO	-2.25	0	-4.5	-3.38	0	-4.5
Dist	0	1.8	2.7	1.69	1.69	0
End Moment	**-50.25**	**-38.7**	**38.7**	**29.81**	**-29.81**	**-38.25**

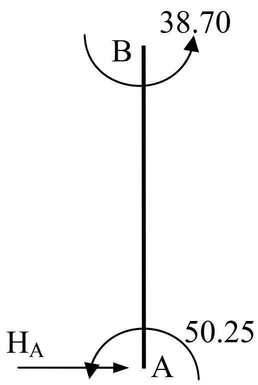
✓ Calculation for Q value



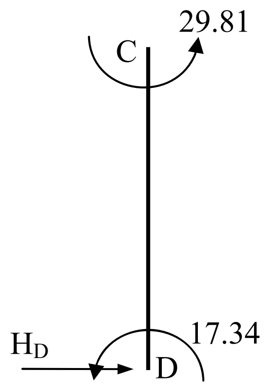
$$\sum Fx = 0 \quad ; \quad Q + H_A + H_D = 0$$

$$Q = -H_A - H_D$$

- Column AB



Column CD



$\sum M_B = 0$

$-38.7 - 50.25 - H_A(4) = 0$

$H_A = -22.24$ kN

$= 22.24$ kN (←)

$\sum M_C = 0$

$-29.81 - 38.25 - H_D(8) = 0$

$H_D = -8.51$ kN

$= 8.51$ kN (←)

* $Q = -(-22.24) - (-8.51) = 30.75$

✓ Correction Value : $X = \frac{F}{Q} = \frac{43.65}{30.75} = 1.42$

✓ Actual Moment : = M without sway + X. Moment with sway

Member	Actual Moment (kNm)	
AB	13.34 + 1.42 (-50.25)	= -58.02
BA	28.27 + 1.42 (-38.70)	= -26.68
BC	-28.27 + 1.42 (38.70)	= 26.68
CB	36.66 + 1.42 (29.81)	= 79.00
CD	-36.67 + 1.42 (-29.81)	= -79.00
DC	-17.34 + 1.42 (-38.25)	= -71.66

✓ Actual Reaction : = Reaction without sway + X. reaction with sway

$H_A = 10.40 + 1.42(-22.4) = -21.18$ kN (←)

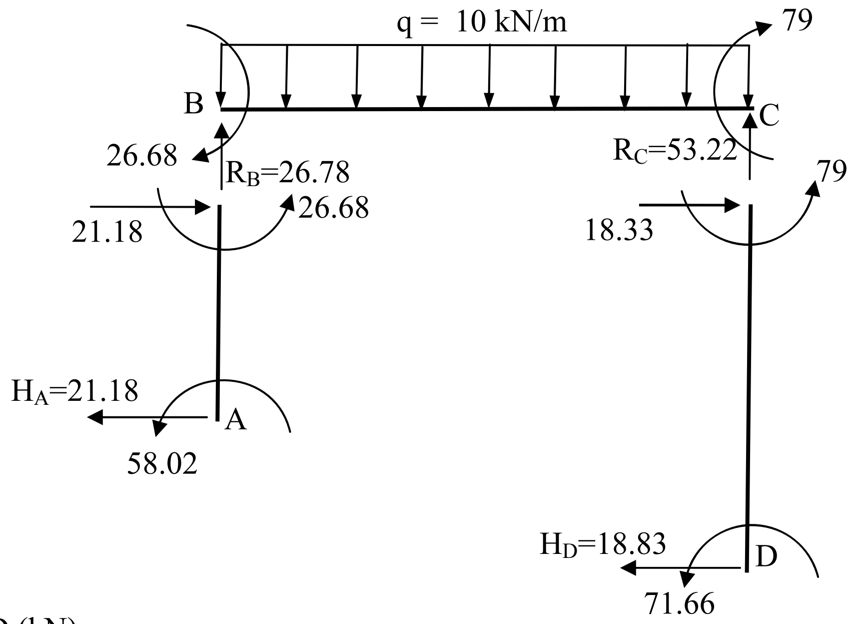
$H_D = -6.75 + 1.42(-8.51) = -18.83$ kN (←)

✓ Reaction at Support BC :

