Chapter 3

PLANE TRUSSES
STATICALLY INDETERMINATE

Bambang Prihartanto
Shahrul Niza Mokhatar
Noorli Ismail

3.1 INTRODUCTION

- Statically indeterminate structures are the ones where the independent reaction components and/or internal forces cannot be obtained by using the equations of equilibrium only (ΣF_X, ΣF_Y and ΣM). The concept of equilibrium with compatibility is applied to solve indeterminate systems. A structure is statically indeterminate when the number of unknowns exceeds the number of equilibrium equations in the analysis. Indeterminancy may arise as a result of added supports or members.

- It can be two types of statically indeterminate:

  1. External Indeterminate
     - ✓ It related with the reactions, it could be indeterminate if the number of reactions of the structures exceed than determinate structures by using static equation.
     - ✓ This surplus reaction known as **external redundant.**

  2. Internal Indeterminate
     - ✓ It related with the framework construction. Some of framework or trusses should have an adequate number of members for stability intentions. If inadequate members were detected, structure is classified as unstable, meanwhile, while the redundant number of members were determined, the structures is classified as statically indeterminate.
     - ✓ Therefore, the internal forces known as **internal redundant.**

- Advantages using redundant structures (statically indeterminate).

  1) Economical materials usage.
  2) Reducing deflections.
  3) Enhancing stiffness.
  4) Beautifying.

- How to define whether the truss is external statically indeterminate or internal statically indeterminate? Use this classification;

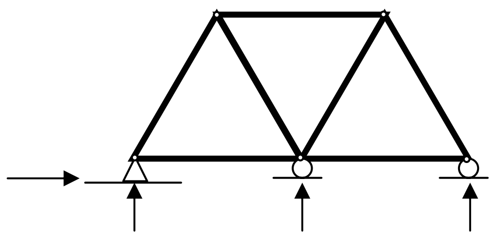
> 1) If $m = 2j - 3$ and $r > 3$,statically external indeterminate
> 2) If $m > 2j - 3$ and $r = 3$,statically internal indeterminate
> 3) If $m > 2j - 3$ and $r > 3$,statically external and internal indeterminate

If (i) is degree of determination of trusses, so: $i = (m + r) - 2j$

- **Virtual Work Method or Unit Load Method** are used for solving the analysis of plane trusses indeterminate whether it is **external redundant** or **internal redundant.**

EXAMPLE 3.1:

Prove the truss is statically external indeterminate.



Solution;

$m = 7$
$r = 4$ $(r > 3)$
$j = 5$

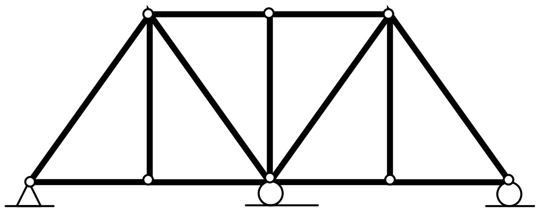
Using $m + r = 2j$ …the truss is statically indeterminate with **1th degree of indeterminacy.**

Using truss classification;
$m = 2j - 3$
$7 = 2(5) - 3$
$7 = 7$ ….**m = 2j – 3 and r > 3 ..proven external statically indeterminate.**

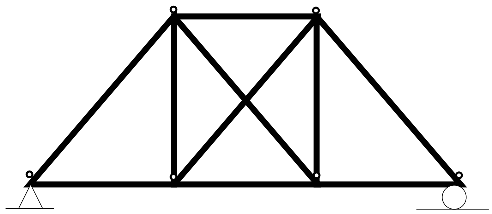
EXERCISE 3.1:

Define whether the truss is statically determinate or indeterminate. If the truss is statically indeterminate, state the degrees on indeterminancy. Then, define whether the truss is external or internal or both statically indeterminate.

a)



b)



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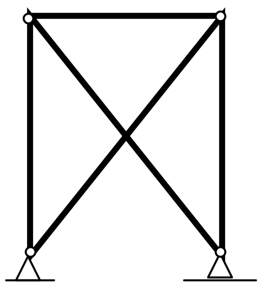
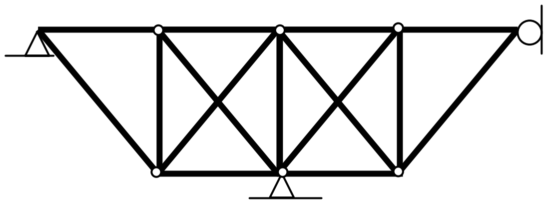
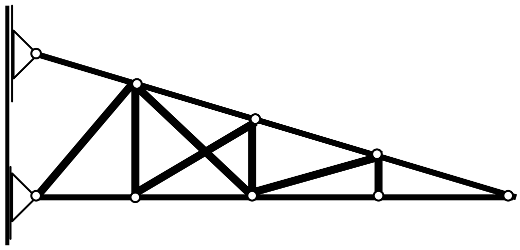


d)

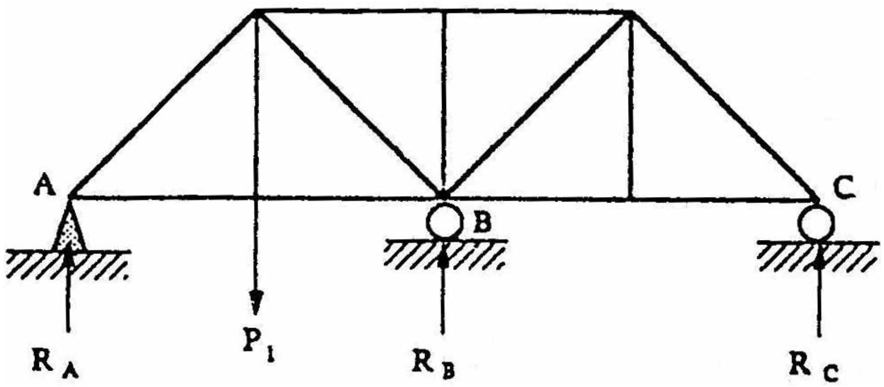


e)



3.2 ANALYSIS OF PLANE TRUSSES WITH EXTERNAL REDUNDANT

- Consider statically indeterminate truss as below:



- This truss can be changed to statically determinate trusses plane with chose support at 'B' or 'C' as the redundant; this example chooses 'purging' the support at 'B'. Consequently, displacement (δ_B) can be discovered at joint B. Furthermore, the external loading will be subjecting at joint B by using unit load method.

- Continually, external loading (P1) was removed and virtual unit load (1 kN) is subjected to the joint B as in which the direction of the deflection required. Unit vertical load then cause the joint B to be displaced (δ_{bb}).

- If the reactions forces at support 'B' is $R_{B,}$ whereby, it was subjected to the joint B for re-originate point B, therefore, the displacement should be made by $(R_B)(\delta_{bb})$, and can be derived an equation of displacement at point B while subjected by external load and virtual unit load:

$$\delta_B + R_B\delta_{bb} = 0$$

$$R_B = -\frac{\delta_B}{\delta_{bb}}$$

- If the external loading (F) subjected to the trusses, whereupon, δ_b can be derived as:

$$\delta_B = \sum \frac{F'\mu L}{AE}$$

With μ is an internal force in the trusses members while subjected virtual unit load (1 kN).

- To find the displacement (δ_{bb}) at point B while subjected virtual unit load, therefore, one unit load must be imposed at point B, however, all loading that forced to the member of the trusses is still identical, μ, δ_{bb} can be derived as:

$$\delta_{bb} = \sum \frac{\mu.\mu L}{AE} = \sum \frac{\mu^2 L}{AE}$$

- Furthermore, $R_B = -\frac{\delta_B}{\delta_{bb}}$ can be derived as:

$$R_B = -\frac{\sum F'\mu L / AE}{\sum \mu^2 L / AE}$$

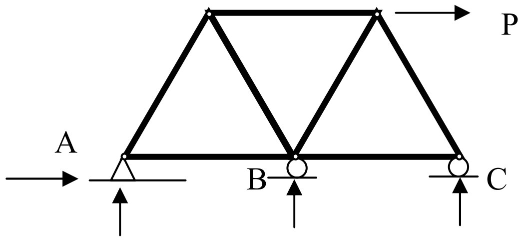
- Once reactions forces R_B was determined, all the reactions forces can be obtained by using static equilibrium equation.