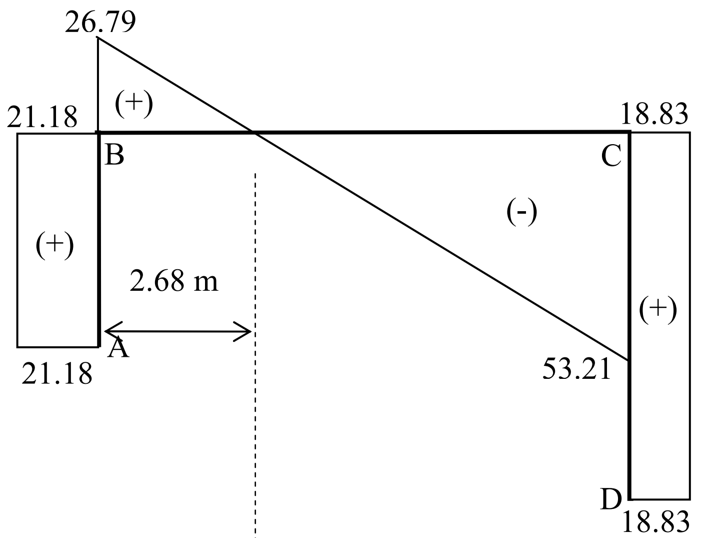
$\sum M_B = 0 \rightarrow 10(8)(4) + 26.68 + 79 - R_C(8) = 0 \rightarrow R_C = 53.21$ kN ; $(R_D = R_C)$

$\sum M_C = 0 \rightarrow -10(8)(4) + 26.68 + 79 + R_B(8) = 0 \rightarrow R_B = 26.79$ kN ; $(R_A = R_B)$

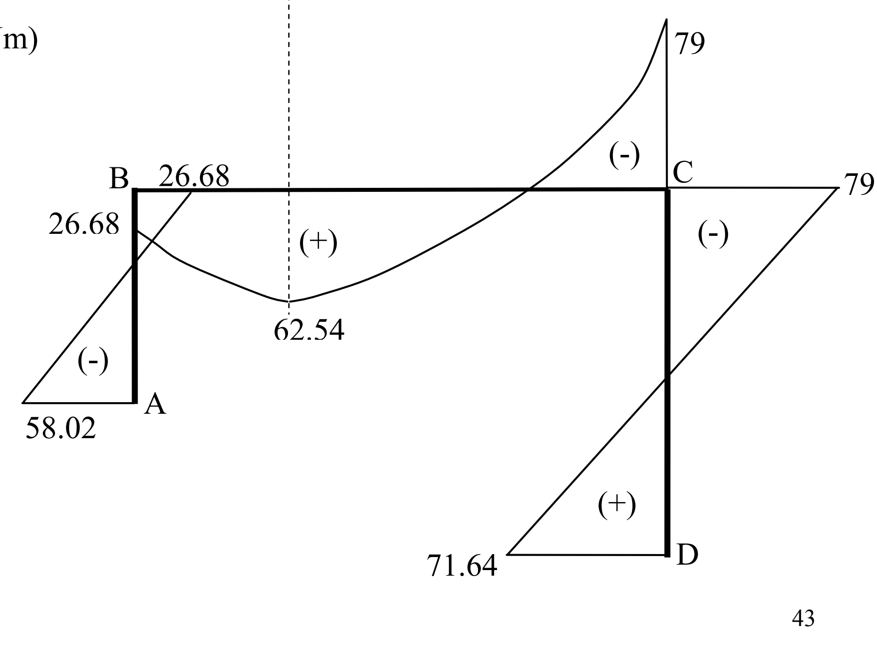
✓ Shear Force Diagram and Bending Moment Diagram



SFD (kN)



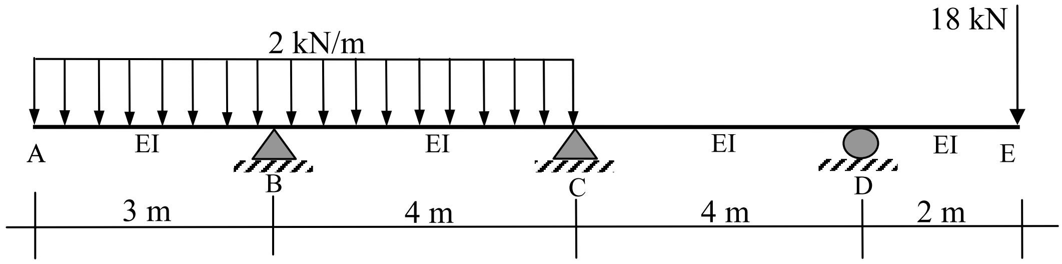
BMD (kNm)