- As the reactions forces at B, R_B was imposed to the trusses. Accordingly, the internal forces will be changing as R_B multiply to the load μ until actual internal forces F determined as an equation:

$$\mathbf{F = F' + R_B.\mu}$$

EXAMPLE 3.2:

After the determination of truss classification, the knowledge of reduced the system is very important to identify the forces that will be involved in the analysis. Reduced the system for the shown truss.



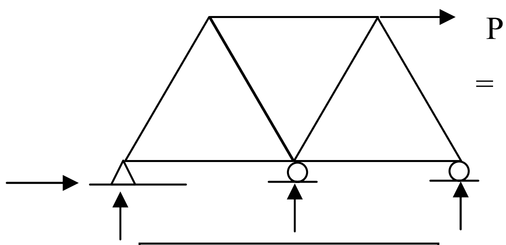
Solution;

Referring to **EXAMPLE 3.1**, the truss is external statically indeterminate to the 1st degree.

Hints to reduce the system;

- If the truss is external statically indeterminate, reduced the support reaction, r to 3.
- If the truss is internal statically indeterminate, reduced the members to the same number degree of indeterminancy.
  Eg; if 1th degree, reduced one member.
- If the truss is external and internal statically indeterminate, reduced the support reaction, r = 3 and member truss for the balance of degree.

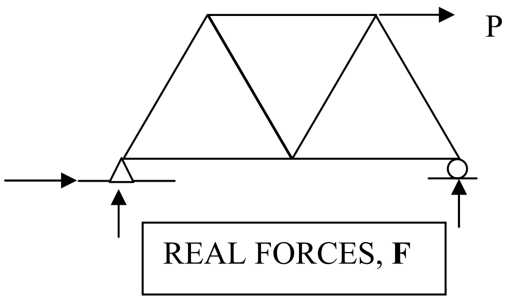
Reduced the system:
The truss is external statically indeterminate, reduced the support reaction, r to 3.



ACTUAL TRUSS

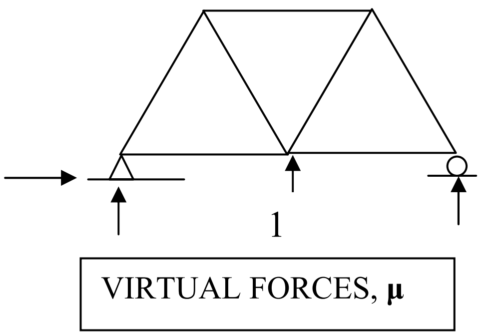
- Since the truss is external statically indeterminate, choose one of the support reaction to be the redundant.
- Remember it must satisfy 2 conditions;

1) It must be statically determinate.
2) It must be stable.



When the reaction at B is removed, there will be a vertical deflection at B called δ. This deflection is prevented by the real support at B. The type of truss is determinate. Find member forces.

+



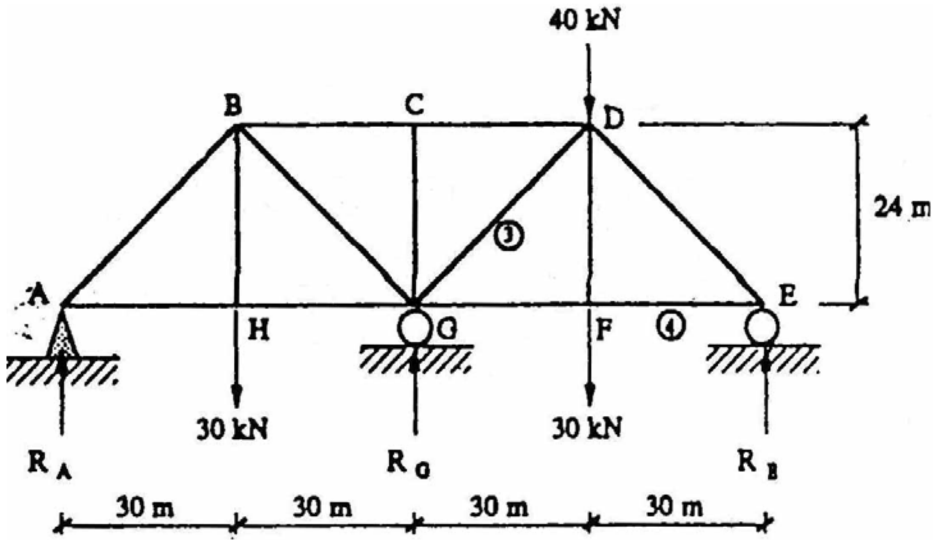
Then, 1 unit load is applied at B. Removed original load. Find new member forces.

There are two types of forces that involved in the analysis, namely are real forces, F and virtual forces,μ.

3.3 PROBLEM OF EXTERNAL REDUNDANT

EXAMPLE 3.3

Prove the truss is external statically indeterminate. Determine the reactions and internal forces of the trusses. The modulus of elasticity (E) of each member is constant.



Solution:

$n = m + r - 2j$

$= 13 + 4 - 2(8)$

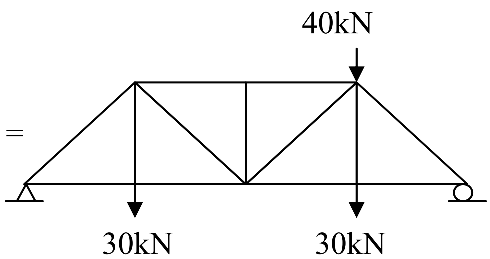
$= 1$ ……statically indeterminate to the **1st degree.**

$m = 2j - 3$

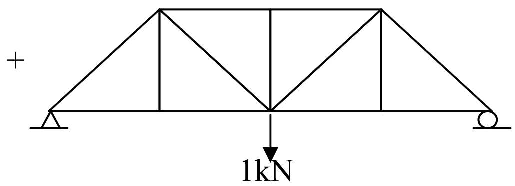
$13 = 2(8) - 3$

$13 = 13$ … $\mathbf{m = 2j - 3}$ **and** $\mathbf{r > 3}$ ….external statically indeterminate.

- ✓ The truss is external statically indeterminate, reduced the support reaction. In reduced the system, support at G or E can be selected as (redundant).