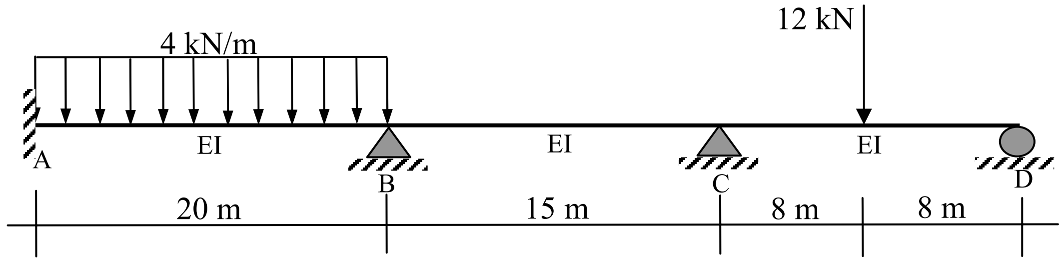
TUTORIAL 4

1. By comparing the result between Slope Deflection Equation and Modified Moment Distribution Method, determine end moment for all member and reaction at the support and then draw SFD and BMD for each beam.

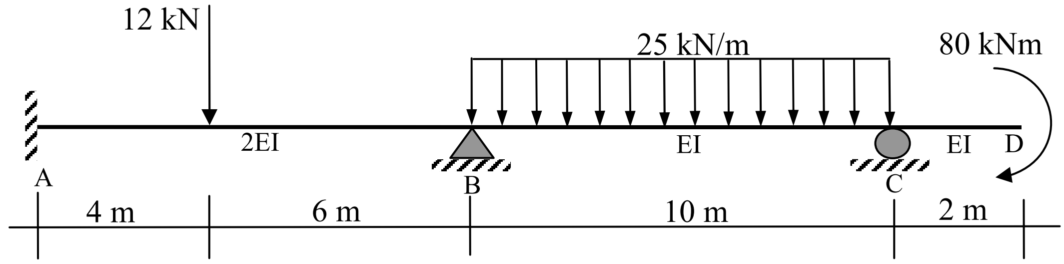
a.



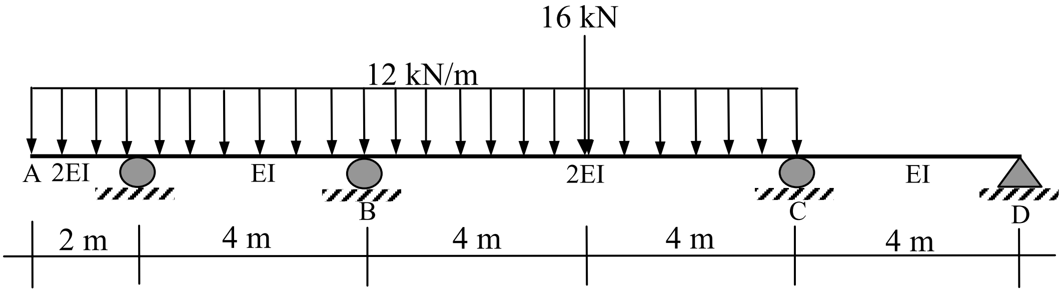
b.



c.

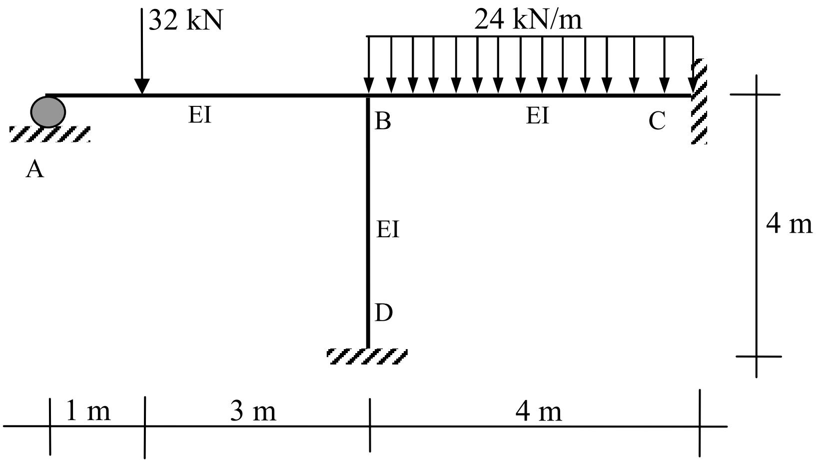


d.