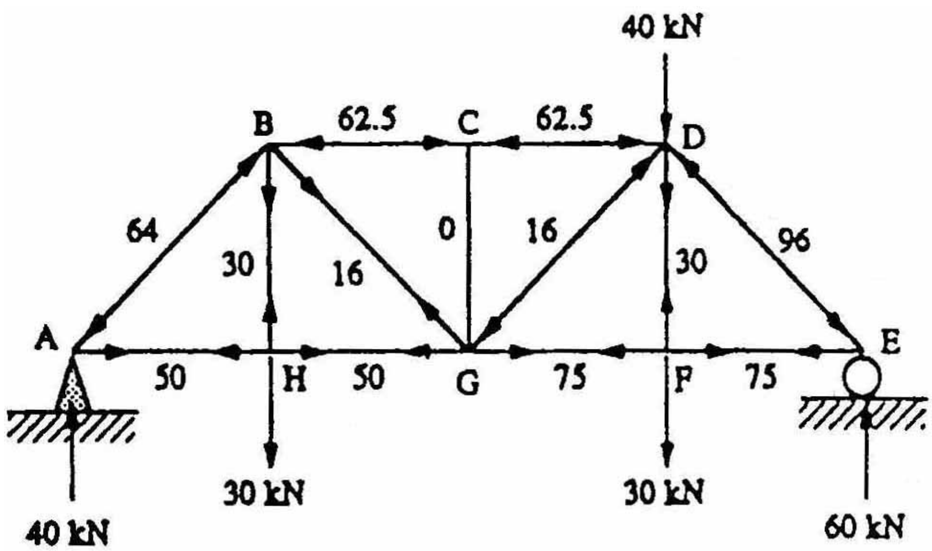


Real Forces, F

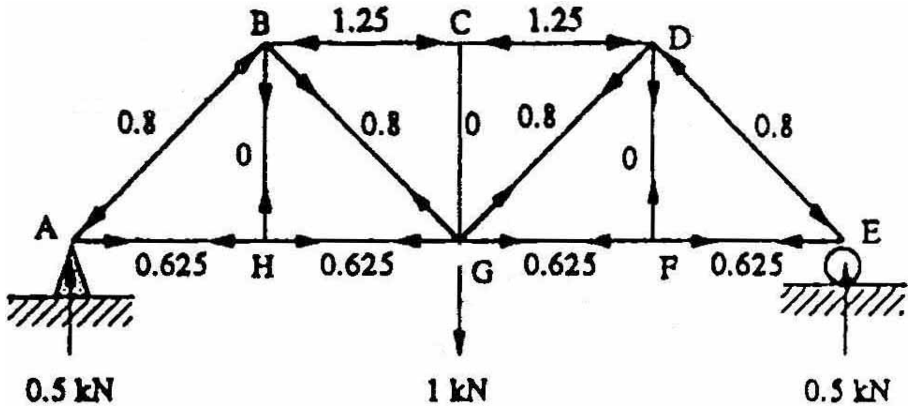


Virtual Forces, μ

- ✓ In this example, support G was removed and **Real Forces, F** will be determining as below:



- ✓ To determine **Virtual Forces,μ** ,all external load (original load) was removed and the virtual unit load (1kN) is applied at point G. Ultimately, every single one internal forces could be determined:



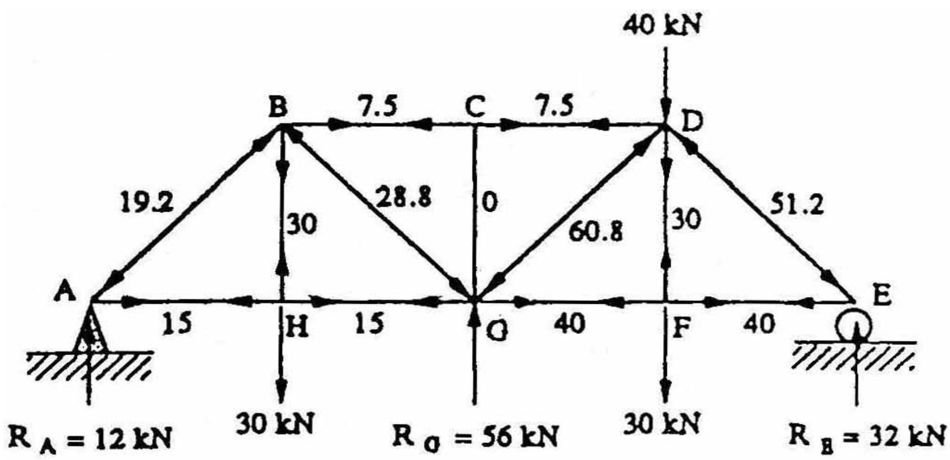
- ✓ This table expedient to simplify the calculation. The internal forces of the truss are listed in the last column by using the formula of **F= F' +R_G(μ)**.

Ang.	L (10^2 mm)	A (10^2 mm^2)	L/A	F' (kN)	u	FuL/A	u^2L/A	F = F' + $R_G u$
AH	300	4	75	50	0.625	2340	29.2	15
HG	300	4	75	50	0.625	2340	29.2	15
GF	300	4	75	75	0.625	3510	29.2	40
FE	300	4	75	75	0.625	3510	29.2	40
AB	384	4	96	- 64	- 0.8	4920	61.4	- 19.2
BC	300	4	75	- 62.5	- 1.25	5850	117.0	7.5
CD	300	4	75	- 62.5	- 1.25	5850	117.0	7.5
DE	384	4	96	- 96	- 0.8	7370	61.4	- 51.2
BH	240	3	80	30	0	0	0	30
BG	384	3	128	16	0.8	1640	82	- 28.8
CG	240	3	80	0	0	0	0	0
GD	384	3	128	- 16	0.8	- 1640	82	- 60.8
DF	240	3	80	30	0	0	0	30
						35690	637.6	

Reactions at G:

$$R_G = -\frac{\sum F'\mu L/AE}{\sum \mu^2 L/AE} = -\frac{35690}{637.6} = -56\,\text{kN}\,(\uparrow)$$

After that, the reaction of the truss can be calculated by using the equilibrium equation.



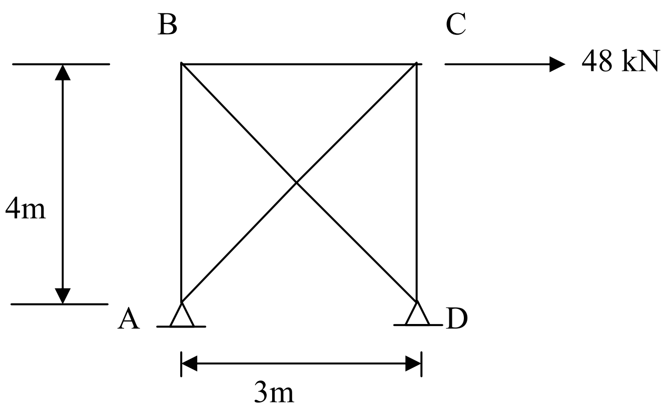
EXAMPLE 3.4

The figure shows the indeterminate truss is pinned supported at A and D. Modulus elasticity, E and cross sectional area, A is given.

a) Determine the classification of the truss.

b) Determine the reaction and member forces for all members using the Method of Virtual Work if the horizontal reaction at A is selected as the redundant.

MEMBER	E(kN/mm^2)	A (mm^2)
AB	200	625
BC	200	500
CD	200	625
AC	30	400
BD	30	400



Solution:

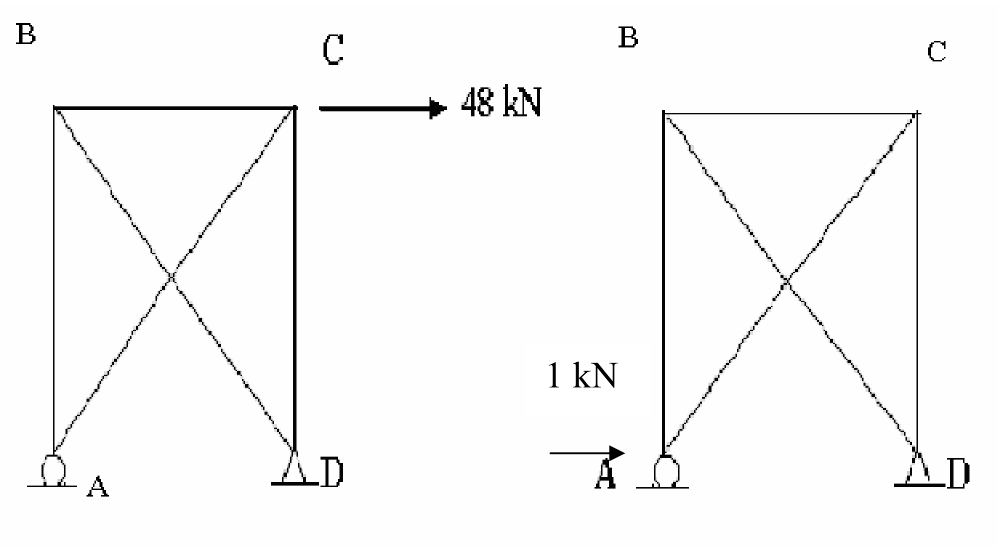
a) Truss classification:

$n = m + r - 2j$
$= 5 + 4 - 2(4)$
$= 1$ ……statically indeterminate to the **1st degree**.

$m = 2j - 3$
$13 = 2(8) - 3$
13 =13 … **m = 2j – 3 and r > 3** ….external statically indeterminate.

b) Reactions and internal forces for each member:

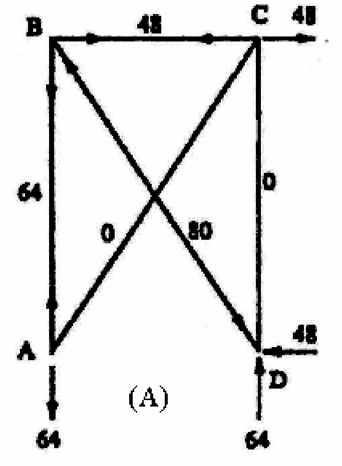
- ✓ The truss is external statically indeterminate, reduced the horizontal support reaction at A as the redundant. Reduced the system.



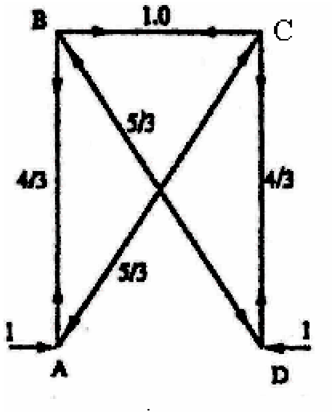
Real Forces, F

Virtual Forces, μ

- ✓ **Real Forces, F** will be determining as below:



✓ To determine **Virtual Forces,μ** , the original load of the truss is eliminates and the virtual load unit (1kN) is applied at A horizontally.

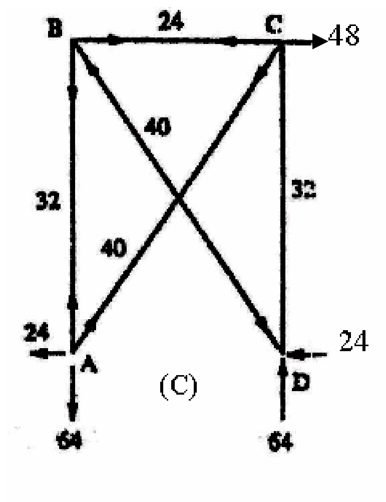


✓ This table expedient to simplify the calculation at A and actual internal forces in each member is listed in last column.

Ang.	L (10^3 mm)	A (mm^2)	E (kN/mm^2)	F (kN)	u	FuL/AE	u^2L/AE	F = F + H_Au
AB	4	625	200	64	4/3	2.731	0.057	32
BC	3	500	200	48	1.0	1.44	0.030	24
CD	4	625	200	0	4/3	0	0.057	- 32
AC	5	400	30	0	- 5/3	0	1.157	40
BD	5	400	30	- 80	- 5/3	55.55	1.157	- 40
						59.721	2.458	

$$H_A = -\frac{\sum F'\mu L/AE}{\sum \mu^2 L/AE} = -\frac{59.721}{2.458} = -24\text{ kN } (\leftarrow)$$

✓ The reaction of the truss can be calculated by using the equilibrium equation.

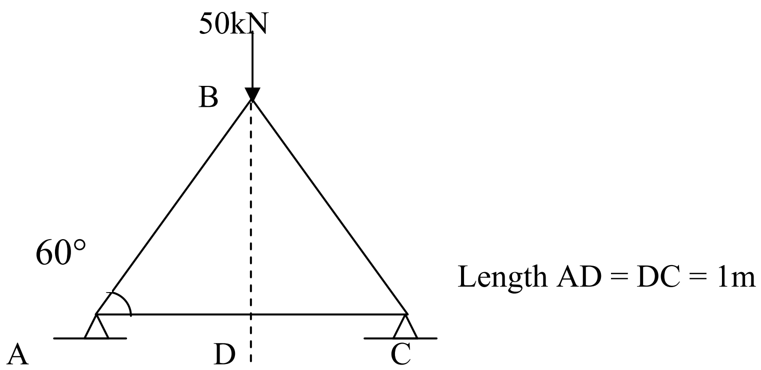


EXERCISE 3.2:

The indeterminate truss is subjected to the concentrated load of 50kN is acting at joint B. Given EA is constant.

a) Prove the truss is external statically indeterminate.

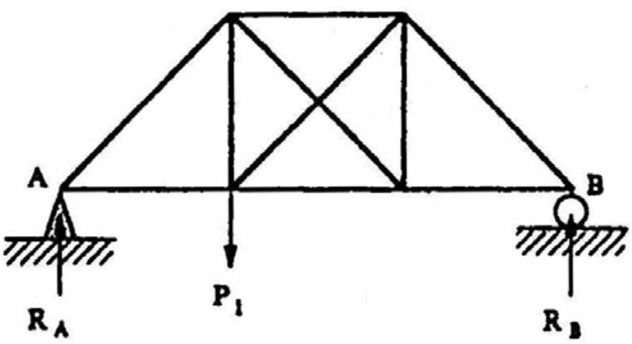
b) Determine all the member forces if horizontal reaction at C is taken as the redundant



[**Ans**: $H_C = 14.43$kN]

3.4 ANALYSIS OF PLANE TRUSSES WITH INTERNAL REDUNDANT

- The truss has 1 redundant member and classified as statically internal indeterminate to the first degree.



= m + r – 2j
= 10 + 3 – 2(6)
= 1 ……statically indeterminate to the **1st degree**.

m = 2j – 3
10 = 2(6) – 3
10 = 9 … **m > 2j – 3 and r = 3** ….internal statically indeterminate.

- The trusses those have ‘internal redundant members’ can be determined by using the same concept as ‘external redundant members’ trusses.

- The internal forces can be determined by impose the external load (point load, etc). Furthermore, this load can cause both joints that connecting to the eliminated member undergoes deformation\deflection. $\sum \frac{F'\mu L}{AE}$.

- Then, the external load is removed and 1 virtual unit load that causes tension forces is applied on the eliminated member. However, the internal member μ caused by 1 virtual unit load can be determined.

- In addition, the internal member μ caused by 1 virtual unit load, can causes the both points that connecting to the redundant members to be displaced. $\sum \frac{\mu^2 L}{AE}$.

- Consider the redundant members as X; two points will be dragged by the displacement as $X.\sum \frac{\mu^2 L}{AE}$

- As the results, an equation of displacement can be derived as below:

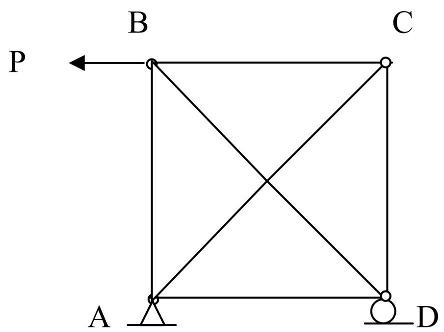
$$\sum \frac{F'\mu L}{AE} + X.\sum \frac{\mu^2 L}{AE} = 0 \quad \text{---------} > \quad X = -\frac{\sum F'\mu L / AE}{\sum \mu^2 L / AE}$$

After X’s forces recognized, the internal forces, F, can be finally determine:

F = F’ + X.μ

<u>EXAMPLE 3.5</u>

Reduced the system for the shown truss.