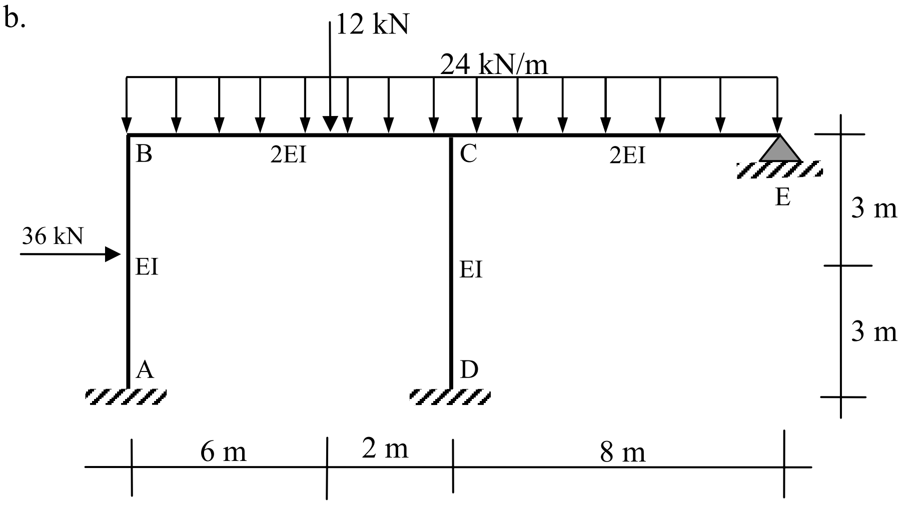


2. Determine end moment for all member and reaction at the support, and then draw SFD and BMD for frame below.

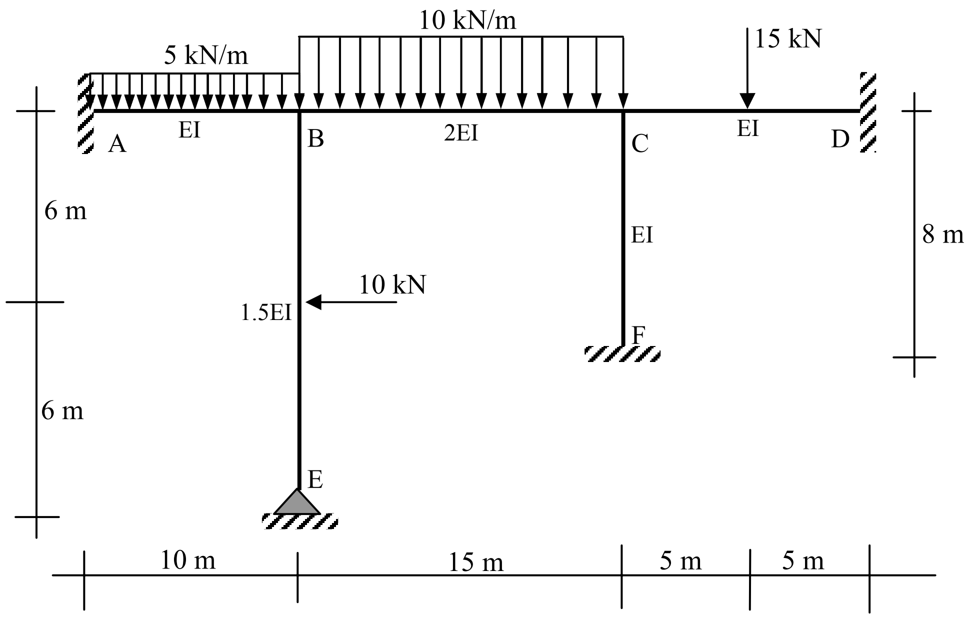
a.



b.



c.



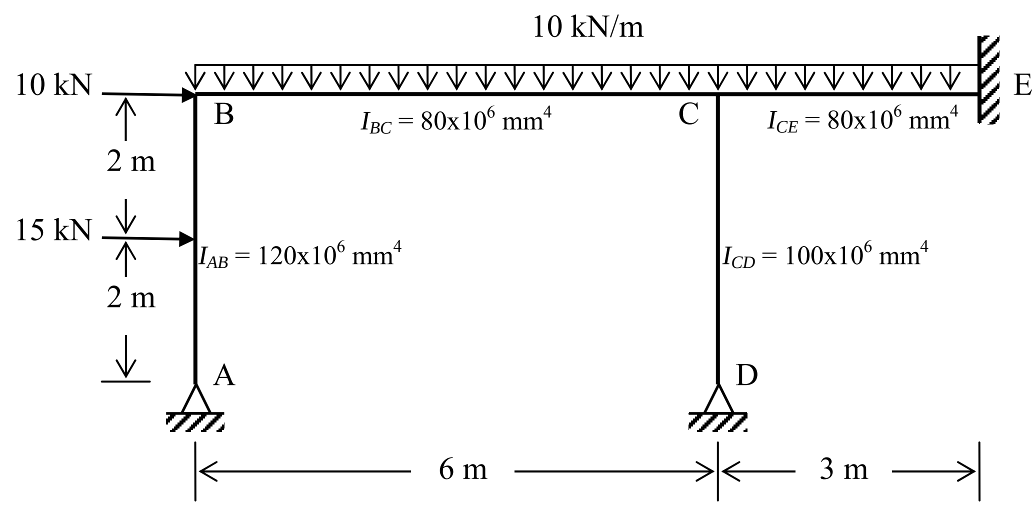
3. Figure below shows a frame which is subjected to uniformly distributed load of 10 kN/m along BCE and a concentrated horizontal load of 10 kN and 15 kN acting on point B and mid span of member AB respectively. The supports at A and D are pins and E is fixed. E value is constant and I for each member is stated in the figure.