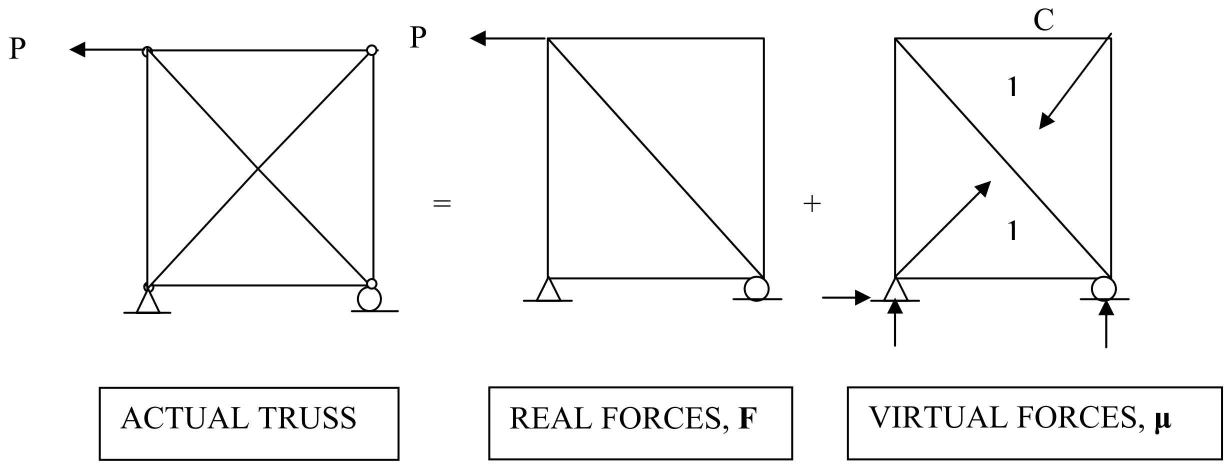


Solution;

$n = m + r - 2j$
$= 6 + 3 - 2(4)$
$= 1$ ……statically indeterminate to the **1st degree**.

$m = 2j - 3$
$6 = 2(3) - 3$
$6 = 3$, ….**m> 2j – 3 and r =3**….internal statically indeterminate.

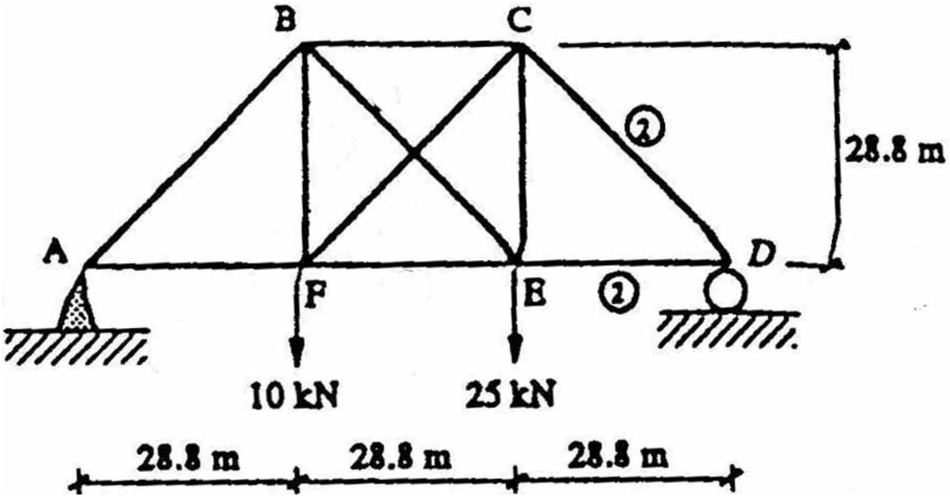
- If the truss is internal statically indeterminate, reduced the members to the same number degree of indeterminacy. In this case, reduce 1 member.



3.5 PROBLEM OF INTERNAL REDUNDANT

EXAMPLE 3.6

Classify the trusses below and determine the internal forces. The modulus of elasticity (E) of each member is constant.



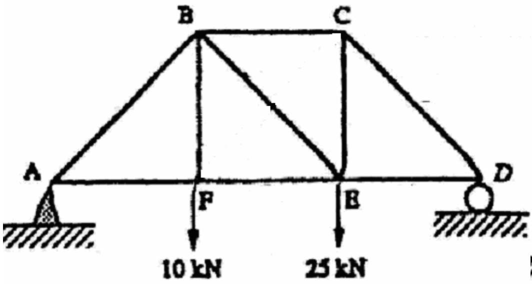
Solution:

- Truss classification;

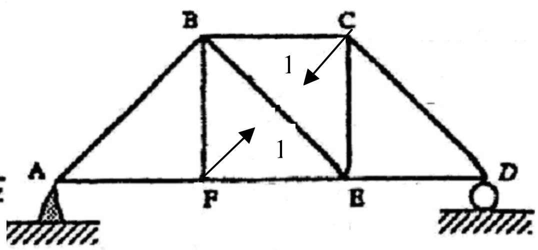
  $n = m + r - 2j$
  $= 10 + 3 - 2(6)$
  $= 1$ ……statically indeterminate to the **1st degree**.

  $m = 2j - 3$
  $10 = 2(6) - 3$
  $10 = 9$, ….**m> 2j – 3 and r =3**….internal statically indeterminate.

- The truss is internal statically indeterminate, reduced 1 member (BE or CF) as the redundant. Reduced the system.

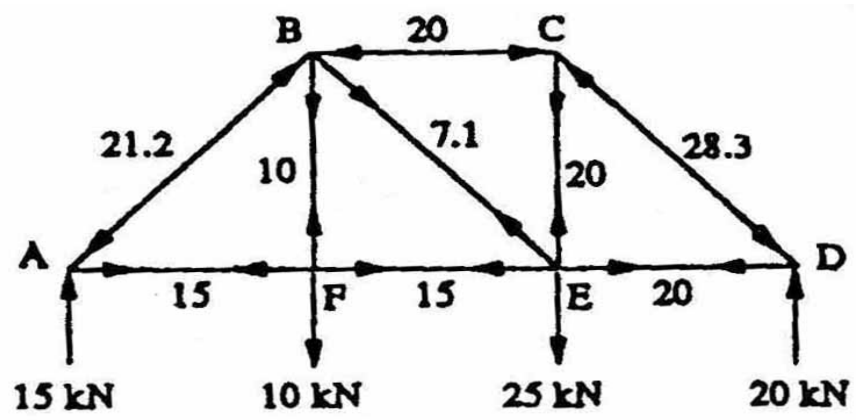


REAL FORCES, **F**

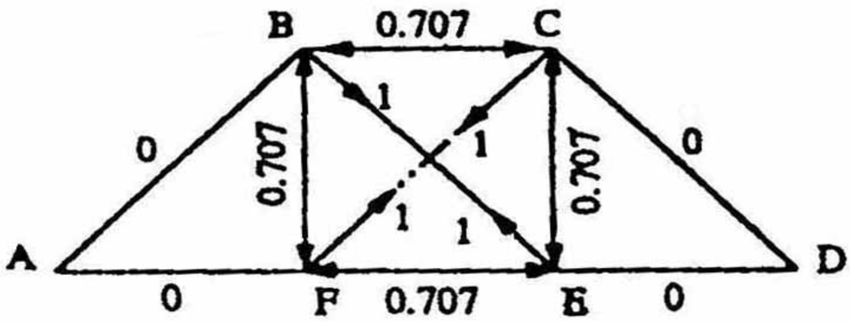


VIRTUAL FORCES, **μ**

- Find **Real Forces, F**, eliminate the FC member as a redundant member. The internal forces, F caused by external load can be determined.



- Determine **Virtual Forces, μ**, the internal forces caused by vertical virtual unit load (1kN) can be determined.



- Calculation of X value as the redundant member (FC) and the real internal forces of each member.

Ang.	L (10^2 mm)	A (10^2 mm^2)	L/A	F'(kN)	u	FuL/A	u^2L/A	F = F' + Xu
AF	288	2	144	15	0	0	0	15
FE	288	2	144	15	- 0.707	- 1530	72	13.47
ED	288	2	144	20	0	0	0	20
AB	408	2	204	- 21.2	0	0	0	- 21.2
BC	288	2	144	- 20	- 0.707	2040	72	- 21.53
CD	408	2	204	- 28.3	0	0	0	- 28.3
BF	288	1	288	10	- 0.707	- 2040	144	8.47
BE	408	1	408	7.1	1	2900	408	9.26
FC	408	1	408	0	1	0	408	2.16
CE	288	1	288	20	- 0.707	- 4070	144	18.47
						- 2700	1248	

X value, internal forces FC =

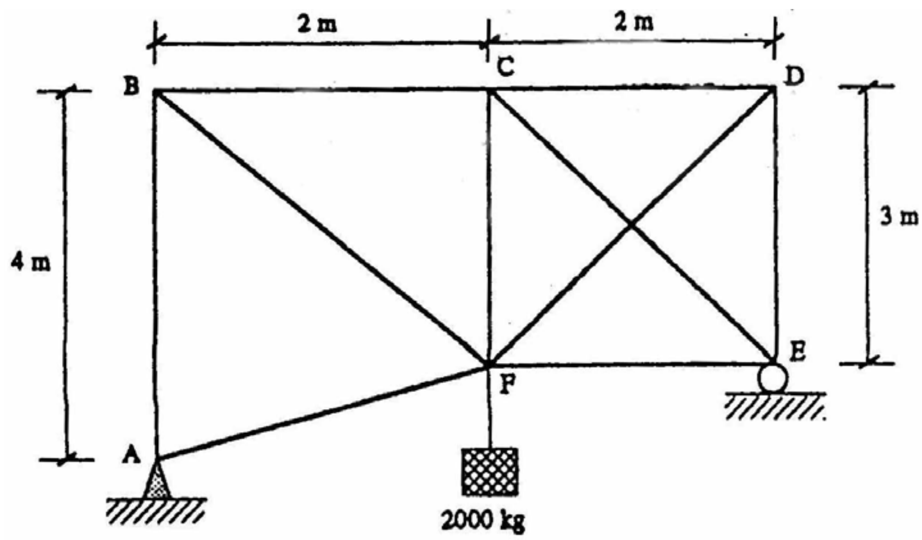
$$X = -\frac{\sum F'\mu L/AE}{\sum \mu^2 L/AE} = -\frac{-2700}{1248} = 2.16\,\text{kN (tension)}$$

EXAMPLE 3.7

A truss carry of 2000 kg load as shown below. The modulus of elasticity (E) of each member is 200 kN/mm^2.

1. Verify the truss is statically internal indeterminate.
2. Determine the internal forces of each member with assuming CE member as redundant.

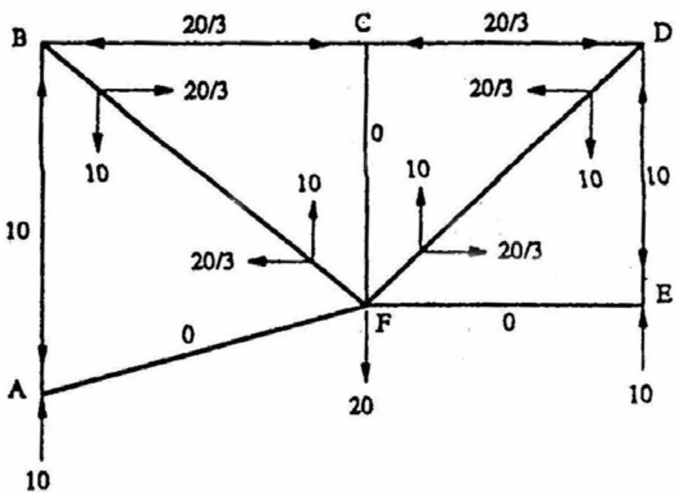
Member	AB	BC	CD	DE	EF	FA	BF	CF	CE	DF
Area (cm^2)	10	12	12	10	12	8	8	10	8	8



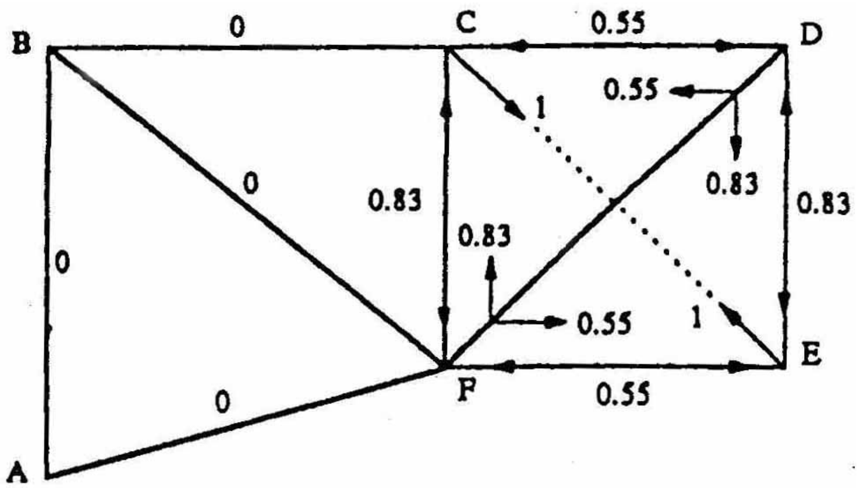
Solution:

1.

2. Assume that the CE member is redundant member and it was eliminated. Therefore, internal forces consequences from external loading, F', as finally could be determined.



- Then, the internal forces consequences from virtual unit load (1 kN) can be determined.



- The calculation of X value as an internal force of redundant member (CE) and all internal members.

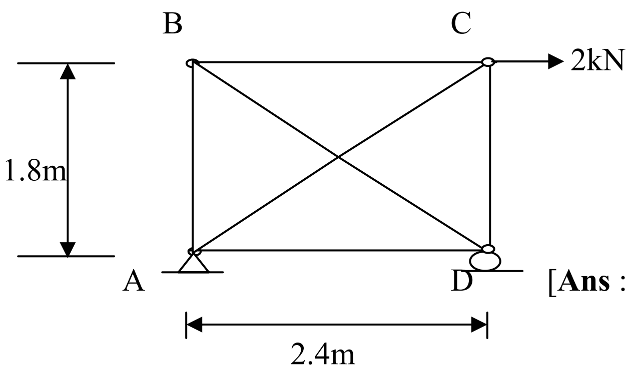
Ang.	L (10^3 mm)	A (mm^2)	F (kN)	u	FuL/A	u^2L/A	F = F' + Xu
AB	4.0	1000	- 10.0	0	0	0	-10
BC	2.0	1200	- 6.67	0	0	0	- 6.67
CD	2.0	1200	- 6.67	- 0.55	6.11	0.5	- 3.36
DE	3.0	1000	- 10.0	- 0.83	24.9	2.07	- 5.0
EF	2.0	1200	0	- 0.55	0	0.5	3.31
FA	2.24	800	0	0	0	0	0
BF	3.61	800	12.0	0	0	0	12.0
CF	3.0	1000	0	- 0.83	0	2.07	5.0
CE	3.61	800	0	1.0	0	4.51	- 6.02
DF	3.61	800	12.0	1.0	54.2	4.51	5.98
					85.21	14.16	

X value, internal force CE :

$$X = -\frac{\sum F'\mu L / AE}{\sum \mu^2 L / AE} = -\frac{85.21}{14.16} = -6.02\text{kN (compression)}$$

EXERCISE 3.3:

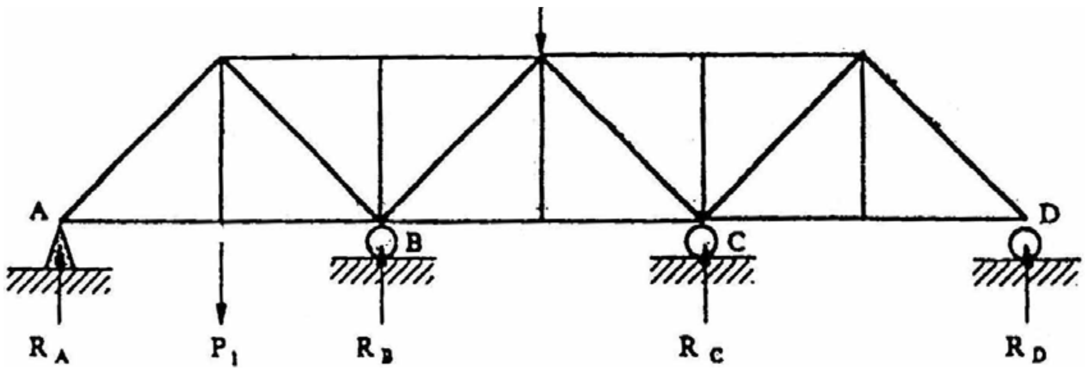
The steel truss subjected to horizontal force,2kN at C. Find the member forces of AC using the Method of Virtual Work.