(a) Calculate all end moments and reactions by using the moment distribution method.

(b) Draw the shear force and bending moment diagrams for the beam. Show all the important values.

(c) Explain the benefit to the structure if the pinned support at A and D is replaced by a fixed support.

(Final Sem 2-2007/2008)



4. (a) State **four (4)** cases that will cause sway on the rigid frame and sketch all of them.

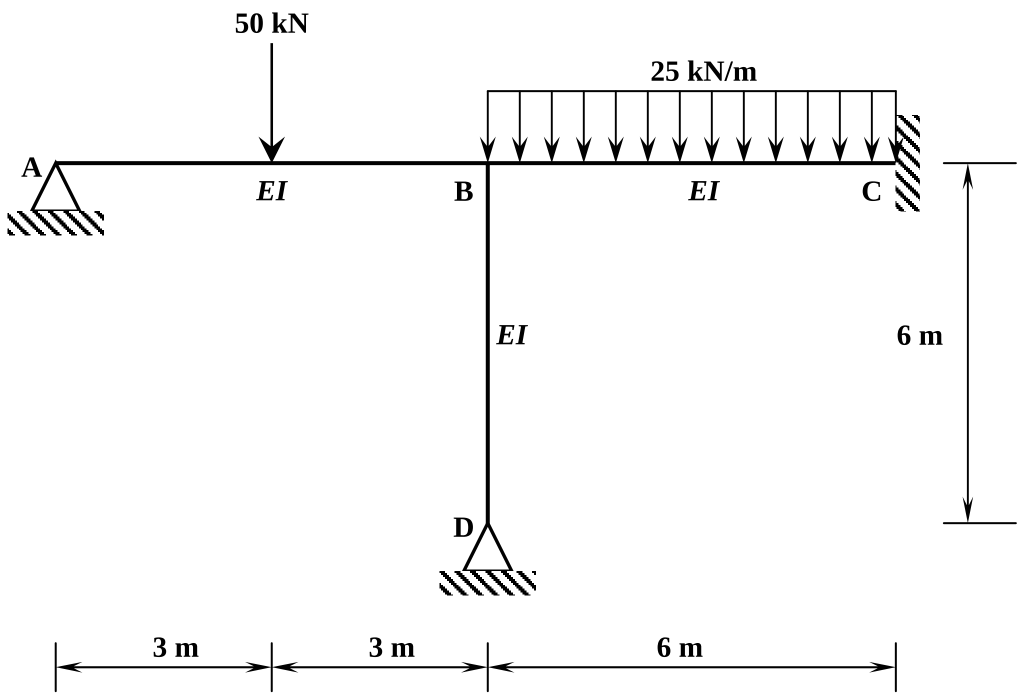
(b) Briefly explain the procedures of analysis of rigid sway frame to determine the end moment of each joint.

(c) Figure below shows the non-sway rigid frame which is pinned at A and D and fixed at C. The modulus of elasticity and second moment of area for each member is as shown.

(i) Calculate the end moment at all joints using modified stiffness of moment distribution method.

(ii) Calculate the reaction at support A, C and D.

(Final Sem 1-2008/2009)



5. Figure below shows the continuous beam with the fixed support and roller support at the end of beam. Two roller supports located at the middle of the beam. The beam subjected to uniform distributed load of 30kN/m between span A and B. Meanwhile, two point loads of 50kN and 70kN acted on the span BC and CD, respectively. By using the modified distribution method;

(a) Define Fixed End Moment (FEM).

(b) Determine the Distribution Factor (DF).

(c) Calculate all end moments and reactions.

(d) Currently there are quite a number of structural analysis software applications available in the market. But we still need the manual calculation to verify the results from the software. As a design engineer, you are assigned to complete designing the continuous beams by using manual calculation. What could you do about it?

(Final Sem 2-2008/2009)