[**Ans** : F_{AC} = 1.62kN (T)]

3.6 ANALYSIS OF PLANE TRUSSES MORE THAN 1 REDUNDANT

- For the trusses that has numerous numbers of reactions of support as diagram below:



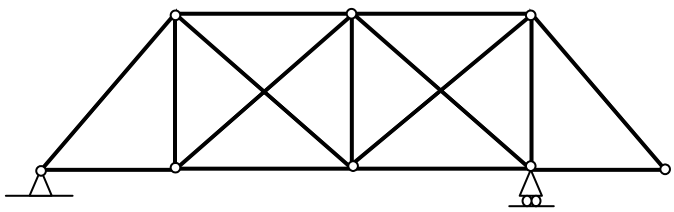
An equation for the displacement occurred can be derived as below:

$$\delta_B + R_B\delta_{bb} + R_C\delta_{bc} = 0$$
$$\delta_C + R_B\delta_{cb} + R_C\delta_{cc} = 0$$
whereby:
$$\Sigma\frac{F'\mu_B L}{AE} + R_B\Sigma\frac{\mu_B^2 L}{AE} + R_C\Sigma\frac{\mu_B\mu_C L}{AE} = 0$$
$$\Sigma\frac{F'\mu_C L}{AE} + R_B\Sigma\frac{\mu_C\mu_B L}{AE} + R_C\Sigma\frac{\mu_C^2 L}{AE} = 0$$

Reactions forces at B and C could be solved by using equilibrium equation.

EXAMPLE 3.8:

Reduced the system for the shown truss.



Solution;

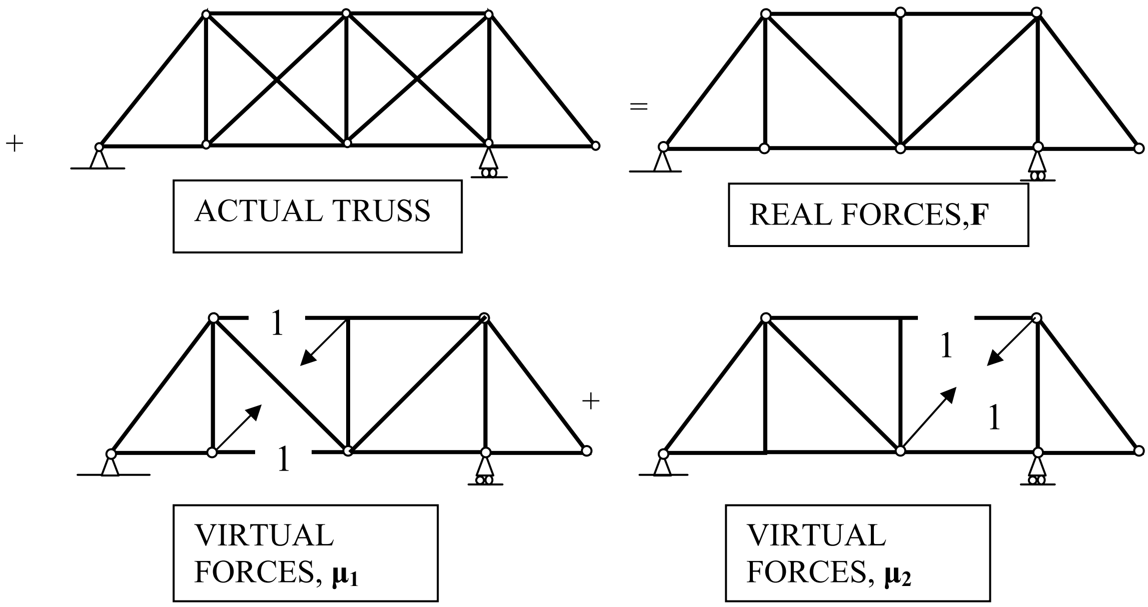
Determine whether the truss is statically or determinate or indeterminate.

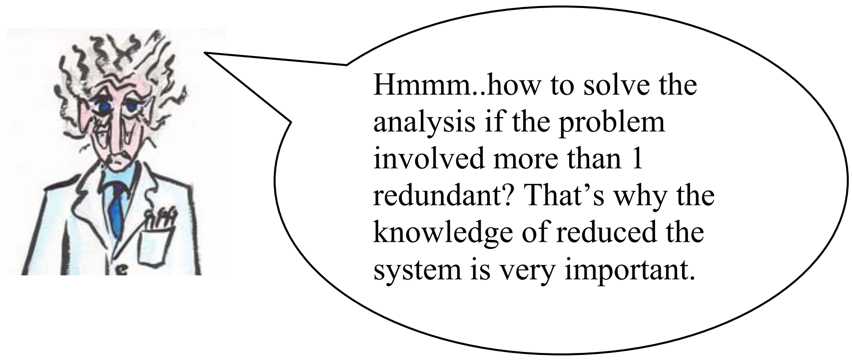
m = 15
r = 3….(r = 3)
j = 8

n = m + r – 2j
= 2 …statically indeterminate with **2nd degree** of indeterminancy.

m = 2j – 3
15 = 2(8) – 3
15 = 13 ….. m > 2j – 3 and r = 3…statically internal indeterminate

Reduced the system
The truss is internal statically indeterminate, reduced 2 members.





Recall: $\{D\} + \{F\}\{\delta\} = 0$ ……….if there are only one redundant.

But there are two redundants;

$$\begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} + \begin{Bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{Bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} \begin{matrix} = 0 \\ = 0 \end{matrix}$$

$$\begin{Bmatrix} \Sigma \dfrac{F\mu_1 L}{AE} \\ \Sigma \dfrac{F\mu_1 L}{AE} \end{Bmatrix} + \begin{Bmatrix} \Sigma \dfrac{\mu_1\mu_1 L}{AE} & \Sigma \dfrac{\mu_1\mu_2 L}{AE} \\ \Sigma \dfrac{\mu_2\mu_1 L}{AE} & \Sigma \dfrac{\mu_2\mu_2 L}{AE} \end{Bmatrix} \begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} \begin{matrix} = 0 \\ = 0 \end{matrix}$$

$$\begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = - \begin{Bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{Bmatrix}^{-1} \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix}$$

Solve by using the matrix.

HINT:

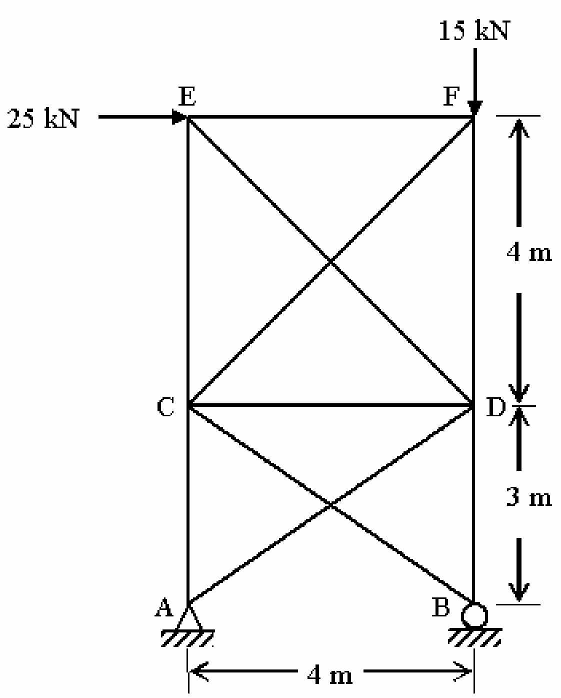
$$\begin{Bmatrix} \delta_1 \\ \delta_2 \end{Bmatrix} = \{A\}^{-1} \{B\}$$

$$\{A\}^{-1} = \frac{1}{\det A} \{\text{adj } A\} \{B\}$$

TUTORIAL 3

1. Figure shows a truss with pin support at point A and roller at point B. Point F and E are subjected to a 15 kN and 25 kN respectively. Given the cross section area of member AD, BC, CF and DE are 1000 mm^2 meanwhile cross section area for other members are 1500 mm^2. Young's Modulus, E for all members taken as 200 kN/mm^2.

   (a) Check the stability and determinacy of the truss.
   (b) By assuming member CF is redundant, determine it's internal force.

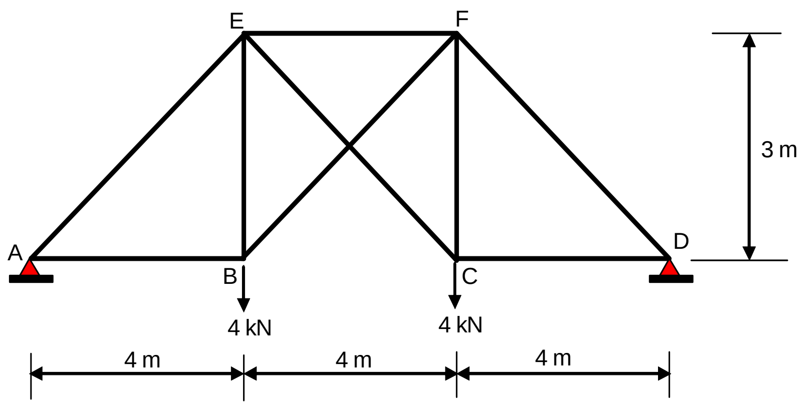


(UTHM SEM II 2007/2008 BFC 3023)

2.

(a) What is the difference between statically indeterminate trusses with external redundant and indeterminate trusses with internal redundant and give one of example for each.

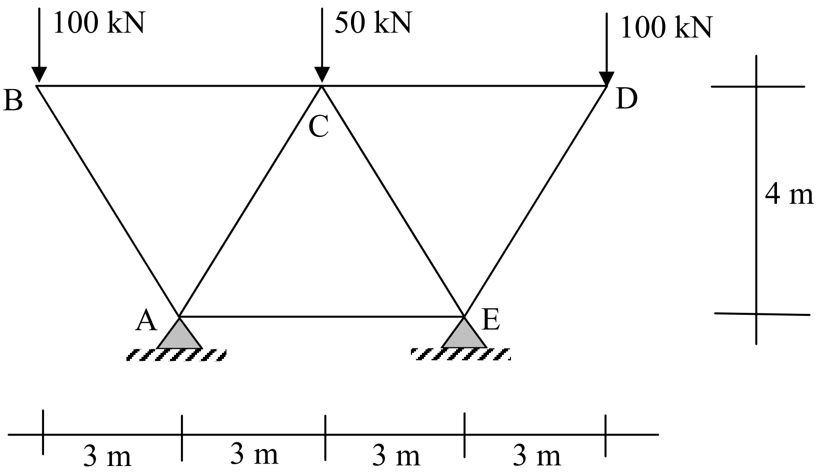
(b) Figure **Q2** shows the trusses supported with pin connection at A and D. Give the value of modulus of elasticity and cross-sectional area of each member are 200 kN/mm^2 and 400 mm^2. Determine the classification of that trusses and internal forces of each member by considering the horizontal reaction at A supports as the redundant.



(UTHM SEM I 2007/2008 BFC 3023)

3.

A truss shown is subjected with pinned support at A and E. The modulus of elasticity of each member and the cross-sectional area of the member are constant. If the support at E assume as an external redundant, determine the all support reaction and internal forces at each member of truss.



4.

A truss with loading applied shown in figure below. The modulus of elasticity of each member is E = 200 kN/mm^2 and the cross-sectional area of the member given:

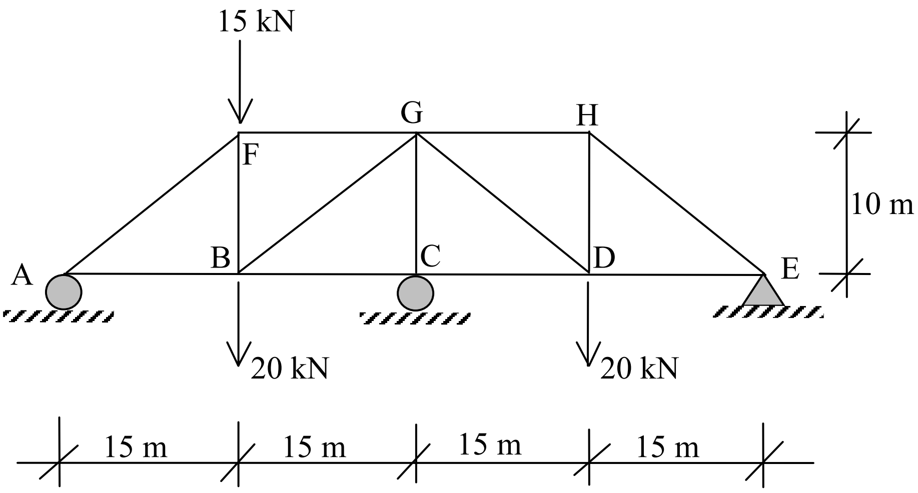
Member	Area (mm^2)	Member	Area (mm^2)
BC	400	DG	250
CD	400	EH	250
DE	400	BF	350
FG	300	CG	350
GH	300	DH	350
AF	250		

AB = BC = CD = DE and DG = BG

If the support at C assume as a external redundant, determine:

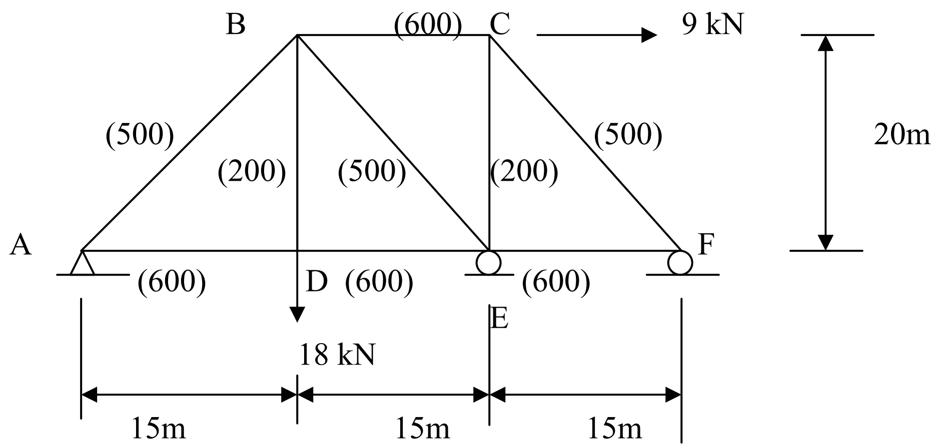
(a) Reaction at the support.

(b) Internal forces in each member of truss.



5.

The truss shown in figure below is pinned supported at A and rolled at E and F. The cross sectional areas for all members is given(in unit mm^2) and E=3 x 10^2 kN/mm^2. Determine all the members force and reaction at all supports using Method of Virtual Work if vertical reaction at E is taken as the redundant.



[**Ans**: V_E = -12.46kN, F_{AB} = -4.81kN(C), F_{AD}= F_{DE}=11.88 kN (T), F_{BE} = 17.69kN (C), F_{CF} = -2.11kN(C), F_{BD}= 18kN (T)]